Quantum Walk as a Generalized Measuring Device

Paweł Kurzyński[1,2,*] and Antoni Wójcik[2]

[1]Centre for Quantum Technologies, National University of Singapore, 3 Science Drive 2, 117543 Singapore, Singapore

[2]Faculty of Physics, Adam Mickiewicz University, Umultowska 85, 61-614 Poznań, Poland

(Received 29 August 2012; revised manuscript received 22 February 2013; published 14 May 2013)

We show that a one-dimensional discrete time quantum walk can be used to implement a generalized measurement in terms of a positive operator value measure (POVM) on a single qubit. More precisely, we show that for a single qubit any set of rank 1 and rank 2 POVM elements can be generated by a properly engineered quantum walk. In such a scenario the measurement of a particle at a position $x = i$ corresponds to a measurement of a POVM element E_i on a qubit. Since the idea of quantum walks originates from the von Neumann model of measurement, in which the change of the position of the pointer depends on the state of the system that is being measured, we argue that von Neumann measurements can be naturally extended to POVMs if one includes the internal evolution of the system in the model.

DOI: 10.1103/PhysRevLett.110.200404 PACS numbers: 03.65.Ta, 03.67.Ac, 03.67.Lx

Introduction.—Discrete time quantum walk is a process in which an evolution of a quantum particle on a lattice depends on a state of an auxiliary system (coin). In the simplest version of a one-dimensional quantum walk the coin is a two-level system. The particle moves either one step to the left or to the right depending on the state of the coin. Between subsequent steps the state of the coin evolves, which after many steps of the walk results in nontrivial correlations between the coin and the particle's position and in a spatial probability distribution that in general cannot be reproduced by classical random walks [1–11].

A single step of an initially localized quantum walk can be considered as a projective von Neumann measurement of the coin, because if one finds the particle at position $x \pm 1$ one knows that the state of the coin corresponds to the "right or left shift." However, in general one can allow the system to evolve for more than one step before the position measurement is done. In this case the particle can be found in more than two different positions and one may ask whether the measurement of the particle at position $x = i$ corresponds to some generalized measurement of the qubit, i.e., a positive operator value measure (POVM) on the coin state. In this work we investigate such a possibility.

POVMs allow one to gain more information from a single measurement than the standard von Neumann projective measurements. Their wide applicability include discrimination of quantum states [12,13] and quantum state tomography in terms of symmetric informationally complete (SIC) POVMs [14,15]. POVMs were proposed and realized for a number of physical systems [16–20]. Physically, POVMs correspond to projective measurements on a joint system of the system of interest and an ancilla whose state is known. Mathematically, POVM elements E_i are given by $E_i = \mathrm{Tr}_{\mathrm{anc}}[(\mathbb{1} \otimes \sigma)\pi_i]$, where $\mathbb{1}$ is the identity operator on the Hilbert space of the system, σ the state of an ancilla, π_i the von Neumann projector on the joint Hilbert space, and one traces out an ancillary system. The probability of measuring the ith POVM element on a state ρ is given by $p_i = \mathrm{Tr}(E_i\rho)$. In addition to the non-negativity condition $E_i \geq 0$, the complete set of measurement operators has to have the resolution of identity; therefore, POVM elements obey $\sum_i E_i = \mathbb{1}$. Finally, the postmeasurement state of the system corresponding to the ith outcome of POVM is given by $M_i\rho M_i^\dagger/p_i$, where $E_i = M_i^\dagger M_i$ and the form of M_i is determined by the state of ancilla and the projector π_i on the joint Hilbert space. In this Letter we propose an implementation of a general single-qubit POVM by means of a quantum walk in which the role of the ancilla is played by the position of the particle in the lattice, while the coin plays the role of the qubit itself.

