Diametrically Driven Self-Accelerating Pulses in a Photonic Crystal Fiber

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We predict the existence of a new class of self-accelerating, exponentially localized pulses consisting of two interacting frequency components propagating at opposite group velocity dispersion. Compared to previous approaches no external force is required and accelerations of both signs can be realized. This seemingly paradoxical behavior resembles an all optical wave realization of a classical diametric drive, where a continuously propulsive effect is achieved by a combination of two fields having effective masses of opposite sign.

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Self-accelerating entities are fascinating, but also irritating, because we expect the center of mass of a closed system to propagate at constant velocity. Any other behavior violates energy and momentum conservation. In fact when self-acceleration was described for the first time for charged particles interacting with their own electromagnetic field and being subject to the Abraham-Lorentz force those runaway solutions were regarded as mathematical artifacts [1]. However, Berry and Balazs [2] theoretically showed that even the free, linear Schrödinger equation exhibits self-accelerating solutions. This surprising finding was recently used in optics to create self-bending so-called Airy beams [3–6]. But, these nondiffracting beams are not exponentially localized. Their mean intensity decays only as the inverse square root of the distance. Hence, they require an infinite amount of energy to exist and neither a center of mass nor a total momentum can be defined.

Nonlinearity is required to create truly accelerated exponentially localized field distributions. The nonlinear Schrödinger equation exhibits localized soliton solutions but, as it is momentum conserving, an acceleration either requires an external perturbation or the emission of some radiation [7–9]. In contrast a non-Hamiltonian perturbation as the Raman effect in optical fibers can cause true acceleration by inducing a redshift, which is transferred into a velocity shift of pulses by group velocity dispersion [10]. Recently Gorbach and Skryabin showed that such a strong Raman accelerated pulse can even track and accelerate weak dispersive waves propagating at normal dispersion via cross-phase modulation [11,12]. Thus the weak dispersive wave is blueshifted. This effect, first observed in Ref. [13], contributes profoundly to supercontinuum generation in photonic crystal fibers [12]. In this Letter we take this idea one pivotal step further and consider a scenario where even without the Raman effect a uniform self-acceleration of arbitrary sign is achieved, although the momentum and energy (Hamiltonian) are finite and remain conserved.

It is known that in a classical two body system the introduction of hypothetical negative masses has such an irritating effect [14]. There one considers two gravitational interacting particles, where one has a positive mass m_A, while the other one has a negative mass m_B. Then, the gravity of the positive mass m_A will attract the negative mass m_B while the negative one is repelled. If properly arranged, one could construct a pair of masses moving with constant pitch and uniform acceleration. This so-called diametric drive was originally proposed as the ultimate propulsion for space drives, but could up to now not be realized because of the obvious lack of particles with negative mass [15]. The total momentum of such a self-accelerating pair

$$M = m_A v_A - |m_B| v_B \tag{1}$$

can indeed be conserved, even if both particles accelerate into the same direction. Hence, a constant pitch requires $m_A = -m_B$. Such a condition is obviously odd for mechanical systems. However, in solid state physics particles with negative masses are well known as this requires only a negative curvature of the respective band structure. The same can happen in photonic crystal fibers, where the dispersion relation can have positive or negative curvature, resulting in normal or anomalous group velocity dispersion. To translate the scenario of self-accelerating masses to nonlinear fiber optics we consider two optical pulses A and B with different frequencies copropagating in a photonic crystal fiber and interacting via the cubic Kerr nonlinearity. The optical field A has a frequency ω_A in the anomalous group velocity dispersion (GVD) regime, i.e., $\beta_2(\omega_A) < 0$, which corresponds to an effectively positive mass. In the presence of a positive Kerr nonlinearity it forms a Schrödinger soliton, which is known to behave like a robust and particlelike object even in the presence of perturbations. Negative mass is achieved by choosing the frequency ω_B of pulse B in the normal dispersion regime, i.e., $\beta_2(\omega_B) > 0$ (see Fig. 1). Both fields are coupled by cross-phase modulation, which resembles an analog of the gravitational interaction in the classical two body case.

