Dynamical Critical Phenomena in Driven-Dissipative Systems

L. M. Sieberer,[1,2] S. D. Huber,[3,4] E. Altman,[4,5] and S. Diehl[1,2]

[1]*Institute for Theoretical Physics, University of Innsbruck, A-6020 Innsbruck, Austria*
[2]*Institute for Quantum Optics and Quantum Information of the Austrian Academy of Sciences, A-6020 Innsbruck, Austria*
[3]*Theoretische Physik, Wolfgang-Pauli-Strasse 27, ETH Zurich, CH-8093 Zurich, Switzerland*
[4]*Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot 76100, Israel*
[5]*Department of Physics, University of California, Berkeley, California 94720, USA*
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We explore the nature of the Bose condensation transition in driven open quantum systems, such as exciton-polariton condensates. Using a functional renormalization group approach formulated in the Keldysh framework, we characterize the dynamical critical behavior that governs decoherence and an effective thermalization of the low frequency dynamics. We identify a critical exponent special to the driven system, showing that it defines a new dynamical universality class. Hence critical points in driven systems lie beyond the standard classification of equilibrium dynamical phase transitions. We show how the new critical exponent can be probed in experiments with driven cold atomic systems and exciton-polariton condensates.

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Recent years have seen major advances in the exploration of many-body systems in which matter is strongly coupled to light [1]. Such systems include for example polariton condensates [2], superconducting circuits coupled to microwave resonators [3,4], cavity quantum electrodynamics [5], as well as ultracold atoms coupled to high finesse optical cavities [6]. As in traditional quantum optics settings, these experiments are subject to losses, which may be compensated by continuous drive, yet they retain the many-body character of condensed matter. This combination of ingredients from atomic physics and quantum optics in a many-body context defines a qualitatively new class of quantum matter far from thermal equilibrium. An intriguing question from the theoretical perspective is what new universal behavior can emerge under such conditions.

A case in point is exciton-polariton condensates. Polaritons are short lived optical excitations in semiconductor quantum wells. Continuous pumping is required to maintain their population in steady state. But in spite of the nonequilibrium conditions, experiments have demonstrated Bose condensation [2] and, more recently, have even observed the establishment of a critical phase with power-law correlations in a two dimensional system below a presumed Kosterlitz-Thouless phase transition [7]. At a fundamental level, however, there is no understanding of the condensation transition in the presence of loss and external drive, and more generally of continuous phase transitions under such conditions.

In this Letter we develop a theory of dynamical critical phenomena in driven-dissipative systems in three dimensions. Motivated by the experiments described above we focus on the case of Bose condensation with the following key results. (i) Low-frequency thermalization: the microscopic dynamics of a driven system is incompatible with an equilibriumlike Gibbs distribution at steady state. Nevertheless a scale independent effective temperature emerges at low frequencies in the universal regime near the critical point, and all correlations in this regime obey a classical fluctuation-dissipation relation (FDR). Such a phenomenon of low frequency effective equilibrium has been identified previously in different contexts [8–13]. (ii) Universal low-frequency decoherence: in spite of the effective thermalization, the critical dynamics is significantly affected by the nonequilibrium conditions set by the microscopic theory. Specifically we show that all coherent dynamics, as measured by standard response functions, fades out at long wavelengths as a power law with a new universal critical exponent. The decoherence exponent cannot be mimicked by any equilibrium model and places the critical dynamics of a driven system in a new dynamical universality class beyond the Halperin-Hohenberg classification of equilibrium dynamical critical behavior [14].