For a one-dimensional discrete time quantum walk the state of the system is described by two degrees of freedom $|x, c\rangle$, the position of the particle $x = \ldots, -1, 0, 1, \ldots$ and the coin $c = \rightarrow, \leftarrow$. Since the dynamics of the system is discrete, one step is given by the unitary operator $U(t, t+1) = TC(x, t)$, where

$$T = \sum_x |x+1, \rightarrow\rangle\langle x, \rightarrow| + |x-1, \leftarrow\rangle\langle x, \leftarrow| \quad (1)$$

is the conditional translation operator and $C(x, t)$ is a coin operator whose action in general can depend on position and time

$$C(x,t)|x,\rightarrow\rangle = c(x,t)|x,\rightarrow\rangle + s(x,t)e^{i\varphi}|x,\leftarrow\rangle,$$
$$C(x,t)|x,\leftarrow\rangle = s^*(x,t)|x,\rightarrow\rangle - c^* e^{i\varphi}(x,t)|x\leftarrow\rangle, \quad (2)$$

where the above complex parameters obey $|c(x,t)|^2 + |s(x,t)|^2 = 1$ for all x and t and $e^{i\varphi}$ is a complex phase factor. Throughout the Letter we use the notation $|\rightarrow\rangle = (1, 0)^T$ and $|\leftarrow\rangle = (0, 1)^T$.

Quantum walks are computationally more efficient than their classical counterparts and were shown to be able to efficiently solve a number of problems [21]. In particular, it was shown that they are capable of universal quantum computation [22,23]. Moreover, quantum walks have been implemented in the laboratory in many different physical systems and by now experimentalists have substantial control over the evolution of the walker [24–35]. It is therefore of great importance to investigate and to exploit all the possibilities that quantum walks can offer.

Unambiguous state discrimination.—We start with an example of a simple quantum walk whose few steps can be interpreted as the unambiguous state discrimination protocol (an additional example of SIC-POVM generation is provided in the Supplemental Material [36]). Imagine that one is given one of two nonorthogonal pure qubit states with equal *a priori* probabilities. The goal is to find which of the two states was given. We consider three possible answers to this test: it is definitely state 1, it is definitely state 2, or I do not know.

Now, let us introduce a quantum walk capable of achieving this goal. First, we note that it is always possible to encode two nonorthogonal pure states as $|\psi_\pm\rangle = \cos(\theta/2)|\rightarrow\rangle \pm \sin(\theta/2)|\leftarrow\rangle$, where $\theta \in [0, (\pi/2)]$. We set the given state as the initial coin state and we initialize the walk with the particle located at the origin. For the first step we choose the coin operator to be trivial $C(x,0) = \mathbb{1}$; hence, the state after one QW step is

$$|\psi_\pm(1)\rangle = \cos\frac{\theta}{2}|1,\rightarrow\rangle \pm \sin\frac{\theta}{2}|-1,\leftarrow\rangle. \tag{3}$$

For the next step the coin operators are

$$C(-1,1) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad C(1,1) = \begin{pmatrix} \sqrt{1-z^2} & z \\ z & -\sqrt{1-z^2} \end{pmatrix}, \tag{4}$$

where $z = \tan(\theta/2)$, and identity elsewhere; therefore, the state after coin operation is

$$C(x,1)|\psi_\pm(1)\rangle = \sqrt{\cos\theta}|1,\rightarrow\rangle + \sin\frac{\theta}{2}|1,\leftarrow\rangle \pm \sin\frac{\theta}{2}|-1,\rightarrow\rangle \tag{5}$$

and, after translation,

$$|\psi_\pm(2)\rangle = \sqrt{\cos\theta}|2,\rightarrow\rangle + \sin\frac{\theta}{2}|0,\leftarrow\rangle \pm \sin\frac{\theta}{2}|0,\rightarrow\rangle. \tag{6}$$

Finally, for the third step the coin operator is the identity everywhere except for position $x = 0$, for which it is the Hadamard operator

$$C(0,2) = H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}; \tag{7}$$

therefore,

$$|\psi_\pm(3)\rangle = \sqrt{\cos\theta}|3,\rightarrow\rangle \pm \sqrt{2}\sin\frac{\theta}{2}|\pm 1,\rightarrow\rangle. \tag{8}$$

As a consequence, if one measures a particle at position $x = 1$, one immediately knows that the coin was in the $|\psi_+\rangle$ state, if at position $x = -1$ it wasin the $|\psi_-\rangle$ state, and if at position $x = 3$ one learns nothing.