We start with the simplest model for the envelope of two optical pulses propagating under the influence of

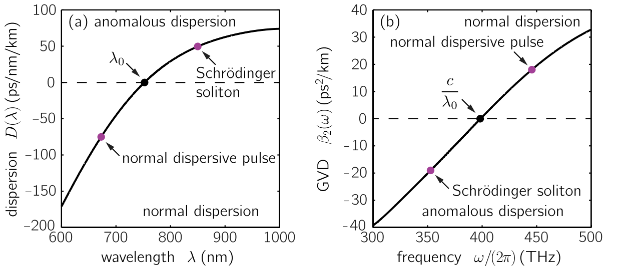

FIG. 1 (color online). Panel (a) shows the dispersion curve of a typical commercial photonic crystal fiber (NL-PM-750), with $\gamma_{A,B} \approx 0.1\ (\mathrm{Wm})^{-1}$ and a zero group velocity point at $\lambda_0 \approx 750$ nm. Panel (b) shows the corresponding GVD $\beta_2(\omega)$. The Schrödinger soliton is launched around $\lambda_A \approx 850$ nm in the anomalous dispersion regime. For the normal dispersive pulse we choose the group velocity matched wavelength, at $\lambda_B \approx 673$ nm. This leads to $\beta_A \approx -20\ \mathrm{ps}^2/\mathrm{km}$ and $\beta_B \approx 20\ \mathrm{ps}^2/\mathrm{km}$.

cross-phase modulation in an optical fiber with a frequency far away from the zero dispersion point [16]:

$$i\frac{\partial A}{\partial z} = \frac{\beta_A}{2}\frac{\partial^2 A}{\partial \tau^2} - \gamma_A(|A|^2 + 2|B|^2)A,$$
$$i\frac{\partial B}{\partial z} = \frac{\beta_B}{2}\frac{\partial^2 B}{\partial \tau^2} - \gamma_B(|B|^2 + 2|A|^2)B. \quad (2)$$

Here z is the propagation distance along the fiber, τ is the time in the retarded frame, $\beta_{A,B} = \beta_2(\omega_{A,B})$ are the GVD values, and $\gamma_{A,B} = \gamma(\omega_{A,B})$ are the nonlinear coefficients at the center frequencies of the pulses. For now we exclude all higher order effects as Raman scattering or third order dispersion, and assume that the frequencies of the pulses are chosen such that their group velocities are matched (see Fig. 1).

If we compare Eqs. (2) with the Schrödinger equations for massive particles, we find that they resemble exactly the scenario of interacting particles with positive ($\beta_A < 0$) and negative ($\beta_B > 0$) mass. Moreover we see that A is attracted by the normal dispersive pulse B which itself is repelled by A. Hence, A pushes B whereas B pulls A and thus all classical ingredients for a diametric drive are present.

To find self-accelerating solutions of Eqs. (2) with constant pitch we switch to a uniformly accelerated frame by means of the coordinate transformation $s = \tau - gz^2/2$ with acceleration g and apply the Gagnon-Bélanger phase transformation $A(\tau, z) = A_0(\tau - gz^2/2)\exp[-igz(\tau - gz^2/6)/\beta_A + i\mu z]$ and $B(\tau, z) = B_0(\tau - gz^2/2)\exp[-igz(\tau - gz^2/6)/\beta_B + i\nu z]$ of the optical fields [17]. Here μ and ν are the propagation constants of the soliton and of the normal dispersive pulse, respectively. Every stationary solution $A_0(s)$, $B_0(s)$ in the new frame correlates to a uniformly accelerated solution in the lab frame. An acceleration $g > 0$ corresponds to a growing redshift of the Schrödinger soliton and a permanently increasing blueshift of the normal dispersive pulse, a situation quite similar to the Raman effect [11,12]. On the other hand, for $g < 0$ the Schrödinger soliton is blueshifted and the normal dispersive pulse is redshifted, which has no Raman analog.

