

Octet Spin Fractions and the Proton Spin Problem

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The relatively small fraction of the spin of the proton carried by its quarks presents a major challenge to our understanding of the strong interaction. Traditional efforts to explore this problem have involved new and imaginative experiments and QCD based studies of the nucleon. We propose a new approach to the problem that exploits recent advances in lattice QCD. In particular, we extract values for the spin carried by the quarks in other members of the baryon octet in order to see whether the suppression observed for the proton is a general property or depends significantly on the baryon structure. We compare these results with the values for the spin fractions calculated within a model that includes the effects of confinement, relativity, gluon exchange currents, and the meson cloud required by chiral symmetry, finding a very satisfactory level of agreement given the precision currently attainable.

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There have been decades of careful experimental investigation since the original discovery of the so-called proton spin crisis by the European Muon Collaboration (EMC) [1–4]. The fraction of the spin of the proton carried by its quarks currently stands at [5] $33 \pm 3 \pm 5\%$, if one relies on SU(3) symmetry for the octet axial charge g_A^8 . This is a dramatic suppression with respect to the value of 100% expected in a naive quark model or even the 65% expected in a relativistic quark model. It increases only marginally, to $36 \pm 3 \pm 5\%$, if g_A^8 is reduced by 20%, as suggested by model calculations [6] and a recent lattice simulation [7].

A number of possible theoretical explanations have been offered, ranging from a key role for the axial anomaly [8–14] to the effect of gluon exchange currents [15–17], the effects of chiral symmetry [18,19] and, in the light of insights gained from lattice QCD studies, a combination of both of these effects [20]. The relatively small values of ΔG , the gluon spin in the proton, found in both fixed target and collider experiments [21,22], imply that the contribution to the spin by polarized gluons through the axial anomaly is small.

It is clearly of great interest to find new ways to shed light on the origin of this remarkable phenomenon. Considerable attention is being directed at the measurement and interpretation of the generalized parton distributions (GPDs) [23]. The moments of these GPDs can be related to the quark angular momentum [24], although there is a lively debate over the physical interpretation of those moments [25,26]. Studies of transverse momentum distributions may also offer insight into the orbital angular momentum carried by quarks in the nucleon [27]. In parallel with these experimental plans, lattice QCD has reached a level of sophistication such that a recent calculation of g_A by the QCDSF Collaboration, after a careful

chiral extrapolation, gave a value of 1.24 ± 0.04 [28], in satisfactory agreement with experiment. Furthermore, one can relatively accurately determine the low moments of the GPDs, which yield the total angular momentum carried by various flavors of quarks in the proton [29,30], $J^{u,d,s}$. The comparison of these results with the predictions of quark models after QCD evolution [31,32] appears promising.

In this Letter we explore a fascinating new line of investigation into the spin puzzle. In particular, we extract the quark spin content of the octet baryons from recent lattice QCD calculations. These are compared with the predictions of a relativistic quark model [33–35] that includes gluon exchange currents [15,36,37] and the meson cloud required by chiral symmetry [18,38]. The variation of the suppression of the fraction of the spin carried by quarks across the octet is striking and within the relatively large uncertainties in the lattice results (which should improve significantly in the near future) this is reproduced by the model.

The lattice QCD calculations of the moments of the spin-dependent PDFs used here were based upon simulations involving $2 + 1$ flavors of dynamical quarks, using the Symanzik improved gluon action and nonperturbatively $\mathcal{O}(a)$ improved Wilson fermions [39,40]. These simulations were performed on a $24^3 \times 48$ volume with lattice spacing $a = 0.083 \pm 3$ fm; more details of the specific simulation may be found in Refs. [40,41].

Because these simulations were performed at π and K masses between (334, 460) and (401, 463) MeV, respectively, it is necessary to extrapolate them to the physical masses. This problem of extrapolation has been studied in great detail in the literature [42–45], with the generalization to include charge symmetry breaking reported most recently in Ref. [46]. Our analysis follows that of Ref. [46],

where the spin-dependent moment relevant to the proton spin problem is

$$\Delta q_B \equiv \langle 1 \rangle_{\Delta q}^B = \int_0^1 dx [\Delta q^B(x) + \Delta \bar{q}^B(x)], \quad (1)$$

corresponding to the matrix element in the proton of the twist-2 operator:

$$\mathcal{O}_{\Delta q}^\mu = \bar{q} \gamma_5 \gamma^\mu q. \quad (2)$$

To lowest order in an SU(3) expansion the matrix element of the operator in Eq. (2) in a member of the baryon octet can be expressed in terms of three coefficients, $\Delta\alpha$, $\Delta\beta$, and $\Delta\sigma$:

$$\begin{aligned} \langle B(\vec{p}) | \mathcal{O}_{\Delta q}^\mu | B(\vec{p}) \rangle = & [\Delta\alpha (\bar{B} S^\mu B \lambda_q) + \Delta\beta (\bar{B} S^\mu \lambda_q B) \\ & + \Delta\sigma (\bar{B} S^\mu B) \text{Tr}(\lambda_q)]. \end{aligned} \quad (3)$$