Next, we show that the quantum walk corresponding to the unambiguous state discrimination problem generates proper POVM elements. We are going to consider only one POVM element, since the construction of the other two follows from this example. The initial state of the ancilla (position) is $\sigma = |x=0\rangle\langle x=0|$, whereas the projector π_i corresponds to $\pi_i = U^\dagger(|x=i\rangle\langle x=i| \otimes \mathbb{1})U$. The unitary operator U generates the three steps of the above quantum walk, and the identity operator acts on the coin space. In order to evaluate the form of the projector π_i one has to consider the reversed quantum walk evolution due to $U^\dagger$ on both states $|i,\rightarrow\rangle$ and $|i,\leftarrow\rangle$. Finally, in order to obtain the POVM element E_i one has to consider the overlap of π_i with the ancilla state and then trace over the ancilla.

Let us consider the POVM element E_{-1}, i.e., the element corresponding to finding the particle at position $x = -1$. We start with the state $|-1,\leftarrow\rangle$. The first step of the reversed evolution corresponds to $C(x,2)^\dagger T^\dagger$. The reversed translation results in the state $|0,\leftarrow\rangle$ and the application of the coin operator gives $1/\sqrt{2}(|0,\rightarrow\rangle - |0,\leftarrow\rangle)$. For the second step, the translation gives $1/\sqrt{2}(|-1,\rightarrow\rangle - |1,\leftarrow\rangle)$ and the coin operation gives $1/\sqrt{2}(|-1,\leftarrow\rangle - \tan(\theta/2)|1,\rightarrow\rangle + \sqrt{1-\tan^2(\theta/2)}|1,\leftarrow\rangle)$. Finally, the last step is just reversed translation which results in $1/\sqrt{2}(|0,\leftarrow\rangle - \tan(\theta/2)|0,\rightarrow\rangle + \sqrt{1-\tan^2(\theta/2)}|2,\leftarrow\rangle)$. Taking the overlap with the ancilla state $|x=0\rangle$ one finds that the contribution of the above state to the POVM is $1/\sqrt{2}(|\leftarrow\rangle - \tan(\theta/2)|\rightarrow\rangle)$.

On the other hand, it is easy to see that the state $|-1,\rightarrow\rangle$ does not contribute to the POVM element, since the reverse evolution generates a state $|-4,\rightarrow\rangle$ that has no overlap with position $x = 0$. Therefore, the POVM element is given by

$$E_{-1} = \frac{1}{2\cos^2\frac{\theta}{2}}\left(\cos\frac{\theta}{2}|\leftarrow\rangle - \sin\frac{\theta}{2}|\rightarrow\rangle\right) \times \left(\cos\frac{\theta}{2}\langle\leftarrow| - \sin\frac{\theta}{2}\langle\rightarrow|\right), \tag{9}$$

which is the correct POVM element for unambiguous state discrimination since $E_{-1}|\psi_+\rangle = 0$.

Generation of arbitrary rank 1 POVM elements.—Let us focus on rank 1 POVMs, since higher rank POVMs can be constructed as a convex combination of rank 1 elements. We recall that rank 1 elements are of the form $E_i = a_i|\psi_i\rangle\langle\psi_i|$, with $0 < a_i \leq 1$; i.e., they are proportional to the projectors onto pure states. We will come back to the problem of higher rank POVMs at the end of this section.

We propose the following algorithm for the generation of an arbitrary rank 1 POVM $\{E_1, \ldots, E_n\}$: (1) Initiate the quantum walk at position $x = 0$ with the coin state corresponding to the qubit state one wants to measure. (2) Set $i := 1$. (3) While $i < n$ do the following: (a) Apply coin operation $C_i^{(1)}$ at position $x = 0$ and identity elsewhere and then apply translation operator T. (b) Apply coin operation $C_i^{(2)}$ at position $x = 1$,

$$\mathrm{NOT} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

at position $x = -1$ and identity elsewhere and then apply translation operator T. (c) $i := i + 1$. (4) Apply coin operations $C_k^{(3)}$ at positions $x = 2k$, $k = 0, \ldots, n-1$, and identity elsewhere. The particular form of the operators $C_i^{(1)}$, $C_i^{(2)}$, and $C_k^{(3)}$ depends on the POVM that one wants to implement. In the following we give a few more details about these, and in the Supplemental Material [36] we prove that they can always be chosen in such a way as to implement any desired POVM.