Localized pulses, which are stationary in the uniformly accelerated frame, are determined by

$$-\mu A_0 = \frac{\beta_A}{2}\frac{\partial^2 A_0}{\partial s^2} - \gamma_A(|A_0|^2 + 2|B_0|^2)A_0 - \frac{gs}{\beta_A}A_0,$$
$$-\nu B_0 = \frac{\beta_B}{2}\frac{\partial^2 B_0}{\partial s^2} - \gamma_B(|B_0|^2 + 2|A_0|^2)B_0 - \frac{gs}{\beta_B}B_0. \quad (3)$$

Hence, a uniformly accelerated motion is equivalent to the presence of a linearly growing potential or a constant force as postulated by Einstein's equivalence principle [18].

To derive an approximate solution of Eq. (3) we regard the pulse propagating at anomalous dispersion as a Schrödinger soliton of fixed form, i.e., $A_0(s) = \sqrt{P_A}\mathrm{sech}(s/T_A)$. Then $T_A = (|\beta_A|L_{\mathrm{NL}}^A)^{1/2}$ is the temporal width of the Schrödinger soliton with the nonlinear length scale $L_{\mathrm{NL}}^A = (\gamma_A P_A)^{-1}$ and P_A its peak power. The propagation constant is given by $\mu = (2L_{\mathrm{NL}}^A)^{-1}$. As Schrödinger solitons are known to be very robust and to behave almost like a particle we consider the additional potential $V_A(s) = -2\gamma_A|B_0(s)|^2 - gs/\beta_A$ for the Schrödinger soliton in Eq. (3) to be a small perturbation, which only exerts a force on it, but does not influence its shape [17,19]. Next we examine the second of Eqs. (3) for the normal dispersive pulse, which according to Eq. (1) should have a power comparable with that of its companion. Hence, contrary to Refs. [11,12] we consider the case where the nonlinearity of the normal dispersive pulse dominates over its dispersion, i.e., $L_{\mathrm{NL}}^B \ll L_D^B$. Here, $L_{\mathrm{NL}}^B = (\gamma_B P_B)^{-1}$ is the nonlinear length scale and $L_D^B = T_B^2/\beta_B$ the dispersion length of B with typical duration T_B and peak power P_B. In this case we can apply the Thomas-Fermi approximation to the second of Eqs. (3) thus neglecting the kinetic term (dispersion). In what follows we will use this approximation to obtain some insight into the structure of the solution and to derive some analytical estimates as, e.g., for the field shape $B_0(s) = \sqrt{[\nu - V_B(s)]/\gamma_B}$. Within this approximation it is entirely defined by the effective potential $V_B(s) = 2\gamma_B|A(s)|^2 + gs/\beta_B$ experienced by the normal dispersive pulse and by its propagation constant ν. To ensure a localized solution the latter must be chosen such that $V_B(s) \le \nu < 2\gamma_B P_A$ holds in the whole area covered by the normal dispersive pulse (see Fig. 2). For $g < 0$ ($g > 0$) the normal dispersive pulse is localized on the left-hand side (right-hand side) of the Schrödinger soliton. For a fixed power P_A we are left with two dynamical parameters: the acceleration g and the propagation constant ν, which are not independent for a diametric drive with constant pitch.

A necessary condition for the Schrödinger soliton to be adiabatically stationary in the accelerated frame is that the net force exerted on it