Here we have defined

$$\lambda^q = \frac{1}{2} (\xi \bar{\lambda}^q \xi^\dagger + \xi^\dagger \bar{\lambda}^q \xi), \quad (4)$$

with ξ the usual 3×3 matrix constructed from the π , η , and K pseudo-Goldstone bosons:

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta}{\sqrt{6}} - \frac{\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}, \quad (5)$$

and

$$\Sigma = \exp\left(\frac{2i\Phi}{f}\right) = \xi^2. \quad (6)$$

In addition, the octet baryon tensor B_{abc} is defined through

$$\mathbf{B} = \begin{pmatrix} \frac{\Lambda}{\sqrt{2}} + \frac{\Sigma^0}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & \frac{\Lambda}{\sqrt{6}} - \frac{\Sigma^0}{\sqrt{2}} & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda \end{pmatrix}, \quad (7)$$

with

$$B_{abc} = \frac{1}{\sqrt{6}} (\epsilon_{abd} \mathbf{B}_{dc} + \epsilon_{acd} \mathbf{B}_{db}). \quad (8)$$

Finally, S^μ is the dimensionless spin operator, with $\bar{B} \gamma_5 \gamma^\mu B = -2\bar{B} S^\mu B$.

The chiral corrections involving π , η , and K loops, which are nonanalytic in the quark mass, are illustrated in Fig. 1. These appear at the next order in the formal expansion in quark masses. We include both octet and decuplet intermediate states (the latter indicated by double lines), while allowing for the mass difference between them in the numerical work. As explained in detail in Ref. [46], the inclusion of these loops adds six new

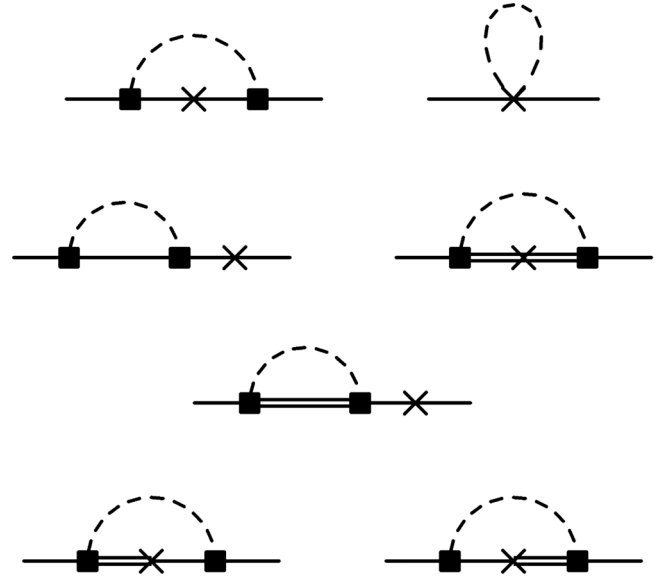


FIG. 1. Chiral loops included in the present calculation. The dashed lines represent the octet of pseudo-Goldstone bosons, the solid lines (double lines) octet (decuplet) baryons, and the cross the external spin current.

$\mathcal{O}(m_q)$ fitting parameters, $\Delta n_{i=(1,6)}^{(0)}$, in addition to $\Delta\alpha$, $\Delta\beta$, and $\Delta\sigma$. These are fit to the 24 available lattice data points [41,47–49], with the quality of the fit illustrated in Fig. 2.