Let us analyze the action of the algorithm. The detailed proof that the algorithm generates arbitrary POVM elements is given in the Supplemental Material [36]. The walk is initialized at the origin and the coin state corresponds to the qubit state that is going to be measured $|0\rangle \otimes |\Psi_0\rangle$. Here, we assume that the initial qubit state $|\Psi_0\rangle$ is pure; however, our analysis works for mixed states as well (one has to consider evolution of two pure parts of the mixture). Next, we set $i := 1$ and apply the coin operation $C_1^{(1)}$ and the subsequent translation T evolves the system into a superposition

$$\alpha_1|1, \rightarrow\rangle + \beta_1|-1, \leftarrow\rangle. \tag{10}$$

The amplitudes in the superposition depend on the initial state of the coin and on the operator $C_1^{(1)}$, i.e., $\alpha_1 = \langle\rightarrow|C_1^{(1)}|\Psi_0\rangle$ and $\beta_1 = \langle\leftarrow|C_1^{(1)}|\Psi_0\rangle$.

Next, we consider the step 3(b) ($i = 1$). At position $x = -1$ we swap the coin state (NOT operation); therefore, the particle cannot go to the left beyond $x = -1$ and is reflected back to the origin. At $x = 1$ we apply the coin operation $C_1^{(2)}$. After the translation the resulting state is $\alpha_1'\alpha_1|2, \rightarrow\rangle + \beta_1'\alpha_1|0, \leftarrow\rangle + \beta_1|0, \rightarrow\rangle$, where $\alpha_1' = \langle\rightarrow|C_1^{(2)}|\rightarrow\rangle$ and $\beta_1' = \langle\leftarrow|C_1^{(2)}|\rightarrow\rangle$. We define an unnormalized vector $|\Psi_1\rangle = \beta_1'\alpha_1|\leftarrow\rangle + \beta_1|\rightarrow\rangle$; therefore, the state after the step 3(b) ($i = 1$) is of the form

$$\alpha_1'\alpha_1|2, \rightarrow\rangle + |0\rangle \otimes |\Psi_1\rangle. \tag{11}$$

Next we set $i := 2$ and go to step 3(a). After the step 3(a) ($i = 2$) we have

$$\alpha_1'\alpha_1|3, \rightarrow\rangle + \alpha_2|1, \rightarrow\rangle + \beta_2|-1, \leftarrow\rangle, \tag{12}$$

where $\alpha_2 = \langle\rightarrow|C_2^{(1)}|\Psi_1\rangle$ and $\beta_2 = \langle\leftarrow|C_2^{(1)}|\Psi_1\rangle$. After the step 3(b) the state is

$$\alpha_1'\alpha_1|4, \rightarrow\rangle + \alpha_2'\alpha_2|2, \rightarrow\rangle + |0\rangle \otimes |\Psi_2\rangle, \tag{13}$$

where we introduced $|\Psi_2\rangle = \beta_2'\alpha_2|\leftarrow\rangle + \beta_2|\rightarrow\rangle$ in an analogical way to $|\Psi_1\rangle$, and set $\alpha_2' = \langle\rightarrow|C_2^{(2)}|\rightarrow\rangle$ and $\beta_2' = \langle\leftarrow|C_2^{(2)}|\rightarrow\rangle$.

The two iterations of steps 3(a) and 3(b) show that apart from the parts of wave function that travel to the right, the relevant evolution of the walk takes place between positions $x = \pm 1$. Moreover, this evolution can be described in a recursive way using parameters $\alpha_j = \langle\rightarrow|C_j^{(1)}|\Psi_{j-1}\rangle$, $\beta_j = \langle\leftarrow|C_j^{(1)}|\Psi_{j-1}\rangle$, $\alpha_j' = \langle\rightarrow|C_j^{(2)}|\rightarrow\rangle$, $\beta_j' = \langle\leftarrow|C_j^{(2)}|\rightarrow\rangle$ and unnormalized vectors $|\Psi_j\rangle = \beta_j'\alpha_j|\leftarrow\rangle + \beta_j|\rightarrow\rangle$. It follows, that once we reach step 4 the state of the system is

$$|0\rangle \otimes |\Psi_{n-1}\rangle + \sum_{j=1}^{n-1} \alpha_{n-j}'\alpha_{n-j}|2j, \rightarrow\rangle. \tag{14}$$