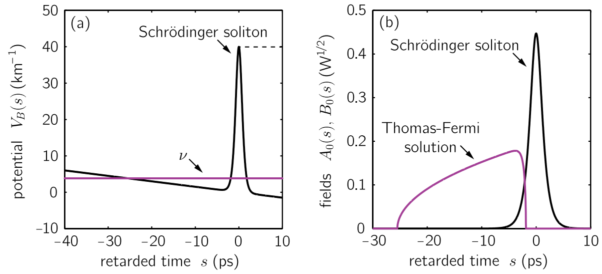

FIG. 2 (color online). Panel (a) shows the effective potential $V_B(s)$ for the normal dispersive pulse (for a numerical evaluation a potential following the dashed line on the $s > 0$ side of the soliton is used). The corresponding solution of a diametric drive is displayed in panel (b). We used $\beta_A = -20\ \mathrm{ps}^2/\mathrm{km}$, $\beta_B = 20\ \mathrm{ps}^2/\mathrm{km}$, $\gamma_{A,B} = 0.1\ (\mathrm{W\,m})^{-1}$, $P_A = 200$ mW, and $g = -3\ \mathrm{ps/km}^2$ ($\nu \approx 3.82\ \mathrm{km}^{-1}$). This results in a duration of $T_A = 1$ ps for the Schrödinger soliton. The condition for a diametric drive, here $E_A = E_B = 0.4$ pJ, is satisfied.

$$F_A^{\mathrm{acc}} = 2\gamma_A \int_{-\infty}^{\infty} |A_0(s)|^2 \frac{\partial |B_0(s)|^2}{\partial s} ds + \frac{gE_A}{\beta_A} \tag{4}$$

vanishes, where $E_A = 2P_A T_A$ is its energy. This implies a constant force $F_A = -gE_A/\beta_A$ in the laboratory frame. A similar expression $F_B = gE_B/\beta_B$ based on the energy E_B of the highly nonlinear pulse in the normal dispersive regime holds in the lab frame. Hence, for a diametric drive the backaction is crucial. Since our system is closed, we find that for all localized solutions the inverted interaction principle $F_A/\gamma_A = F_B/\gamma_B$ is valid. Therefore, the condition for a diametric drive is satisfied if

$$\frac{E_B(g,\nu)}{\gamma_B \beta_B} = -\frac{E_A}{\gamma_A \beta_A} \tag{5}$$

holds (see Fig. 2). This statement implicitly defines the propagation constant ν as a function of the acceleration g or vice versa. Hence, for fixed soliton peak power P_A a single free parameter is left thus allowing us to choose a desired acceleration within a certain range. As the above requirement depends on the pulse energies the optical diametric drive is a highly nonlinear phenomenon.

To clarify the physical meaning of the above constraint, we consider the Schrödinger soliton and the localized normal dispersive pulse as particles. The Schrödinger soliton moves with the momentary velocity $v_A(z) = \beta_A \Omega_A(z) = gz$, where $\omega_A(z) = \omega_A + \Omega_A(z)$ is the momentary frequency in the lab frame. Equivalently we have $v_B(z) = \beta_B \Omega_B(z) = gz$. Then the conserved total momentum M of Eqs. (2) is just

$$\frac{M}{\sqrt{\gamma_A \gamma_B}} = \frac{E_A}{\gamma_A |\beta_A|} v_A - \frac{E_B}{\gamma_B \beta_B} v_B. \tag{6}$$

This relates directly to the classical Eq. (1). Hence, the requirement [Eq. (5)] is the fiber optical analog of the classical constraint of equal mass to form a diametric drive. It is possible to evaluate the condition [Eq. (5)] within the Thomas-Fermi approximation and to obtain an approximate analytic expression

$$g(\nu) \approx -\frac{\beta_B \nu^2}{2\gamma_B E_A}\left(1 - \frac{\beta_B \gamma_B}{\beta_A \gamma_A} - \sqrt{1 - \frac{\nu}{2\gamma_B P_A}} + \frac{\nu}{2\gamma_B P_A} \operatorname{arctanh}\sqrt{1 - \frac{\nu}{2\gamma_B P_A}}\right)^{-1}, \tag{7}$$

for an acceleration $g < 0$. Summarizing, we found a self-accelerating pair of two nonlinear optical pulses, although their respective total momentum [Eq. (6)] is conserved. Note that the center of mass is self-accelerated and not only each individual pulse. We have to add that strictly speaking the investigated bound state is only quasistationary as the normal dispersive pulse is not completely bound by the effective potential $V_B(s)$. There is always a small leakage towards the Schrödinger soliton. Due to this tunneling loss every bound state has in principle a finite lifetime, but for any realistic propagation length it is surprisingly robust and even survives different types of perturbations.

