Predictive Model for Radiatively Induced Neutrino Masses and Mixings with Dark Matter

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(Received 5 March 2013; published 22 May 2013)

A minimal extension of the standard model to naturally generate small neutrino masses and provide a dark matter candidate is proposed. The dark matter particle is part of a new scalar doublet field that plays a crucial role in radiatively generating neutrino masses. The symmetry that stabilizes the dark matter also suppresses neutrino masses to appear first at three-loop level. Without the need of right-handed neutrinos or other very heavy new fields, this offers an attractive explanation of the hierarchy between the electroweak and neutrino mass scales. The model has distinct verifiable predictions for the neutrino masses, flavor mixing angles, colliders, and dark matter signals.

DOI: 10.1103/PhysRevLett.110.211802 PACS numbers: 14.60.Pq, 12.60.Fr, 95.35.+d

The existence of a large amount of nonbaryonic dark matter in the Universe and the observation of nonzero neutrino masses may be regarded as the most direct and compelling evidence of particle physics beyond the standard model (SM). However, both the origin of neutrino masses and the nature of dark matter are still unknown. A scenario of being able to incorporate both phenomena in a unified framework would then be very attractive.

One of the best motivated dark matter scenarios is that of stable weakly interacting massive particles (WIMPs), produced as a thermal relic from the early Universe. Among the simplest realizations of this WIMP scenario is the inert doublet model [1,2]. The SM is extended by a scalar doublet Φ_2, and the dark matter scalar is made stable due to an exact Z_2 symmetry under which the new field has odd parity. The inert doublet model is currently constrained by results from dark matter searches as well as by particle collider data, but still a large region of its parameter space is allowed.

From the perspective of neutrino physics, currently the most popular way to generate small neutrino masses is the seesaw mechanism (see Ref. [3] for a review). In its simplest variant, it postulates the existence of very massive $SU(2)_L \times U(1)_Y$ singlet right-handed neutrinos. Other realizations involve the existence of very heavy scalar or fermionic triplets. Although elegant, this mechanism is difficult (if not impossible) to test, as the masses of the new states are typically much larger than can be experimentally probed.

Small neutrino masses can also be generated via radiative corrections. This has been explored by explicit lepton number violation in extensions of the scalar sector of the SM. As opposed to the seesaw mechanism, this approach generates small neutrino masses without relying on new particles at a very high energy scale. One simple realization of this idea is the Zee model [4], in which the SM field content is enlarged by a second scalar doublet Φ_2 and a charged scalar singlet S^+. Another simple scenario is the Zee-Babu model [5], which replaces the scalar doublet Φ_2 in the Zee model by a doubly charged singlet scalar field ρ^{++}.

However, these scenarios of radiative neutrino mass generation in Refs. [4,5] (together with many others, such as Refs. [6,7]) cannot at the same time contain a viable dark matter candidate. A stabilizing symmetry for some of the new fields would in fact forbid the very terms responsible for the generation of neutrino masses (see, however, Refs. [1,8] for interesting scenarios including right-handed neutrinos and dark matter particle candidates). This is for example the case in the Zee model if an odd Z_2 parity is assigned to Φ_2. Finding a unified scenario for radiative neutrino mass generation and a dark matter particle candidate is then a nontrivial task.

In this Letter we construct a minimal model, that generates neutrino masses radiatively and provides a stable dark matter candidate, via an extended scalar sector with an exact Z_2 symmetry. We do this by adding to the SM two scalar singlets, ρ^{++} and S^+, and a scalar doublet Φ_2 with masses around the electroweak (EW) scale. The fields S^+ and Φ_2 have odd Z_2 parity (while all other fields do not transform under this symmetry), and therefore a variation of the mentioned inert doublet model of dark matter is automatically embedded into the scenario. Due to the Z_2 symmetry and the field content of the model, Majorana neutrino masses are first generated at the three-loop level, naturally explaining the large hierarchy $m_\nu/v \sim 10^{-13}$ as due to the loop suppression $(g^2/16\pi^2)^3 \sim 10^{-13}$ (g being an EW-sized coupling and with all masses at the EW scale v) [9]. This scenario then provides an intrinsic and interesting link between the stability of the dark matter candidate and the smallness of the neutrino mass scale.