The operator C_{n-1}^2 is simply identity (see supplementary material), therefore $|\Psi_{n-1}\rangle = \beta_{n-1}|\rightarrow\rangle$ and after the step 4 we have

$$\beta_{n-1}|0\rangle \otimes |\psi_n\rangle + \sum_{j=1}^{n-1} \alpha_{n-j}'\alpha_{n-j}|2j\rangle \otimes |\psi_{n-j}\rangle, \tag{15}$$

where $\alpha_{n-1}' = 1$ and we have chosen $C_k^{(3)} = |\psi_{n-k}\rangle \times \langle\rightarrow| + |\psi_{n-k,\perp}\rangle\langle\leftarrow|$, being $|\psi_{n-k,\perp}\rangle$ the state orthogonal to $|\psi_{n-k}\rangle$, so that we get the proper postmeasurement states. Note that the probability of finding the walker at positions $\{0, 2, \ldots, 2(n-1)\}$ is given by $N^{-1}\{|\beta_{n-1}|^2, |\alpha_{n-1}'\alpha_{n-1}|^2, \ldots, |\alpha_1'\alpha_1|^2\}$, where $N = |\beta_{n-1}|^2 + \sum_{j=1}^{n-1}|\alpha_j'\alpha_j|^2$; also, note that in the Supplemental Material [36] we prove that, irrespective of the initial state $|\Psi_0\rangle$, the coin operators $C_k^{(1)}$ and $C_k^{(2)}$ can be chosen in such a way that these probabilities equal the probabilities $\{p_n, p_{n-1}, \ldots, p_1\}$ of any desired POVM. This way, we arrive to the main result of the Letter: for any single-qubit POVM, one can engineer a quantum walk in which a measurement of the walker's position is equivalent to that generalized measurement.

Finally, let us discuss the generation of rank 2 POVM elements. These elements can be constructed as a convex combination of two orthogonal rank 1 elements. The above algorithm can be modified in the following way. Imagine that before the final step 4 the algorithm generated N POVM elements corresponding to the measurement of the quantum walker at positions $x = 0, 2, 4, \ldots, 2(N-1)$ and that we want to construct a rank 2 element that is a combination of two orthogonal rank 1 elements $a|\psi\rangle\langle\psi|$ and $b|\psi_\perp\rangle\langle\psi_\perp|$ corresponding to positions $x = 2i$ and $x = 2(i+1)$, respectively. It is enough to apply NOT coin operation at position $x = 2(i+1)$ and then to apply conditional translation. As a result, the two probability amplitudes originating from positions $x = 2i$ and $x = 2(i+1)$ meet at position $x = 2i + 1$ and the respective position measurement corresponds to the rank 2 POVM element $a|\psi\rangle\langle\psi| + b|\psi_\perp\rangle\langle\psi_\perp|$. The other rank 1 POVM elements are also shifted to the right by a single position; however, their structure remains unchanged. Finally, note that the step 4 is analogical and that the coin for the rank 2 element is $C^{(3)} = |\psi\rangle\langle\leftarrow| + |\psi_\perp\rangle\langle\rightarrow|$.

Discussion.—We proved that discrete-time quantum walks are capable of performing generalized measurements on a single qubit. The main physical effect employed in this process is the interference between the probability amplitudes of the quantum walker. This interference effect is not present in the standard von Neumann projective measurement. However, note that the original idea of discrete-time quantum walks proposed in Ref. [1] is based on the von Neumann model of measurement. In von Neumann's model a pointer of a measuring device is coupled to an observable on a measured system. The Hamiltonian of this coupling is given by $H_{\rm int} = g(t)AP$, where P is the momentum of the pointer, A is the observable one wants to measure and $g(t)$ is the time-dependent coupling strength that is nonzero only during the period of the measurement. In addition, the system and the pointer of the device can undergo their own evolution due to Hamiltonians H_s and H_d, respectively. Nevertheless, the coupling strength $g(t)$ is usually assumed to be much greater than H_s and H_d, whereas the time of interaction T is assumed to be very short $T \rightarrow 0$; therefore, the measurement is considered to be *impulsive* and the evolution of the pointer and the system due to H_s and H_d is ignored [37]. The pointer state $|\psi(x)\rangle$, that is initially localized around the origin, and the state of the system $|\varphi\rangle$ evolves into