Unfortunately, as there are no lattice calculations for the Λ hyperon, we are unable to present results in that case. However, for the other members of the octet one can sum the values for Δu_H , Δd_H , and Δs_H to obtain the spin fractions carried by the quarks in each octet baryon. Note that because the analysis of the renormalization of the lattice operators is not yet complete, the absolute values

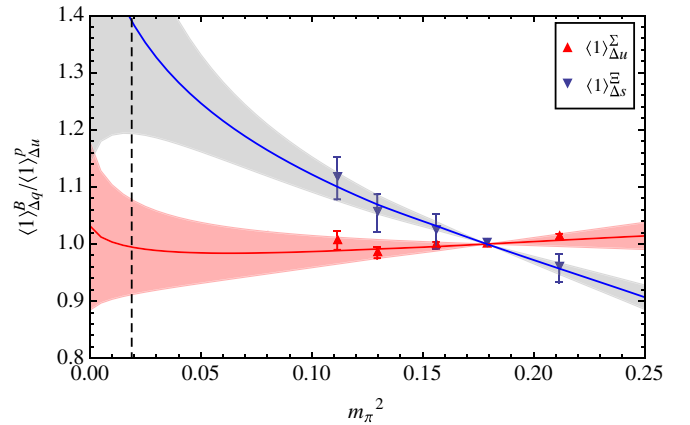


FIG. 2 (color online). Illustration of the fit to the ratio of the lattice moment of the doubly represented valence quarks (u in the proton and Σ^+ , s in the Ξ) and in the Σ and Ξ hyperons to that in the nucleon. The vertical dashed line indicates the physical pion mass.

of the spin fractions are not known. However, one *can* compute the ratios of the spin fractions for the Σ and Ξ to that of the nucleon, and these values are shown in the final column of Table I.

In spite of the fact that at this stage the uncertainties are substantial, there is a remarkable degree of variation with the structure of the baryon, with the ratio of spin fractions equal to 0.92 ± 0.13 for $\Sigma:N$, while it is equal to 1.61 ± 0.33 for $\Xi:N$. These results clearly do not support the hypothesis that the spin suppression observed for the proton might be a universal property. It is therefore of considerable interest to investigate the predictions of models in which the suppression of the spin carried by quarks is dependent on structure. With this in mind, we now apply the cloudy bag model (CBM), as developed in Refs. [15,16,18,20], to this problem.

There are three major ingredients of that calculation: (i) the relativistic suppression of the spin of a confined quark because of the orbital angular momentum in the lower component of its Dirac wave function; (ii) color exchange current corrections associated with the hyperfine interaction mediated by one-gluon exchange, and (iii) corrections arising from the interchange of spin and orbital angular momentum when the cloud of pseudo-Goldstone bosons required by chiral symmetry is included. We briefly outline the calculation of each of these corrections within the CBM [33–35].

(i) *Relativity*.—Within the MIT bag model we take $m_s = 250$ MeV as a representative value required to yield the observed hyperon masses [50]. Using this value we find the spin suppression arising from the lower Dirac component of the $1s$ wave function with a bag radius $R = 1$ fm (0.8 fm) to be 0.77 (0.78), compared with the well-known 0.65 for a massless quark. Thus the most naive expectation is that there should be less spin suppression for the hyperons than for the nucleon.

(ii) *Gluon exchange current correction*.—The one-gluon-exchange (OGE) force is an essential component of spectroscopic studies in most quark models, including the MIT bag model. For example, it provides a very natural explanation for the Δ - N and Σ - Λ mass differences. As originally observed by Hogassen and Myhrer [51], it also leads to important corrections to spin-dependent

observables, such as magnetic moments and axial charges, through the processes illustrated in Fig. 3. The corresponding correction to the nucleon spin was calculated by Myhrer and Thomas [15], who showed that it reduced the quark spin content from 0.65 to around 0.5. We have repeated that calculation for the baryon octet. Recall that the dominant terms for a spin-dependent external probe are those involving an intermediate q - \bar{q} pair (i.e., with the intermediate quark line in Fig. 3 traveling backwards in time). In the case of the hyperons there are four different sets of matrix elements, labeled f_{ij} with $(ij) \in (q, s)$ (with q and s a light or strange quark, respectively). The second subscript refers to the quark hit by the external operator and the first to the quark emitting the exchanged gluon. In terms of these matrix elements the corrections to the spins of the octet baryons $\delta\Sigma^H$ are

$$\begin{aligned}\delta\Sigma^p &= 2f_{qq}, \\ \delta\Sigma^\Lambda &= 2f_{sq}, \\ \delta\Sigma^\Sigma &= \frac{4}{3}f_{qq} - \frac{2}{3}f_{sq} + \frac{4}{3}f_{qs}, \\ \delta\Sigma^\Xi &= \frac{4}{3}f_{ss} - \frac{2}{3}f_{qs} + \frac{4}{3}f_{sq},\end{aligned}\tag{9}$$

where for $R = 1$ fm (0.8 fm) our numerical evaluation gives $f_{qq} = -0.058(-0.058)$, $f_{sq} = -0.049(-0.040)$, $f_{qs} = -0.047(-0.049)$, $f_{ss} = -0.039(-0.042)$. This yields the values for the spin fractions shown in the column of Table I labelled MIT bag + OGE. There is only a small variation of the size of the correction across the octet, with values varying from 12% in the proton to 8% in the Ξ .

