Protecting Conditional Quantum Gates by Robust Dynamical Decoupling

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Dephasing—phase randomization of a quantum superposition state—is a major obstacle for the realization of high fidelity quantum logic operations. Here, we implement a two-qubit controlled-NOT gate using dynamical decoupling (DD), despite the gate time being more than 1 order of magnitude longer than the intrinsic coherence time of the system. For realizing this universal conditional quantum gate, we have devised a concatenated DD sequence that ensures robustness against imperfections of DD pulses that otherwise may destroy quantum information or interfere with gate dynamics. We compare its performance with three other types of DD sequences. These experiments are carried out using a well-controlled prototype quantum system—trapped atomic ions coupled by an effective spin-spin interaction. The scheme for protecting conditional quantum gates demonstrated here is applicable to other physical systems, such as nitrogen vacancy centers, solid state nuclear magnetic resonance, and circuit quantum electrodynamics.

Quantum information science has grown into an interdisciplinary research field encompassing the investigation of fundamental questions of quantum physics [1], metrology [2,3] as well as the quest for a quantum computer, or quantum simulator. The latter would allow unprecedented insight into scientific problems relevant, for instance, for physics and chemistry [4–7]. In order to exploit the principles of quantum physics for such purposes, it is necessary to preserve quantum coherence while carrying out gate operations. Dynamical decoupling (DD) [8,9] was successfully employed to extend the coherence time of quantum states [10–16], of single-qubit operations [17], and of two-qubit quantum gates [18,19] using pulsed fields. Also, dynamical control approaches that rely on shaped continuous fields have been suggested (e.g., Refs. [20,21]) and implemented (e.g., Ref. [22]).

The most eminent conditional quantum gate is the two-qubit controlled-NOT (CNOT) gate, since it is a basic ingredient for arbitrary quantum algorithms [23]. Physical systems with which, in principle, such gates may be realized often do not possess coherence times long enough compared to the time necessary to carry out a gate. The coherence time may be limited by undesired interactions of qubits with their environment and among themselves. This reduces the achievable gate fidelity or prevents conditional quantum gates altogether. It is, therefore, desirable to protect the quantum system during its coherent evolution while carrying out a conditional quantum gate. One way to achieve this would be through the use of dynamical decoupling (DD) techniques [24,25].

DD techniques, developed in the framework of nuclear magnetic resonance (NMR), were originally intended to be used in high precision magnetic spectroscopy [26,27]. In the modern field of quantum information processing, it was investigated how the dephasing of a qubit, which

would cause loss of information during processing, could be suppressed by the use of DD techniques [8]. New DD pulse sequences were proposed that are optimal in particular environments or robust against operational imperfections [9]. The performance of DD sequences in protecting the state of qubits (a quantum memory) was successfully demonstrated, for example, with ensembles of trapped ions [10], individual ions [14], solid state NMR [11], and quantum dots [12]. In addition to such single-qubit quantum memory investigations, experimental steps have been undertaken towards an entangled two-qubit quantum memory, whose coherence time is enhanced by DD pulses [15,16]. A conditional gate protected by DD was demonstrated using hybrid two-qubit systems with the qubits dephasing at different time scales [18]. A two-qubit gate with quantum dots was performed using a single spin echo pulse [19], and a two-qubit gate with trapped ions was made robust against variations of the driving fields' detuning using shaped pulses [28].

Our experiments are carried out using two hyperfine qubits of trapped atomic $^{171}\text{Yb}^+$-ions (a spin pseudomolecule [29]). The Hamiltonian describing this system reads

$$H = \frac{\hbar}{2}\omega_0^{(1)}\sigma_z^{(1)} + \frac{\hbar}{2}\omega_0^{(2)}\sigma_z^{(2)} - \frac{\hbar}{2}J_{12}\sigma_z^{(1)}\sigma_z^{(2)}, \quad (1)$$

where $\omega_0^{(i)}$ is the resonance frequency of qubit i and $\sigma_z^{(i)}$ is a Pauli matrix. We realize a physical system described by such a generic Hamiltonian with two laser cooled $^{171}\text{Yb}^+$ ions, forming a pseudomolecule [29–31] (see the Supplemental Material [32]). This Ising spin-spin coupling together with single qubit rotations can be used to realize a conditional NOT gate (CNOT) between a target and a control qubit [32,33]. This realization can be viewed as a Ramsey-type experiment on the target qubit where first, a clockwise $\pi/2$ rotation around the x axis of the Bloch sphere is

applied and finally the same rotation around the y axis (both driven by microwave radiation). During the conditional evolution time T_g between these two pulses the target acquires a phase shift conditioned on the state of the control qubit. For the experiments described below the J coupling between the control and target qubit yields a necessary gate time of $T_g = 5$ ms.