Open system dynamics.—A microscopic description of driven open systems typically starts from a Markovian quantum master equation or an equivalent Keldysh action (see the Supplemental Material [15]). However, the novel aspects in the critical dynamics of driven dissipative systems discussed below can be most simply illustrated by considering an effective mesoscopic description of the order parameter dynamics using a stochastic Gross-Pitaevskii equation [16]

$$i\partial_t \psi = [-(A - iD)\nabla^2 - \mu + i\chi + (\lambda - i\kappa)|\psi|^2]\psi + \zeta. \tag{1}$$

As we show below, this equation can be rigorously derived from a fully quantum microscopic description of the condensate when including only the relevant terms near the critical point. The different terms in Eq. (1) have a clear physical origin. $\chi = (\gamma_p - \gamma_l)/2$ is the effective gain, which combines the incoherent pump field minus the local single-particle loss terms. κ, $\lambda > 0$ are, respectively, two-body loss and interaction parameters. The diffusion term D is not contained in the original microscopic model, and is not crucial to describe most nonuniversal aspects of, e.g., exciton-polariton condensates [17] (but see Ref. [18]). In a systematic treatment of long-wavelength universal critical behavior, however, such a term is generated upon integrating out high frequency modes during the renormalization group (RG) flow, irrespective of its microscopic value. We therefore include it at the mesoscopic level with a phenomenological coefficient. Finally ζ is a Gaussian white noise with correlations $\langle \zeta^*(t, \mathbf{x})\zeta(t', \mathbf{x}')\rangle = \gamma\delta(t - t')\delta(\mathbf{x} - \mathbf{x}')$ where $\gamma = \gamma_p + \gamma_l$. Such noise is necessarily induced by the losses and sudden appearances of particles due to pumping.

The stochastic Gross-Pitaevskii equation describes a mean field transition from a stationary condensate solution with density $|\psi|^2 = \chi/\kappa$ for $\chi > 0$ to the vacuum state when χ crosses zero. Dynamical stability [19] determines the chemical potential as $\mu = \lambda|\psi|^2$. Similar to a temperature, the noise term in Eq. (1) can drive a transition at finite particle density, thereby inducing critical fluctuations.

As the equation of motion is cast in Langevin form, one might suspect that it can be categorized into one of the well-known models of dynamical critical phenomena classified by Hohenberg and Halperin [14]. However, this is not true in general. Crucially coherent [real parts of the couplings in Eq. (1)] and dissipative (imaginary parts) dynamics have different physical origins in driven-dissipative systems. In particular, the dissipative dynamics is determined by the intensity of the pump and loss terms, independently of the intrinsic Hamiltonian dynamics of the system. Equilibrium models [14], on the other hand, are constrained to have a specific relation between the reversible and dissipative terms to ensure a thermal Gibbs ensemble in steady state [20,21] (see below). The unconstrained dynamics in driven systems is the key feature that can lead to novel dynamic critical behavior.

Microscopic model.—Having illustrated the nature of the problem with the effective classical Eq. (1) we turn to a fully quantum description within the Keldysh framework. Our starting point is a nonunitary quantum evolution described by a many-body master equation in Lindblad form, or equivalently by the following dissipative Keldysh action (see the Supplemental Material [15] for details of the correspondence):

$$\mathcal{S} = \int_{t,\mathbf{x}} \Big\{ (\phi_c^*, \phi_q^*) \begin{pmatrix} 0 & P^A \\ P^R & P^K \end{pmatrix} \begin{pmatrix} \phi_c \\ \phi_q \end{pmatrix} + i4\kappa \phi_c^* \phi_c \phi_q^* \phi_q - [(\lambda + i\kappa)(\phi_c^{*2}\phi_c\phi_q + \phi_q^{*2}\phi_c\phi_q) + \text{c.c.}] \Big\}. \quad (2)$$

Here ϕ_c, ϕ_q are the "classical" and "quantum" fields, defined by the symmetric and antisymmetric combinations of the fields on the forward and backward parts of the Keldysh contour [22,23]. The microscopic inverse Green's functions are given by $P^R = i\partial_t + A\nabla^2 + \mu - i\chi$, $P^A = P^{R\dagger}$, $P^K = i\gamma$.