$$\sum_i \alpha_i |\psi(x + g a_i)\rangle \otimes |a_i\rangle, \tag{16}$$

where $g = \int_0^T g(t)dt$ is an average coupling strength, a_i are eigenvalues, and $|a_i\rangle$ are eigenvectors of A and $\alpha_i = \langle a_i|\varphi\rangle$. This model indeed resembles the quantum walk, since the translation of the pointer depends on the other system.

Apart from the discrete nature of the evolution, the main difference between quantum walks and *impulsive* von Neumann measurements is the fact that the evolution of quantum walks is extended in time and that the system that is coupled to the particle or pointer also evolves. Using the analogy to the von Neumann model, one can say that in the case of quantum walks the Hamiltonian H_s is not ignored. On the other hand, the Hamiltonian H_d does not appear in quantum walks. One of the possible solutions to this problem can be based on an assumption that the particle or pointer is very heavy; hence, its natural dispersion requires much more time than the time of the quantum walk. Because of the internal evolution the measured system does not remain in an eigenstate of A and the interaction $H_{\rm int}$ causes the pointer to further evolve in time which eventually leads to an interference that is analogical to the effect we observe in quantum walks.

The above discussion shows that quantum walks approximate the von Neumann model that is extended in time and for which the internal evolution of the measured system affects the evolution of the pointer of the measuring device. Furthermore, we showed that if the measured system is a qubit and if one can control its internal dynamics, one can implement an arbitrary POVM. Therefore, one can naturally extend the von Neumann measurement model to an arbitrary generalized measurement. Note, that the standard approach to POVM assumes that von Neumann measurements are performed on an extended system; i.e., the Hilbert space of the system has to be enlarged. Here, we show that in principle the extension of the Hilbert space is not needed. It is natural to ask if this is also true in case if the measured system is more than two-dimensional. Another interesting question is, how much control over the evolution of the measured system is needed in order to implement an arbitrary POVM. It would be also interesting to experimentally verify our results. One of the main problems of experimental setups testing quantum mechanics is how they scale with the dimensionality of the system that is tested. However, recent quantum walk implementations use new methods that allow for an efficient scaling of the experimental setup (see, for example, [27]); hence, these setups would be natural candidates for an implementation of our algorithm.

We acknowledge stimulating discussions with Ravishankar Ramanathan. This work is supported by the National Research Foundation and Ministry of Education in Singapore. P. K. is also supported by the Foundation for Polish Science.

*cqtpkk@nus.edu.sg

[1] Y. Aharonov, L. Davidovich, and N. Zagury, Phys. Rev. A **48**, 1687 (1993).
[2] J. Kempe, Contemp. Phys. **44**, 307 (2003).
[3] S. E. Venegas-Andraca, Quantum Inf. Process. **11**, 1015 (2012).
[4] I. Carneiro, M. Loo, X. Xu, M. Girerd, V. Kendon, and P. L Knight, New J. Phys. **7**, 156 (2005).
[5] G. Abal, R. Siri, A. Romanelli, and R. Donangelo, Phys. Rev. A **73**, 042302 (2006).
[6] G. Abal, R. Donangelo, and H. Fort, *Annals of the 1st Workshop on Quantum Computation and Information* (Universidade Católica de Pelotas, Pelotas, RS, 2006), p. 189.
[7] O. Maloyer and V. Kendon, New J. Phys. **9**, 87 (2007).
[8] A. Romanelli, Phys. Rev. A **81**, 062349 (2010).
[9] Y. Ide, N. Konno, and T. Machida, Quantum Inf. Comput. **11**, 855 (2011).