When performing experiments fast magnetic field fluctuations are present. We use DD pulses to probe the shape of the noise spectrum [34], which yields a power-law $S(f) \propto f^{-2}$ in the range between 1 and 50 kHz. These components, which are much faster than the coupling, cause dephasing of quantum superposition states within 200 ± 100 μs during the gate operation [Fig. 1(a)]. This time scale, after which we cannot expect to observe any *quantumness*, is more than 1 order of magnitude shorter than the necessary gate time. Therefore, it seems impossible to implement a quantum gate as the one described above.

In addition to fast fluctuating magnetic fields, there are also slowly varying stray fields present causing drifts of the qubits' resonance frequency. A detuning of δ causes the spin vector to rotate around an axis tilted by $\arctan(\delta/\Omega)$ out of the x-y plane, where Ω is the Rabi frequency of the qubit transition [33]. The nutation angle of the pulse is also relatively boosted by a factor of $\sqrt{1 + (\delta/\Omega)^2}$. The drift between consecutive experiments can be described by a Gaussian distribution with a width of 20 kHz [Fig. 1(b)], which is substantial in comparison with the Rabi frequency of about $\Omega = 2\pi \times 60$ kHz. Therefore, the robustness of DD techniques against instrumental errors is an important feature to be considered, not only in our trapped ion setup, but also, for example, for nitrogen vacancy centers in diamond [12] or other solid state systems [13].

DD usually refocuses dephasing due to the qubits' interaction with the environment but also couplings between qubits that are needed for conditional quantum gates. We apply sequences of DD to both qubits simultaneously. The simultaneity is necessary, because this allows us to enhance the coherence time of each individual qubit while not refocusing their common spin-spin interaction. DD as demonstrated here allows for the exchange of the roles of control and target qubit with minimal modifications.

To demonstrate conditional dynamics while DD pulses are applied, Ramsey interference experiments are carried out on the target qubit while the control is first prepared in $|0\rangle$. Between the two Ramsey pulses, during the time interval T where the two qubits interact, a DD sequence is applied. Ramsey fringes are observed by varying the phase of the second Ramsey pulse [Fig. 2(a), here $T = 5$ ms or $T = 8$ ms]. The target qubits phase coherence is now protected from dephasing during $T = 8$ ms. This time has to be compared to the coherence time of 0.2 ms without DD [Fig. 1(a)].

If the conditional evolution time T between two Ramsey pulses is zero, the minima of the fringes are found at $\varphi = \pi$ [not shown in Fig. 2(a)]. When the time T is increased, the Ramsey fringes are shifted accordingly, which reveals precession of the target spin and is visualized in Fig. 2(b). The conditionality of these dynamics is demonstrated by repeating the experiment with the control qubit prepared in $|1\rangle$. If the conditional evolution time T equals the gate time T_g (8 ms for the sequence used here; see below), the minima are either to be found at $\varphi = 3\pi/2$ or at $\varphi = \pi/2$. Therefore, a Ramsey pulse with a chosen phase of $\varphi = 3\pi/2$ would leave the target in the state $|0\rangle$, or flip it to the excited state $|1\rangle$ dependent on the state of the control qubit, thus realizing a conditional spin flip. Discussion of gate fidelities deduced from Ramsey fringe contrast and additional measurements can be found in the Supplemental Material [32].
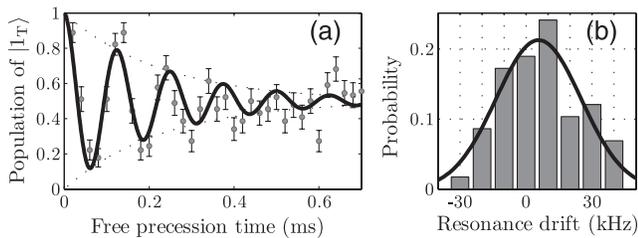