A model for neutrino masses.—In addition to the SM fields, the model includes two $SU(2)_L$ singlet scalars (singly and doubly charged) S^+ and ρ^{++}, and a scalar doublet Φ_2. We introduce a Z_2 symmetry under which the Φ_2 and

S^+ fields are odd, whereas ρ^{++} and the SM fields are even. The Z_2 symmetry should be unbroken after EW symmetry breaking, so that the lightest Z_2-odd state remains stable and can provide a dark matter particle candidate. Given the symmetry and particle content of the model, the Lagrangian will include the following relevant terms leading to lepton number violation:

$$-\Delta\mathcal{L} = \frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)^2 + \kappa_1\Phi_2^T i\sigma_2\Phi_1 S^- + \kappa_2\rho^{++}S^-S^- + \xi\Phi_2^T i\sigma_2\Phi_1 S^+\rho^{--} + C_{ab}\overline{\ell_{aR}^c}\ell_{bR}\rho^{++} + \text{H.c.} \tag{1}$$

Here, a, b denote family indices of the right-handed charged leptons ℓ_R, and the Yukawa couplings C_{ab} form a symmetric and complex matrix, allowing for charge-parity (CP) violation in the leptonic sector.

The SM scalar doublet Φ_1 and the inert scalar doublet Φ_2 can in the unitary gauge be written as

$$\Phi_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ h\end{pmatrix} + \begin{pmatrix}0\\ v\end{pmatrix}, \qquad \Phi_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}\Lambda^+\\ H_0 + iA_0\end{pmatrix}, \tag{2}$$

where $v \simeq 174$ GeV is the vacuum expectation value of Φ_1. After EW symmetry breaking, and for $\kappa_1 \neq 0$, the charged states Λ^+ and S^+ will mix (the mixing angle being β), giving rise to two charged mass eigenstates

$$H_1^+ = s_\beta S^+ + c_\beta\Lambda^+, \qquad H_2^+ = c_\beta S^+ - s_\beta\Lambda^+, \tag{3}$$

with s_β, $c_\beta = \sin\beta, \cos\beta$, respectively. A convenient set of independent variables may be given by the five new scalar masses $m_{\rho,H^0,A^0,H_{1,2}^\pm}$, the mixing angle β, and the couplings ξ and κ_2. All coefficients in the scalar potential should be chosen within their perturbative regime and to make the potential preserve vacuum stability [10].

The Lagrangian in Eq. (1) breaks lepton number explicitly by two units [11], which generates a Majorana mass for the left-handed neutrinos. With the viable matter content neutrino masses can never be generated at one-loop order and the Z_2 symmetry precisely forbids all terms that would have generated neutrino masses at two-loop order. Therefore the leading contributions to neutrino masses appear first at three loops—through the "cocktail diagram" shown in Figure 1.

In the basis where charged current interactions are flavor diagonal and the charged leptons e, μ, τ are mass eigenstates, the summed contributions of the six different finite three-loop diagrams shown in Figure 1 (coming from $H_{1,2}^+$, A_0 and H_0 running in the loop) give the Majorana neutrino mass matrix,

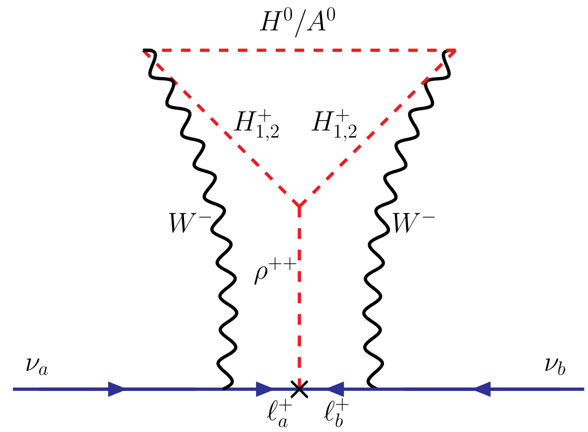

FIG. 1 (color online). The "cocktail diagram."