In order to investigate the dynamical properties of the bound state we determine its shape as exactly as possible by numerically calculating the shape of the normal dispersive pulse as the bound state of the effective potential $V_B(s)$. To find smooth solutions being localized on one side of the Schrödinger soliton we artificially kept the potential at a constant value at the side of the soliton [see the dashed line in Fig. 2(a)]. Figure 3 shows that robust bound states with different accelerations can be formed, provided that Eq. (5) is satisfied. Note that the accelerations in Fig. 3 are negative, which leads to a blueshift of the Schrödinger soliton. Next we simulated an experimentally more relevant initial condition for the normal dispersive pulse,

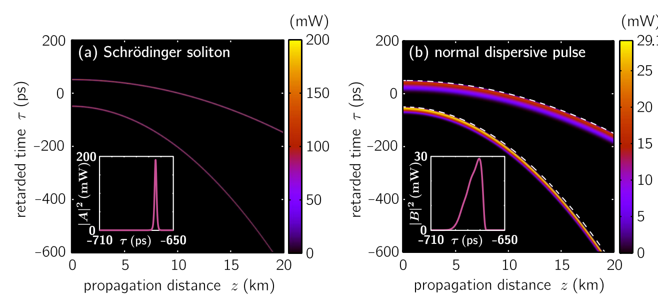

FIG. 3 (color online). Diametric drives for two different accelerations (parameters are the same as in Fig. 2). The white dashed lines indicate the accelerated paths with $g = -1\ \mathrm{ps/km}^2$ and $g = -3\ \mathrm{ps/km}^2$, respectively. Insets show the intensity profiles at $z = 20$ km in the case of $g = -3\ \mathrm{ps/km}^2$. Panel (a) shows the Schrödinger solitons and panel (b) the corresponding normal dispersive pulses.

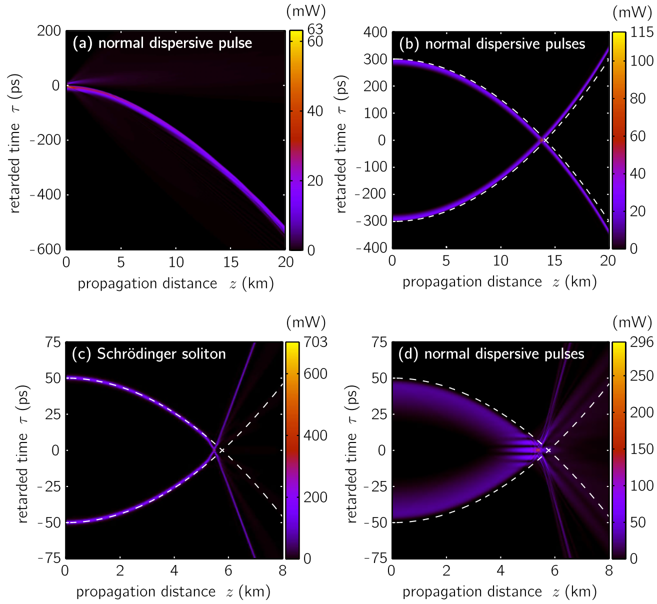

FIG. 4 (color online). Panel (a) shows the Gaussian excitation of a diametric drive with peak power $P_B = 29.41$ mW, width $T_B = 10$ ps, and delay $\Delta\tau = -6$ ps. In panels (b)–(d) we see collisions of two diametric drives. We used the same parameters as in Fig. 2.