The importance of the various terms in the microscopic action [Eq. (2)] in the vicinity of the critical point can be inferred from canonical power counting, which serves as a valuable guideline for the explicit evaluation of the problem. Vanishing of the mass scale χ defines a Gaussian fixed point with dynamical critical exponent $z = 2$ ($\omega \sim k^z$, where k is a momentum scale). Canonical power counting determines the scaling dimensions of the fields and interaction constants with respect to this fixed point: At criticality, the spectral components of the Gaussian action scale as $P^{R/A} \sim k^2$, while the Keldysh component generically takes a constant value, i.e., $P^K \sim k^0$. Hence, to maintain scale invariance of the quadratic action, the scaling dimensions of the fields must be $[\phi_c] = \frac{d-2}{2}$ and $[\phi_q] = \frac{d+2}{2}$. From this result we read off the canonical scaling dimensions of the interaction constants. This analysis shows that in the case of interest $d = 3$, local vertices containing more than two quantum fields or more than five classical fields are irrelevant. For the critical problem, the last terms in both lines of Eq. (2) can thus be skipped, massively simplifying the complexity of the problem. The only marginal term with two quantum fields is the Keldysh component of the single-particle inverse Green's function, i.e., the noise vertex. In this sense, the critical theory is equivalent to a stochastic classical problem [24,25], as previously observed in Refs. [8,26]. But as noted above it cannot be *a priori* categorized in one of the dynamical universality classes [14] subject to an intrinsic equilibrium constraint.

Functional RG.—In order to focus quantitatively on the critical behavior we use a functional RG approach formulated originally by Wetterich [27] and adapted to the Keldysh real time framework in Refs. [28,29] (see the Supplemental Material [15] for details). At the formal level this technique provides an exact functional flow equation for an effective action functional $\Gamma_\Lambda[\phi_c, \phi_q]$, which includes information on increasingly long-wavelength fluctuations (at the microscopic cutoff scale $\Gamma_{\Lambda_0} \approx \mathcal{S}$). In practice one works with an ansatz for the effective action and thereby projects the functional flow onto scaling equations for a finite set of coupling constants. For the description of general equilibrium [30–35] and Ising dynamical [36] critical behavior the functional RG gave results that are competitive with high-order epsilon expansion and with Monte Carlo simulations already in rather simple approximation schemes.

Our ansatz for the effective action is motivated by the power counting arguments introduced above. We include in Γ_Λ all couplings that are relevant or marginal in this scheme,

$$\Gamma_\Lambda = \int_{t,\mathbf{x}} \Big\{ (\phi_c^*, \phi_q^*) \begin{pmatrix} 0 & iZ\partial_t + \bar{K}\nabla^2 \\ iZ^*\partial_t + \bar{K}^*\nabla^2 & i\bar{\gamma} \end{pmatrix} \times \begin{pmatrix} \phi_c \\ \phi_q \end{pmatrix} - \Big(\frac{\partial \bar{U}}{\partial \phi_c}\phi_q + \frac{\partial \bar{U}^*}{\partial \phi_c^*}\phi_q^* \Big) \Big\}. \quad (3)$$

The dynamical couplings Z and $\bar{K}$ have to be taken complex valued in order to be consistent with power counting, even if the respective imaginary parts vanish (or are very small) at the microscopic scale: Successive momentum mode elimination implemented by the RG flow generates these terms due to the simultaneous presence of local coherent and dissipative couplings in the microscopic model. The fact that the spectral components of the effective action depend only linearly on ϕ_q allowed us to introduce an effective potential $\bar{U}$ determined by the complex static couplings. $\bar{U}(\rho_c) = 1/2\bar{u}(\rho_c - \rho_0)^2 + 1/6\bar{u}'(\rho_c - \rho_0)^3$ is a function of the $U(1)$ invariant combination of classical fields $\rho_c = \phi_c^*\phi_c$ alone. It has a Mexican hat structure ensuring dynamical stability. With this choice we approach the transition from the ordered side, taking the limit of the stationary state condensate $\rho_0 = \phi_c^*\phi_c|_{\rm ss} = \phi_0^*\phi_0 \to 0$.