$$m_{ab}^\nu \simeq C_{ab}x_a x_b s_{2\beta}^2\frac{\mathcal{I}}{(16\pi^2)^3}\mathcal{A}, \tag{4}$$

where $s_{2\beta} = \sin(2\beta)$, $x_a = m_a/v$ for $a = e, \mu, \tau$, and

$$\mathcal{A} = \frac{(\Delta m_+^2)^2\Delta m_0^2}{\mu_0\mu_+}\frac{(\kappa_2 + \xi v)}{m_\rho^2 v^2}. \tag{5}$$

The factor $\mathcal{I}$ is a dimensionless $\mathcal{O}(1)$ number emerging from the three-loop integral after all generic factors have been factorized out. Its exact value depends on the specific mass spectrum, and we have estimated its value using the numerical code SECDEC [12]. The reduced masses are $\mu_0^{-1} = m_{H_0}^{-1} + m_{A_0}^{-1}$ and $\mu_+^{-1} = m_{H_1^+}^{-1} + m_{H_2^+}^{-1}$.

The dependence of m_{ab}^ν on the mass differences $\Delta m_0^2 = m_{A_0}^2 - m_{H_0}^2$ and $\Delta m_+^2 = m_{H_2^+}^2 - m_{H_1^+}^2$ signals a Glashow-Iliopoulos-Maiani–like (GIM-like) mechanism [13] at play in Eq. (4), which can be easily understood noticing that $\Delta m_0^2 \propto \lambda_5$ and $\Delta m_+^2 \propto \kappa_1$. In the limit $\lambda_5 \to 0$ the Lagrangian in Eq. (1) conserves the lepton number and no Majorana neutrino mass can be generated, while in the limit $\kappa_1 \to 0$, the leading contribution to m_{ab}^ν will appear at a higher loop order.

We now analyze the ability of the model to reproduce the observed pattern of neutrino masses and mixings. The standard parametrization for the neutrino mass matrix in terms of three masses $m_{1,2,3}$, three mixing angles θ_{12}, θ_{23}, θ_{13}, and three phases δ, α_1, α_2 reads

$$m^\nu = U^T m_D^\nu U \quad \text{with} \quad m_D^\nu = \text{Diag}(m_1, m_2, m_3), \tag{6}$$

$$U = \text{Diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1) \times \begin{pmatrix} c_{13}c_{12} & -c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta} & s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta} \\ c_{13}s_{12} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i\delta} \\ s_{13}e^{-i\delta} & s_{23}c_{13} & c_{23}c_{13} \end{pmatrix},$$

where $s_{ij} \equiv \sin(\theta_{ij})$ and $c_{ij} \equiv \cos(\theta_{ij})$. A global fit to neutrino oscillation data after the recent measurement of θ_{13} (see for example Ref. [14]) gives $\Delta m^2_{21} \equiv m_2^2 - m_1^2 = 7.62^{+0.19}_{-0.19} \times 10^{-5}$ eV^2, $|\Delta m^2_{31}| \equiv |m_3^2 - m_1^2| = 2.55^{+0.06}_{-0.09} \times 10^{-3}$ eV^2, $s^2_{12} = 0.320^{+0.016}_{-0.017}$, $s^2_{13} = 0.025^{+0.003}_{-0.003}$, and $s^2_{23} = 0.43^{+0.03}_{-0.03}$ ($0.61^{+0.02}_{-0.04}$) when the fit prefers the first (second) octant for θ_{23}. Neutrino oscillations are not sensitive to the Majorana phases α_1 and α_2 nor to the absolute neutrino mass scale, while the value of the CP phase δ is beyond current experimental sensitivity. In the inverted hierarchy scenario ($\Delta m^2_{31} < 0$) these experimental data lead to $|m^\nu_{ee}| \gtrsim 10^{-2}$ eV, which cannot be accommodated by Eq. (4) due to the $x_e^2 \sim 10^{-9}$ suppression of an already three-loop suppressed EW-sized mass scale $\mathcal{A}$. Thus, a normal hierarchy pattern for neutrino masses is predicted.