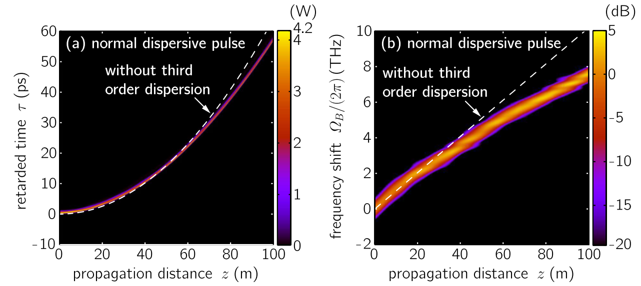

FIG. 5 (color online). Panel (a) shows the temporal dynamics of a diametric drive under the influence of the Raman effect and third order dispersion. Panel (b) shows the spectral evolution of the normal dispersive pulse. We took the same fiber parameters as in Fig. 2, now including the third order dispersion $\beta_3(\omega_{A,B}) = 0.1\ \mathrm{ps}^3/\mathrm{km}$. We used $T_A = 100$ fs for the Schrödinger soliton and $g = 12.8\ \mathrm{fs/m}^2$. The Raman response time was taken to be $T_R = 3$ fs, which leads to $g_R = 6.4\ \mathrm{fs/m}^2$. Hence, the diametric drive doubles the Raman frequency shift.

namely a Gaussian pulse of the form $B(\tau, z = 0) = \sqrt{P_B}\exp[-(\tau - \Delta\tau)^2/(2T_B^2)]$. As the bound state has still a free parameter, i.e., the acceleration, it can easily adapt to a nonperfect initial condition. A stable bound state forms, if peak power P_B, width T_B, and delay $\Delta\tau$ of the Gaussian wave packet are chosen such that they approximately match the ideal configuration displayed in Fig. 2(b). Obviously some power is initially emitted until a stable self-accelerating state is formed [see Fig. 4(a)]. The robustness of our approach is further illustrated by a collision of two bound states having opposite acceleration. Surprisingly, the diametrically driven states survive such a collision provided that their relative velocity is large enough [see Fig. 4(b)]. If their mutual interaction becomes too intense due to a small relative velocity the bound state breaks apart in the course of a violent explosion, where the two Schrödinger solitons get ejected with high speed and the normal dispersive pulse starts spreading immediately [see Figs. 4(c) and 4(d)]. As soon as the normal and anomalous dispersive waves have separated the velocities of all components stay constant, demonstrating that acceleration is a truly collective effect in the system.

For Schrödinger solitons with durations of less than 1 ps we cannot neglect the Raman effect or third order dispersion. However, even in the presence of the Raman effect we are still able to construct a diametric drive. In the framework of our mechanical model the Raman effect just adds an additional external force acting almost exclusively on the Schrödinger soliton and thus changing the requirement [Eq. (5)] to $E_B(g, \nu)/(\gamma_B\beta_B) = -E_A(1 - g_R/g)/(\gamma_A\beta_A)$, where $g_R = 8T_R\beta_A^2/(15T_A^4)$ is the Raman acceleration of a Schrödinger soliton and T_R is the Raman response time [10,16]. Note that for ultrashort solitons the Raman term is dominant and the additional acceleration due to the normal dispersive pulse becomes a small perturbation [12]. In this case the compound state can no longer be termed self-accelerating, since the main contribution originates from the dissipative Raman effect.

Considering the third order dispersion of the fiber presented in Fig. 1, we find that its main influence is to change the resulting acceleration in the same way as it does for a single Schrödinger soliton [16]. As demonstrated in Fig. 5 the diametric drive is in general very robust with respect to the influence of the Raman effect and third order dispersion.

In conclusion, we found a novel self-accelerating bound state of two nonlinear optical pulses. Such a diametric drive is robust under collisions and the influence of small perturbations like the Raman effect and third order dispersion. For pulse durations of $T_A \gtrsim 100$ fs the frequency shift induced by a diametric drive outplays the Raman shift. Moreover, with a diametric drive it is possible to red- and blueshift a normal dispersive pulse or a Schrödinger soliton in a controlled manner. Our findings suggest implications for several applications such as the generation of supercontinua in photonic crystal fibers [11,20] or tunable Raman laser sources [21].

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