[10] P. P. Rohde, A. Fedrizzi, and T. C. Ralph, J. Mod. Opt. **59**, 710 (2012).
[11] S. Salimi and R. Yosefjani, Int. J. Mod. Phys. B **26**, 1250112 (2012).
[12] S. M. Barnett and S. Croke, Adv. Opt. Photonics **1**, 238 (2009).
[13] M. E. Deconinck and B. M. Terhal, Phys. Rev. A **81**, 062304 (2010).
[14] G. Zauner, Ph.D. thesis, Universitat Wien, 1999.
[15] M. Renes, R. Blume-Kohout, A. J. Scott, and C. M. Caves, J. Math. Phys. (N.Y.) **45**, 2171 (2004).
[16] R. B. M. Clarke, A. Chefles, S. M. Barnett, and E. Riis, Phys. Rev. A **63**, 040305(R) (2001).
[17] P. J. Mosley, S. Croke, I. A. Walmsley, and S. M. Barnett, Phys. Rev. Lett. **97**, 193601 (2006).
[18] H. Yang, W. Wei, W. Chun-Wang, D. Hong-Yi, and L. Cheng-Zu, Chin. Phys. Lett. **25**, 4195 (2008).
[19] A. C. Dada, E. Andersson, M. L. Jones, V. M. Kendon, and M. S. Everitt, Phys. Rev. A **83**, 042339 (2011).
[20] G. Waldherr, A. C. Dada, P. Neumann, F. Jelezko, E. Andersson, and J. Wrachtrup, Phys. Rev. Lett. **109**, 180501 (2012).
[21] A. Ambainis, Int. J. Quantum. Inform. **01**, 507 (2003).
[22] A. M. Childs, Phys. Rev. Lett. **102**, 180501 (2009).
[23] N. B. Lovett, S. Cooper, M. Everitt, M. Trevers, and V. Kendon, Phys. Rev. A **81**, 042330 (2010).
[24] C. A. Ryan, M. Laforest, J. C. Boileau, and R. Laflamme, Phys. Rev. A **72**, 062317 (2005).
[25] M. Karski, L. Forster, J.-M. Choi, A. Steffen, W. Alt, D. Meschede, and A. Widera, Science **325**, 174 (2009).
[26] D. Bouwmeester, I. Marzoli, G. P. Karman, W. Schleich, and J. P. Woerdman, Phys. Rev. A **61**, 013410 (1999).
[27] A. Schreiber, K. N. Cassemiro, V. Potocek, A. Gabris, P. J. Mosley, E. Andersson, I. Jex, and C. Silberhorn, Phys. Rev. Lett. **104**, 050502 (2010).
[28] H. B. Perets, Y. Lahini, F. Pozzi, M. Sorel, R. Morandotti, and Y. Silberberg, Phys. Rev. Lett. **100**, 170506 (2008).
[29] A. Peruzzo *et al.*, Science **329**, 1500 (2010).
[30] T. Schaetz, Ch. Schneider, M. Enderlein, T. Huber, and R. Matjeschk, ChemPhysChem **12**, 71 (2011).
[31] M. A. Broome, A. Fedrizzi, B. P. Lanyon, I. Kassal, A. Aspuru-Guzik, and A. G. White, Phys. Rev. Lett. **104**, 153602 (2010).
[32] H. Schmitz, R. Matjeschk, C. Schneider, J. Glueckert, M. Enderlein, T. Huber, and T. Schaetz, Phys. Rev. Lett. **103**, 090504 (2009).
[33] F. Zahringer, G. Kirchmair, R. Gerritsma, E. Solano, R. Blatt, and C. F. Roos, Phys. Rev. Lett. **104**, 100503 (2010).
[34] L. Sansoni, F. Sciarrino, G. Vallone, P. Mataloni, A. Crespi, R. Ramponi, and R. Osellame, Phys. Rev. Lett. **108**, 010502 (2012).
[35] A. Crespi, R. Osellame, R. Ramponi, V. Giovannetti, R. Fazio, L. Sansoni, F. De Nicola, F. Sciarrino, and P. Mataloni, Nat. Photonics **7**, 322 (2013)
[36] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.110.200404 for detailed proof of universality of the quantum walk algorithm for generation of an arbitrary single qubit POVM; an example of a quantum walk implementation of a single qubit SIC-POVM.
[37] Y. Aharonov and D. Rohrlich, *Quantum Paradoxes* (Wiley-VCH, Weinheim, 2005), p. 95.