FIG. 1. Dephasing and drift. (a) A Ramsey experiment is performed to deduce the dephasing time of a qubit. A fit to the data yields a characteristic decay time of the fringes, the dephasing time, of 200 ± 100 μs. We repeat the sequence for every data point ($n = 50$); error bars indicate standard deviation. (b) Drift between consecutive experiments. Before and after an experimental run that takes a few minutes of data acquisition a qubit's addressing frequency is measured by microwave-optical double resonance spectroscopy The drift is Gaussian distributed with a width of 20 kHz, owing to the stochastic origin of this process; $n = 64$.
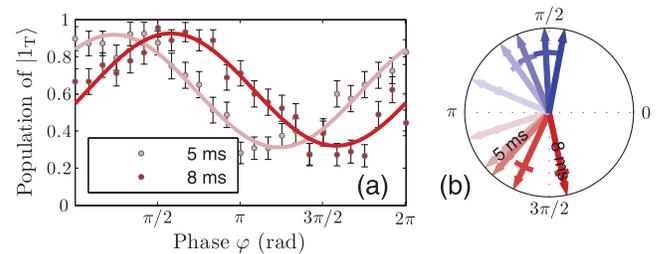

FIG. 2 (color). Conditional quantum dynamics. (a) A Ramsey-type experiment is performed on the target qubit while DD pulses are applied to both qubits simultaneously. The minimum of the Ramsey fringes, which is originally found at π, is shifted when the conditional evolution time is increased to 5 ms and 8 ms. $n = 50$; error bars indicate standard deviation. (b) Ramsey fringe minima after 2 ms, 5 ms, 6 ms, 7 ms, and 8 ms of conditional evolution time. After preparing the two-qubit system in $|00\rangle$ (red arrows), the minima are shifted to higher Ramsey pulse phases φ. If the control qubit is initialized in $|1\rangle$ (blue arrows), the target precesses in the opposite direction; $n = 10$; error bars indicate standard deviation.

In what follows, we first describe four different DD sequences (Fig. 3) that are investigated experimentally, and thereafter discuss their performance in protecting the desired quantum gate dynamics from dephasing. The first sequence is labeled CPMG$_{yy}$ since it is a Carr-Purcell-Meiboom-Gill sequence [26,27] that consists of π pulses rotating the qubit around the y axis only. This sequence protects a quantum state whose Bloch vector lies along the y axis. When considering the described gate, where the target spin precesses, we cannot expect this sequence to perform well. To protect any arbitrary superposition state, one should make use of isotropic [13] sequences that rotate around x and y axes equally. This can be accomplished by alternating the pulse phases between $\varphi = 0$ and π. It is possible to further improve the robustness of a sequence by the use of concatenated sequences [13]. In these sequences individual pulse errors do not accumulate but compensate each other.

We introduce a new kind of concatenated DD sequence, where the phases of the individual pulses are constructed by a concatenation, while their timing still follows CPMG-type sequences. As the basic sequence we define C-CPMG$_1 \equiv [\tau Y \tau \tau \bar{X} \tau]^2$, where Y and $\bar{X}$ denote π pulses rotating the qubit anticlockwise around the y or clockwise around the x axis and $[\ ]^2$ indicates a repetition of the time evolution in parentheses. The next levels of our concatenated CPMG sequence are constructed by the recursion formula

$$\text{C-CPMG}_n = \left[\sqrt{\text{C-CPMG}_{(n-1)}} \tau Y \tau \text{C-CPMG}_{(n-1)} \tau \bar{X} \tau \right.$$
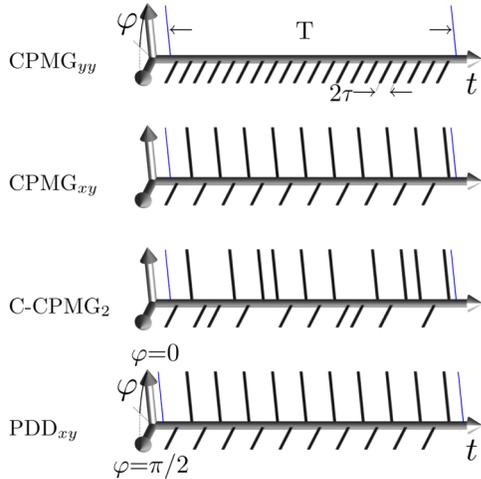$$\left. \times \sqrt{\text{C-CPMG}_{(n-1)}} \right]^2,$$

FIG. 3 (color online).   Four DD sequences employed to protect conditional quantum gate dynamics. π pulses (black, bold lines) are applied on both qubits during a conditional evolution time T between two Ramsey $\pi/2$ pulses (blue, thin lines) applied only to the target qubit. All the sequences show the same pulse interval, except the PDD sequence for the first and last $\pi/2$ pulses. To achieve robust sequences the pulses are applied with different phases ($\varphi = 0$ and $\varphi = \pi/2$).