The fact that the entries $m^\nu_{ee,e\mu}$ in Eq. (4) are parametrically much smaller than the rest (being proportional to x_e^2 and $x_e x_\mu$) results in an approximate neutrino mass texture of the form $m^\nu_{ee} \sim 0$, $m^\nu_{e\mu} \sim 0$. For given values of Δm^2_{21}, Δm^2_{31}, s^2_{12}, and s^2_{23}, the four constraints Re$[m^\nu_{ee}]$, Im$[m^\nu_{ee}] \simeq 0$ and Re$[m^\nu_{e\mu}]$, Im$[m^\nu_{e\mu}] \simeq 0$ can in fact only be satisfied over a certain range of s^2_{13} [7]. If Δm^2_{21}, Δm^2_{31}, s^2_{12} and s^2_{23} are taken at their central values of the global fit in Ref. [14], then the model predicts $0.009 \leq s^2_{13} \leq 0.015$ ($s^2_{13} \geq 0.017$) for $\theta_{23} > \pi/4$ ($\theta_{23} < \pi/4$). It thus gives a correlated prediction for θ_{13} and the deviation of θ_{23} from $\pi/4$ (maximal mixing) towards lower or larger values. Moreover, for fixed values of Δm^2_{21}, Δm^2_{31}, s^2_{12}, s^2_{23}, and s^2_{13} (and when allowed by the mass texture) the above constraints lead to a specific prediction for m_1, α_1, α_2 and δ.

For the mass texture $|m^\nu_{ee,e\mu}| \simeq 0$ discussed above, the experimental pattern of neutrino masses and mixings results in a neutrino mass matrix with the accompanying structure $|m^\nu_{e\tau}| \simeq 1 \times 10^{-2}$ eV, $|m^\nu_{\mu\mu,\mu\tau,\tau\tau}| \simeq 3 \times 10^{-2}$ eV. Compared to Eq. (4) we find that in order to radiatively generate $m^\nu_{e\tau}$ and $m^\nu_{\mu\mu}$ entries of the right size (bounds on $C^\nu_{\mu\tau}$ and $C^\nu_{\tau\tau}$ are lower) we need

$$C_{e\tau} s^2_{2\beta} \mathcal{A} \simeq 1.3 \text{ TeV}, \qquad C_{\mu\mu} s^2_{2\beta} \mathcal{A} \simeq 0.3 \text{ TeV}. \quad (7)$$

Dark matter.—When the lightest Z_2-odd state is electrically neutral the model has a WIMP dark matter candidate. For the remainder of the Letter, this particle will be assumed to be H_0 (taking A_0 would be equivalent). This WIMP scenario resembles the inert doublet model [2] and should share much of its phenomenology (see, e.g., Ref. [15] and references therein).

The relic abundance of H_0 is determined by its annihilation rate at freeze-out. In the mass range $m_{H_0} = 50$–75 GeV [16] or above 520 GeV [17], the correct dark matter abundance can be achieved while being compatible with existing bounds from the Large Electron-Positron collider (LEP), EWPTs, and direct and indirect dark matter searches. The lower WIMP mass range allows us to simultaneously generate neutrino masses of the right size in our model. The correct dark matter abundance can be reached in the following situations: (a) Annihilation into fermions via resonant SM scalars when $m_{H_0} \sim m_h/2$. (b) Coannihilation with either A_0 or $H^+_{1,2}$, if the mass splitting to H_0 is less than some GeV. For fairly large mass splittings Δm^2_+ and Δm^2_0, coannihilations H_0-A_0 are strongly suppressed, while coannihilations H_0-H^+_1 may still be possible. (c) The WIMP mass approaching m_W, where the closeness to the WW threshold regulates the annihilation rate at freeze-out.

Apart from a potential signal in direct dark-matter search experiments, the model could produce a striking monochromatic gamma-ray line [18] detectable by the Fermi Large Area Telescope.