All the parameters appearing in Eq. (3) including the stationary condensate density ρ_0 are functions of the running cutoff Λ. Hence, the functional flow of Γ_Λ is reduced by means of the approximate ansatz to the flow of a finite number of couplings $\mathbf{g} = (Z, \bar{K}, \rho_0, \bar{u}, \bar{u}', \bar{\gamma})^T$ determined by the β functions $\Lambda\partial_\Lambda \mathbf{g} = \beta_{\mathbf{g}}(\mathbf{g})$ (see the Supplemental Material [15]). The critical system is described by a scaling solution to these flow equations. It is obtained as a fixed point of the flow of dimensionless renormalized couplings, which we derive in the following. First we rescale the couplings with Z,

$$K = \bar{K}/Z, \quad u = \bar{u}/Z, \quad u' = \bar{u}'/Z, \quad \gamma = \bar{\gamma}/|Z|^2. \quad (4)$$

Coherent and dissipative processes are encoded, respectively, in the real and imaginary parts of the renormalized coefficients $K = A + iD$, $u = \lambda + i\kappa$, and $u' = \lambda' + i\kappa'$.

We define the first three dimensionless scaling variables to be the ratios of coherent to dissipative coefficients: $r_K = A/D$, $r_u = \lambda/\kappa$, and $r_{u'} = \lambda'/\kappa'$. Another three dimensionless variables are defined by rescaling the loss coefficients κ and κ' and the condensate density ρ_0,

$$w = \frac{2\kappa\rho_0}{\Lambda^2 D}, \qquad \tilde{\kappa} = \frac{\gamma\kappa}{2\Lambda D^2}, \qquad \tilde{\kappa}' = \frac{\gamma^2\kappa'}{4D^3}. \quad (5)$$

The flow equations for the couplings $\mathbf{r} = (r_K, r_u, r_{u'})^T$ and $\mathbf{s} = (w, \tilde{\kappa}, \tilde{\kappa}')^T$ form a closed set,

$$\Lambda\partial_\Lambda \mathbf{r} = \beta_{\mathbf{r}}(\mathbf{r}, \mathbf{s}), \qquad \Lambda\partial_\Lambda \mathbf{s} = \beta_{\mathbf{s}}(\mathbf{r}, \mathbf{s}) \quad (6)$$

(see the Supplemental Material [15] for the explicit form). As a consequence of the transformations of Eqs. (4) and (5), these β functions acquire a contribution from the running anomalous dimensions $\eta_a(\mathbf{r}, \mathbf{s}) = -\Lambda\partial_\Lambda \ln a$ associated with $a = Z, D, \gamma$.

Critical properties.—The universal behavior near the critical point is controlled by the infrared flow to a Wilson-Fisher-like fixed point. The values of the coupling constants at the fixed point, determined by solving $\beta_{\mathbf{s}}(\mathbf{r}_*, \mathbf{s}_*) = 0$ and $\beta_{\mathbf{r}}(\mathbf{r}_*, \mathbf{s}_*) = 0$, are given by

$$\mathbf{r}_* = (r_{K*}, r_{u*}, r_{u'*}) = \mathbf{0},$$
$$\mathbf{s}_* = (w_*, \tilde{\kappa}_*, \tilde{\kappa}'_*) \approx (0.475, 5.308, 51.383). \quad (7)$$

The fact that $\mathbf{r}_* = 0$ implies that the fixed point action is purely imaginary (or dissipative), as in model A of Hohenberg and Halperin [14] [cf. Fig. 1(c)]. We interpret the fact that the ratios of coherent vs dissipative couplings are zero at the fixed point as a manifestation of decoherence at low frequencies in an RG framework. The coupling values $\mathbf{s}_*$ are identical to those obtained in an equilibrium classical $O(2)$ model from functional RG calculations at the same level of truncation [30].

Let us turn to the linearized flow, which determines the universal behavior in the vicinity of the fixed point. We find that the two sectors corresponding to $\mathbf{s}$ and $\mathbf{r}$ decouple in this regime, giving rise to a block diagonal stability matrix,

$$\frac{\partial}{\partial \ln\Lambda} \begin{pmatrix} \delta\mathbf{r} \\ \delta\mathbf{s} \end{pmatrix} = \begin{pmatrix} N & 0 \\ 0 & S \end{pmatrix} \begin{pmatrix} \delta\mathbf{r} \\ \delta\mathbf{s} \end{pmatrix}, \quad (8)$$

where $\delta\mathbf{r} \equiv \mathbf{r}$, $\delta\mathbf{s} \equiv \mathbf{s} - \mathbf{s}_*$, and N, S are 3×3 matrices (see the Supplemental Material [15]).