$n \geq 2$. In contrast to the original concatenated sequences [13], the additional conditional evolution times τ around the pulses still feature the CPMG timing.

Another sequence investigated is an isotropic, strictly periodic PDD$_{xy}$ sequence [8]. Its timing deviates from the CPMG sequence only in the fact that the first and last intervals are of the same length as any other one. At first sight this may seem to be only a slight change, but, in fact, substantially changes the performance of the pulse sequence. This can be understood by considering that a DD sequence can be viewed as a filter function cutting out particular parts of a noise spectrum. Hence, different sequences are described by different filter functions and thus have a different performance under particular noise conditions [10].

We compare the performance of the DD sequences described above by Ramsey interference experiments that are carried out after preparing the system in each of the four input states of a CNOT truth table ($|00\rangle$, $|01\rangle$, $|10\rangle$, and $|11\rangle$). When using a CPMG$_{yy}$ sequence of 24 DD pulses during 5 ms of conditional evolution time, we can observe Ramsey fringes with a clear contrast indicating preserved single qubit phase coherence. This demonstrates the capability of protecting the quantum system from its noisy environment. However, we cannot observe any significant conditional phase shift. This behavior is reproduced by simulating the experiment while taking the detunings, originating from the drift [Fig. 1(b)], explicitly into account. The results of our simulation indicate that the pulse imperfections in the CPMG$_{yy}$ sequence suppress the precession and thus hamper the desired evolution, which renders the conditional gate realization impossible.

Figure 4 presents the results for the three other sequences. When using the CPMG$_{xy}$ sequence with 24 alternating pulses or C-CPMG$_3$ with 84 pulses, the clearly observed Ramsey fringes show the conditional phase shifts as
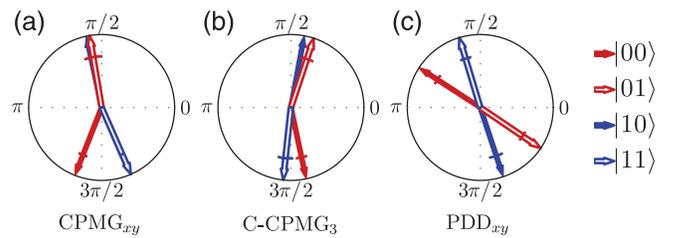

FIG. 4 (color).   Experimental conditional gate evolution under three different DD sequences. Ramsey fringes are recorded and the phase of their minima is plotted. If we apply CPMG$_{xy}$ [(a) 24 pulses] or C-CPMG$_3$ [(b) 84 pulses] during conditional evolution time, the minima are either found around $\varphi = 3\pi/2$ or $\varphi = \pi/2$ depending on the state of the control qubit. If, however, a PDD$_{xy}$ sequence is used the minima gain an additional global shift of 0.8 rad (see text for details). The relative phase shift between different input states of the control is smaller than π. Data are an average of about 10 Ramsey-type experiments (25 phase points and $n = 100$ repetitions); error bars indicate standard deviation.

expected [4(a) and 4(b)]. A Ramsey pulse of phase $\varphi = 3\pi/2$ flips the target qubit only if the control is in $|1\rangle$, which defines the CNOT gate. Both sequences give similar results in terms of the conditional phase shift and the gate fidelity (see the Supplemental Material [32]) when using a J coupling that result in a gate time $T_g = 5$ ms. However, for longer gate times, the performance of CPMG_{xy} sequence degrades rapidly while C-CPMG$_3$ is able to protect longer quantum gates as well (see the Supplemental Material [32]). For the concatenated sequence the time between the two Ramsey pulses is increased from 5 to 8 ms. We explain this by two different effects. First, the duty cycle of 84 pulses reduces the effective time in which the spin may conditionally precess, and second, imperfections of the larger number of pulses still results in a slightly suppressed precession.