Experimental constraints.—Direct searches at LEP for doubly charged scalars ρ^{++} decaying into same-sign dileptons set a lower bound $m_\rho \gtrsim 160$ GeV [19] (which however depends on the value of C_{ee}). Bounds from virtual ρ^{++} exchange in Bhabha scattering lead to $C^2_{ee} \lesssim 9.7 \times 10^{-6}$ GeV$^{-2} m^2_\rho$ [19,20]. More stringent limits from direct searches at the Tevatron and the Large Hadron Collider (LHC) are more subtle to derive (as opposed to doubly charged scalars Δ^{++} from inside $SU(2)_L$ triplets, ρ^{++} does not couple to W bosons). The ATLAS collaboration at the LHC searched for pair produced ρ^{++} decaying into leptons and set a limit $m_\rho \gtrsim 400$ GeV [21]. For the charged states $H^+_{1,2}$, the LEP data from chargino searches can be translated into an approximate bound $m_{H^+} \gtrsim 70-90$ GeV (depending on m_{H_0}) [22]. Moreover, LEP excludes models with $m_{A_0} \lesssim 100$ GeV if $m_{H_0} \lesssim 80$ GeV and $m_{A_0} - m_{H_0} \gtrsim 10$ GeV [23].

The new inert fields also contribute to EWPT observables, such as the oblique parameters S, T, and U [24]. For $m_h \simeq 126$ GeV, the most important constraint is given by $\Delta T \in [-0.04, 0.12]$ at 95% C.L. [25] (contributions to S and U are found to be negligible). The one-loop contribution to T from the new fields is calculated to be

$$\Delta T = \frac{1}{16\pi m^2_W s^2_{\theta_W}} [c^2_\beta (F_{H^+_1,H_0} + F_{H^+_1,A_0}) + s^2_\beta (F_{H^+_2,H_0} + F_{H^+_2,A_0}) - 2c^2_\beta s^2_\beta F_{H^+_1,H^+_2} - F_{H_0,A_0}], \quad (8)$$

where $F_{i,j} = \frac{m_i + m_j}{2} - \frac{m_j m_j}{m_i - m_j} \ln \frac{m_i}{m_j}$, and θ_W is the Weinberg angle. EWPT constraints can be satisfied for a wide range of masses and mixing angles β and, as opposed to the inert doublet model [2], the present scenario allows for large mass splittings (see Figure 2).

In addition, the doubly charged scalar ρ^{++} mediates lepton-flavor violation (LFV) at tree level in processes such as $\mu \to 3e$ and $\tau \to 3e$, 3μ, and at one loop in

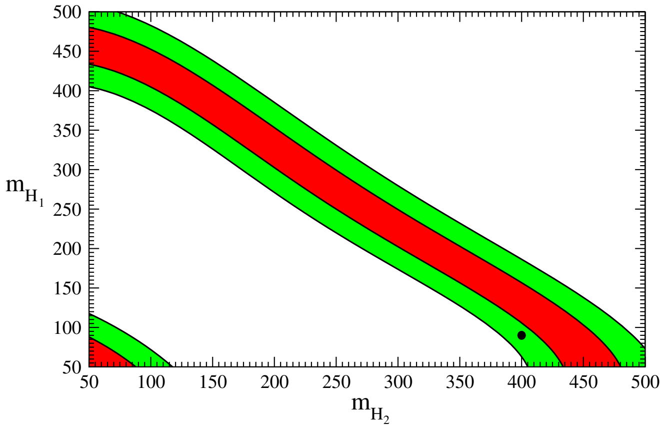

FIG. 2 (color online). Allowed regions for $m_{H_2^+}$ *vs* $m_{H_1^+}$ from electroweak precision constraints on ΔT at the 1σ [red (dark gray)] and 2σ [green (light gray)] confidence levels. For $\beta = \pi/4$, $m_{A_0} = 250$ GeV, $m_{H_0} = 70$ GeV and $m_\rho = 1$ TeV. The solid circle marks our benchmark point (see body text for its definition).

processes like $\mu \to e\gamma$ and $\tau \to e\gamma$, $\mu\gamma$. This constrains the allowed values of C_{ab} as a function of m_ρ^2, with the most stringent bounds being [26–28]