The anomalous dimensions entering this flow are found by plugging the fixed point values $\mathbf{r}_*$, $\mathbf{s}_*$ into the expressions for $\eta_a(\mathbf{r}, \mathbf{s})$. We obtain the scaling relation between

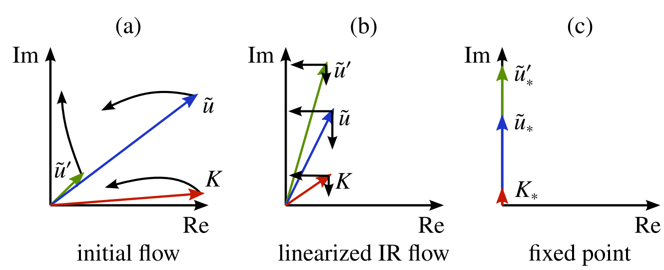

FIG. 1 (color online). Flow in the complex plane of dimensionless renormalized couplings. (a) The microscopic action determines the initial values of the flow. Typically, the coherent propagation will dominate over the diffusion $A \gg D$, while two-body collisions and two-body loss are on the same order of magnitude $\tilde{\lambda} \approx \tilde{\kappa}$, with a similar relation for the marginal complex coupling $\tilde{u}'$. The initial flow is nonuniversal. (b) At criticality, the infrared (IR) flow approaches a universal linear domain encoding the critical exponents and anomalous dimensions. In particular, this regime is independent of the precise microscopic initial conditions. (c) The Wilson-Fisher fixed point describing the interacting critical system is purely imaginary.

the anomalous dimensions $\eta_Z = \eta_{\bar{\gamma}}$, valid in the universal infrared regime. This leads to cancellation of η_Z with $\eta_{\bar{\gamma}}$ in the static sector S (see the Supplemental Material [15]). The critical properties in this sector, encoded in the eigenvalues of S, become identical to those of the standard $O(2)$ transition. This includes the correlation length exponent $\nu \approx 0.716$ and the anomalous dimension $\eta \approx 0.039$ associated with the bare kinetic coefficient $\bar{K}$. These values are in good agreement with more sophisticated approximations [37].

The equilibriumlike behavior in the S sector can be seen as a result of an emergent symmetry. Locking of the noise to the dynamical term implied by $\eta_Z = \eta_{\bar{\gamma}}$ leads to invariance of the long-wavelength effective action (times i) under the transformation $\Phi_c(t, \mathbf{x}) \rightarrow \Phi_c(-t, \mathbf{x})$, $\Phi_q(t, \mathbf{x}) \rightarrow \Phi_q(-t, \mathbf{x}) + \frac{2}{\gamma}\sigma^z \partial_t \Phi_c(-t, \mathbf{x})$, $i \rightarrow -i$ with $\Phi_\nu = (\phi_\nu, \phi_\nu^*)^T$, $\nu = (c, q)$, and the Pauli matrix σ^z. It generalizes the symmetry noted in Refs. [38,39] to models that include also reversible couplings. The presence of this symmetry implies a classical FDR with a distribution function $F = 2T_{\mathrm{eff}}/\omega$, governed by an effective temperature $T_{\mathrm{eff}} = \bar{\gamma}/(4|Z|)$. This quantity becomes scale independent in the universal critical regime where $\bar{\gamma} \sim k^{-\eta_{\bar{\gamma}}}$ and $Z \sim k^{-\eta_Z}$ cancel. We interpret this finding as an asymptotic low-frequency thermalization mechanism of the driven system at criticality. The thermalized regime sets in below the Ginzburg scale where fluctuations start to dominate, for which we estimate perturbatively $\chi_G = (\gamma\kappa)^2/(16\pi^2 D^3)$ (see the Supplemental Material [15]). The values entering here are determined on the mesoscopic scale, and we specify them for exciton-polariton systems in the Supplemental Material [15] based on Ref. [18]. Above the scale χ_G, no global (scale independent) temperature can be defined in general. We note that, unlike Hohenberg-Halperin type models, here the symmetry implied by $\eta_Z = \eta_{\bar{\gamma}}$ is not imposed at the microscopic level of the theory, but rather is emergent at the critical point.