For the strictly periodic sequence PDD_{xy}, 49 pulses yield the best fringe contrast, but the relative phase shift resulting from different control qubit input states is smaller than π. Here, also the duty cycle of the pulses and imperfections impair the spin precession. However, it is not possible to further increase the conditional evolution time because of the sequence's low capability to prevent the spin from dephasing. Increasing the number of pulses does not improve the results as now pulse imperfections strongly harm the dynamics as reflected by a reduced fringe contrast. In addition, the fringes are no longer centered around π but instead, the target qubit is additionally rotated 0.8 rad around the z axis. We explain this spurious extra phase shift by an oscillating magnetic field along the quantization axis. If DD pulses are applied always when the oscillation changes its sign, the pulses will not cancel the acquired precession angles but cause them to add up. This coherent phase pickup may be exploited to realize a single-ion quantum lock-in amplifier [35]. We identified a dominant noise source in the range of a few kHz that is correlated to DD sequences. This noise is picked up by the power supply and the cables connecting to the Helmholtz coils that define the quantization axis. This in turn leads to magnetic field oscillations correlated with the pulse sequence. For the other DD sequences this effect was not observed, because of the different delay between the pulses. Beside this additional shift, also the odd number of DD pulses has to be considered. This results in an additional rotation that shifts the position of all minima by π.

So far, we have investigated the performance of different sequences to protect the conditional precession of a dephasing spin while being coupled to a control qubit. Now, we investigate in more detail how imperfections in successfully tested sequences may impair the gate dynamics. For this purpose, again a Ramsey experiment is carried out on the target qubit. In contrast to the experiments before, DD pulses are addressed to the target only which effectively refocuses the spin-spin coupling [33] and prevents any conditional precession. The fringe contrast as a function
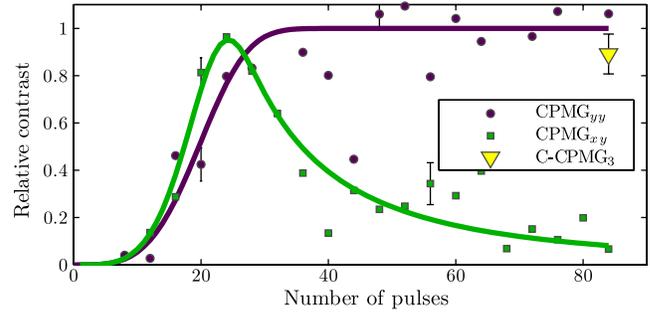

FIG. 5 (color online). Robustness of the DD sequences. On one of the qubits a Ramsey experiment is performed. During a free evolution time of 5 ms DD sequences are applied and the final state is probed. The phase of the final Ramsey pulse is chosen as $\varphi = 0$ in order to detect the fringe maximum. CPMG_{yy} is used as a benchmark (however, does not permit a conditional quantum gate). The C-CPMG$_3$ sequence performs well even when many pulses are used, and thus makes conditional quantum gates possible for gate times long compared with the coherence time. Data are averaged over ten experimental runs with 100 repetitions of each data point. Lines are drawn to guide the eye; error bars indicate standard deviation.

of the number of DD pulses is shown in Fig. 5. The CPMG_{yy} is robust for the particular input state whose Bloch vector lies along the y axis [13], and therefore the contrast grows with the number of applied DD pulses. For 24 pulses the coherence time is 10 ± 1 ms. Then, the contrast reaches a plateau that can be considered as a benchmark. Using the CPMG_{xy} sequence, in contrast, pulse imperfections accumulate and cause a seemingly chaotic behavior for increasing the number of pulses: If more than 24 pulses are used, this random behavior reduces the contrast of Ramsey fringes after several realizations of the experiment. The concatenated sequence C-CPMG$_3$ with 84 pulses, when performed under the same experimental conditions, still shows a high relative contrast. This clearly demonstrates the fact that individual pulse imperfections do not accumulate but compensate each other in this sequence. Since a higher number of dynamical decoupling pulses also yields a longer coherence time, this sequence is also able to protect slower gates, based on a lower J coupling strength. For comparison, by the use of a CPMG_{xy} sequence, realized by simply adding more and more alternating pulses, it is not possible to implement a slower gate.

Because of its generality, the approach demonstrated here for carrying out quantum gates in an environment that causes dephasing is applicable to a large variety of physical systems. Thus, existing quantum gates could be improved to reach the fidelity limit that would allow for scalable fault-tolerant quantum computing. The universal idea of protecting coherent quantum dynamics could be applied as well in other contexts where conditional quantum logic is used, for example, for a quantum repeater, metrology, or spectroscopic applications.

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