$$\begin{aligned}
&\mu^- \to 3e: && |C_{e\mu}C_{ee}| < 2.3\times 10^{-5}(m_\rho/\mathrm{TeV})^2\\
&\tau^- \to 3e: && |C_{e\tau}C_{ee}| < 9.0\times 10^{-3}(m_\rho/\mathrm{TeV})^2\\
&\tau^- \to 3\mu: && |C_{\mu\tau}C_{\mu\mu}| < 8.1\times 10^{-3}(m_\rho/\mathrm{TeV})^2\\
&\tau^- \to \mu^+e^-e^-: && |C_{\mu\tau}C_{ee}| < 6.8\times 10^{-3}(m_\rho/\mathrm{TeV})^2\\
&\tau^- \to \mu^+e^-\mu^-: && |C_{\mu\tau}C_{e\mu}| < 6.5\times 10^{-3}(m_\rho/\mathrm{TeV})^2\\
&\tau^- \to e^+e^-\mu^-: && |C_{e\tau}C_{e\mu}| < 5.2\times 10^{-3}(m_\rho/\mathrm{TeV})^2\\
&\tau^- \to e^+\mu^-\mu^-: && |C_{e\tau}C_{\mu\mu}| < 7.1\times 10^{-3}(m_\rho/\mathrm{TeV})^2\\
&\mu^+ \to e^+\gamma: && \Big|\sum_l C_{l\mu}C_{le}^*\Big| < 3.2\times 10^{-4}(m_\rho/\mathrm{TeV})^2.
\end{aligned}$$

LFV constraints favor $m_\rho \gtrsim 1$ TeV, which combined with Eq. (7) leads to large values of $\kappa_2 \gtrsim 1$ TeV and/or ξ, close-to-maximal mixing $\beta \sim \pi/4$ and fairly large mass splittings Δm_+^2, $\Delta m_0^2 \sim v^2$. For such large mass splittings, satisfying the EWPT constraints requires a mass spectrum $m_{H_{2,1}^+} \gtrsim m_{A_0} \gtrsim m_{H_{1,2}^+}$, resulting in a partial cancellation of the H_1^+ and H_2^+ contributions in Eq. (8).

As a prototypical benchmark model that satisfies EWPTs (see Figure 2) and collider constraints, we take $m_{H_0} = 70$ GeV, $m_{A_0} = 250$ GeV, $m_{H_1^+} = 90$ GeV, $m_{H_2^+} = 400$ GeV and $m_\rho = 1$ TeV, with $\kappa_2 = 2$ TeV and $\beta = \pi/4$. Neutrino masses and mixings of the right size are then obtained for $C_{e\tau} \sim 0.06$, $C_{\mu\mu} \sim 0.01$, $C_{\mu\tau} \sim 9\times 10^{-4}$ and $C_{\tau\tau} \sim 5\times 10^{-5}$, and, together with C_{ee} and $C_{e\mu}$ satisfying $C_{ee} \lesssim 0.1$ plus $C_{ee}C_{e\mu} \lesssim 2\times 10^{-5}$, these fulfil at the same time all the LFV bounds. However, branching ratios for several LFV processes (like $\tau^- \to e^+\mu^-\mu^-$ and $\mu \to e\gamma$) are predicted close to the current experimental bounds, and may be probed in the near future.

In this model, the short-distance contribution to neutrinoless double beta ($0\nu\beta\beta$) decays dominates over the one coming from light-neutrino exchange (since this one is proportional to m_{ee} and thus suppressed by $x_e^2 \sim 10^{-9}$). If the value of C_{ee} is not too small, this could open up the possibility to test this scenario at future $0\nu\beta\beta$ decay experiments.

To conclude, we have put forward a minimal extension of the SM to include neutrino mass generation and dark matter in a unified framework, without introducing right-handed neutrinos. While giving an elegant explanation of the hierarchy m_ν/v, the model predicts a small and nonzero value of θ_{13}, together with a nontrivial relation between θ_{13} and the octant of θ_{23}, to be tested by future neutrino experiments. It also predicts LFV, WIMP dark matter with a mass ~50–75 GeV, and new scalar states to be searched for.

We thank J. M. Frère, T. Hambye, T. Scarnà, and M. Tytgat for useful discussions, as well as S. Borowka, G. Heinrich, and J. Vermaseren for useful correspondence. M. A. R. is supported by Fondecyt Grant No. 11110472, Anillo "Atlas Andino" ACT1102, and DGIP Grant No. 11.12.39. J. M. N. is supported by the Science Technology and Facilities Council (STFC) under Grant No. ST/J000477/1. M. G. is supported by the Belgian Science Policy (IAP VII/37), the IISN, and the ARC project "Beyond Einstein: Fundamental Aspects of Gravitational Interactions."

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