(iii) *Chiral corrections*.—For the nucleon the correction arising from the pion cloud was first discussed by Schreiber and Thomas [18], who found a reduction of the spin fraction carried by quarks by 20%–30%. At the time there was a serious concern about potential double counting if one were to combine the OGE and pion corrections, which was only resolved (in favor of no significant double counting) after recent studies of the Δ - N mass splitting in quenched and full lattice QCD [20,52]. Already in the late 1980s, Kubodera and collaborators [38] combined the

TABLE I. Spin fraction (in %) carried by the valence quarks as the corrections discussed in the text are added. The column “Model” summarizes the full prediction of the ratio of the spin fraction for each hyperon to that of the nucleon (the value in brackets corresponds to $R = 0.8$ fm, rather than the default 1 fm). The final column shows the values obtained from our chiral extrapolation of recent lattice QCD data (where the quoted errors combine the uncertainties on the lattice calculations and the chiral extrapolation) for the ratio of the quark spin in the Σ and Ξ hyperons to that in the nucleon.

	MIT bag	MIT bag + OGE	MIT bag + M. cloud	MIT bag + OGE + M. cloud	Model	Lattice
N	65.4	53.8	51.9	43.8	1.0	1.0
Λ	77.1	67.3	66.4	58.9	1.35 (1.33)	(\dots)
Σ	61.5	50.8	50.5	42.6	0.97 (0.98)	0.92 (13)
Ξ	80.9	72.3	72.0	65.2	1.49 (1.44)	1.61 (33)

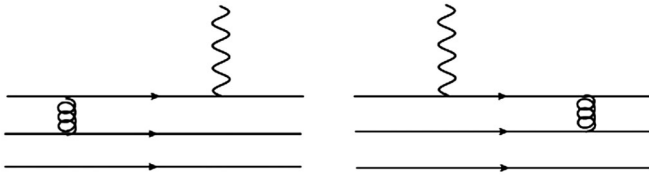


FIG. 3. Gluon exchange current corrections to the spin carried by quarks in the octet baryons. The dominant terms are those with antiquark intermediate states.

OGE corrections with the chiral loops for pions, etas, and kaons under the assumption that there was no double counting problem.

We have repeated that calculation for the full octet of baryons, working strictly within the volume coupling version of the CBM [53] (which reproduces the experimental value of g_A within a few percent) and using a typical bag radius of 1 fm everywhere (including the CBM form factors at the meson-baryon vertices). The effect of the meson cloud on the MIT bag is shown in column ‘MIT bag + M. cloud of Table I, while the final results, including relativity, OGE, and the meson cloud corrections, are shown in the column labeled MIT bag + OGE + M. cloud. We see that once all effects have been included there is a substantial variation in the spin fractions carried by the quarks across the octet. The meson cloud correction is considerably smaller in the Ξ than in the nucleon. That, combined with the less relativistic motion of the heavier strange quark, results in the spin fraction in the Ξ being quite a bit larger than in the nucleon.

The next-to-last column in Table I shows the ratios of the quark spin fractions in the hyperons compared with that in the nucleon. That the dependence on the bag radius is minimal is illustrated in the second last column of Table I, where the number in brackets shows the model result with the bag radius changed from 1 fm to 0.8 fm (the latter unrealistic for hyperons) everywhere in the calculation. At the current level of precision there is clearly very good agreement between the values calculated within the CBM and those extracted from lattice QCD. Since the lattice calculations are unquenched, we note that there could be an additional correction to the model calculations arising from polarized gluons smaller than -6% [2]. This would have only a minor effect on the ratios computed in the model and shown in the second last column of Table I. It will be extremely interesting to investigate the hyperon spin fractions in other models. In addition, this work illustrates the importance of further work to reduce the statistical errors of lattice QCD simulations of the moments of hyperon spin-dependent PDFs and to extend them to quark masses closer to the physical region.

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