The key new element in the driven-dissipative dynamics is encoded in the decoupled "drive" sector (the 3×3 matrix N in our case). It describes the flow towards the emergent purely dissipative model A fixed point [see Fig. 1(b)] and thus reflects a mechanism of low frequency decoherence. This sector has no counterpart in the standard framework of dynamical critical phenomena and is special to driven-dissipative systems. In the deep infrared regime, only the lowest eigenvalue of this matrix governs the flow of the ratios. This means that only one new critical exponent $\eta_r \approx -0.101$ is encoded in this sector. Just as the dynamical critical exponent z is independent of the static ones, the block diagonal structure of the stability matrix ensures that the drive exponent is independent of the exponents of the other sectors.

The fact that the inverse Green's function in Eq. (3) is specified by three real parameters, $\mathrm{Re}\bar{K}$, $\mathrm{Im}\bar{K}$, and $|Z|$ [the phase of Z can be absorbed by a $U(1)$ transformation] allows for only three independent anomalous dimensions: η_D, η_Z, and the new exponent η_r. Hence the extension of critical dynamics described here is maximal; i.e., no further independent exponent will be found. Moreover this extension of the purely relaxational (model A) dynamics leads to a different universality than an extension that adds reversible couplings compatible with relaxation toward a Gibbs ensemble. The latter is obtained by adding real couplings to the imaginary ones with the same ratio of real to imaginary parts for all couplings [40–43]; in this case, the above symmetry is present, while absent in the general nonequilibrium case. The compatible extension adds only an independent 1×1 sector N to the purely relaxational problem, for which we find $\eta_R = -0.143 \neq \eta_r$. This proves that the independence of dissipative and coherent dynamics defines indeed a new nonequilibrium universality class with no equilibrium counterpart. It is rooted in different symmetry properties of the equilibrium vs nonequilibrium situation.

Experimental detection.—The novel anomalous dimension identified here leaves a clear fingerprint in single-particle observables accessible with current experimental technologies on different platforms. For ultracold atomic systems this can be achieved via rf spectroscopy [44] close to the driven-dissipative Bose-Einstein condensation transition. In exciton-polariton condensates, the dispersion relation can be obtained from angle resolved rf spectroscopy as demonstrated in Ref. [45]. Using the RG scaling behavior of the diffusion and propagation coefficients $D \sim D_0\Lambda^{-\eta_D}$, $A = Dr_K \sim A_0\Lambda^{-\eta_r - \eta_D}$, we obtain the anomalous scaling of the frequency and momentum resolved, renormalized retarded Green's function $G^R(\omega, \mathbf{q}) = (\omega - A_0|\mathbf{q}|^{2-\eta_r-\eta_D} + iD_0|\mathbf{q}|^{2-\eta_D})^{-1}$, with A_0 and D_0 nonuniversal constants. Peak position and width are implied by the complex dispersion $\omega \approx A_0|\mathbf{q}|^{2.22} - iD_0|\mathbf{q}|^{2.12}$. The energy resolution necessary to probe the critical behavior is again set by the Ginzburg scale χ_G (see above).

Conclusions.—We have developed a Keldysh field theoretical approach to characterize the critical behavior of driven-dissipative three dimensional Bose systems at the condensation transition. The main result presents a hierarchical extension of classical critical phenomena. First, all static aspects are identical to the classical $O(2)$ critical point. In the next shell of the hierarchy a subclass of the dynamical phenomena is identical to the purely dissipative model A dynamics of the equilibrium critical point. Finally we identify manifestly nonequilibrium features of the critical dynamics, encoded in a new independent critical exponent that betrays the driven nature of the system.

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