P H Y S I C A L   R E V I E W   L E T T E R S

# Anomalous Reduction of the Lorenz Ratio at the Quantum Critical Point in YbAgGe

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We report measurements of the electrical and thermal transport on the hexagonal heavy-fermion metal YbAgGe for temperatures $T \gtrsim 40$ mK and in magnetic fields $H \parallel ab$ up to 14 T. This distorted kagome-lattice system displays a series of magnetic states and a quantum critical point at $H_c = 4.5$ T. The Lorenz ratio $L(T)/L_0$ displays a marked reduction only close to $H_c$. A $T$-linear contribution below 120 mK, present at all different fields, allows us to extrapolate the Lorenz ratio towards $T = 0$. At the critical field this yields $L/L_0 = 0.92 \pm 0.03$, suggesting a violation of the Wiedemann-Franz law due to strong inelastic scattering.

The realization of quantum critical points (QCPs), i.e., continuous transitions at $T = 0$ driven by the change of a nonthermal parameter, has led to the discovery of interesting new phenomena such as non-Fermi liquid (NFL) behavior and unconventional superconductivity in recent years [1–4]. Heavy fermion (HF) systems consist of unstable local magnetic moments (often from Ce, Yb, or U atoms) in a metallic environment and are prototype materials for the investigation of QCPs. Their ground state is determined by the competition between the Kondo- and Ruderman-Kittel-Kasuya-Yosida interaction [5]. As a result of their sensitivity to small changes in pressure, composition, or magnetic field, HF compounds represent an ideal playground for the controlled realization of novel quantum phases [6,7].

Two different scenarios were proposed to describe QCPs in HF metals: the spin-density wave and the Kondo-breakdown scenario [8–10]. A clear distinction between them concerns the role of Kondo singlets when the system is tuned from the paramagnetic side towards the QCP [11]. In the former scenario, the Kondo singlets survive across the QCP and the magnetic order is of spin-density-wave type. This restricts the critical fluctuations to the vicinity of "hot spots" while the main part of the Fermi surface remains intact. In striking contrast, the entire Fermi surface is influenced in the latter case, and the heavy Landau quasiparticles break up due to the disintegration of the Kondo entanglement [11].

Electrical and thermal transport measurements are well-established tools to investigate the nature of QCPs [12–14]. The ultra-low-temperature thermal conductivity $\kappa$ of a metallic sample is related to its electrical conductivity $\sigma$ through the Wiedemann-Franz (WF) law $\kappa/\sigma T = \pi^2 k_B^2/3e^2 = L_0$. Since the WF law is independent of the quasiparticle mass, it can be viewed as a touchstone to verify whether Landau quasiparticles survive at the QCP. The WF law is strictly valid at $T = 0$ K, if the following necessary conditions are fulfilled: (i) the charge and energy

must be carried by the same fermions and (ii) scattering is fully elastic [15]. If, for example, in a one-dimensional material charge-neutral spinon excitations are present, the Lorenz ratio $L(T)/L_0$ with $L(T) = \kappa/\sigma T$ can strongly exceed 1 in the zero-temperature limit. In three-dimensional metals, a violation of the WF law may be expected at the Kondo-breakdown QCP where well-defined heavy quasiparticles disintegrate [12,13].

Experimentally, a violation of the WF law was reported in the two prototype HF metals $CeCoIn_5$ and $YbRh_2Si_2$ [13–16]. In tetragonal $CeCoIn_5$, the WF law was found to be violated with $L/L_0 \approx 0.8$ at the field-induced QCP when the current flows perpendicular to the layers, whereas it turns out to hold for the in-plane transport [15,16]. The violation of the WF law results from linear extrapolation of the thermal and electrical resistivity towards absolute zero, which, however, has been questioned subsequently [17]. For $YbRh_2Si_2$, two recent studies on the thermal and electrical conductivity near the field-tuned QCP have found rather similar experimental data obtained for fields either perpendicular [13] (down to 25 mK) or parallel [14] (down to 40 mK) to the tetragonal $c$ direction at the respective critical fields. However, these studies strongly differ in the extrapolation of the thermal resistance at the critical field for $T \rightarrow 0$. Whereas the former study attributed an observed downturn of the thermal resistance below 0.1 K to overdamped magnons, the latter one treated the same downturn as signature of the quasiparticle formation. Exclusion or inclusion of this feature to the $T \rightarrow 0$ extrapolation leads to either a violation ($L/L_0 \approx 0.9$) [13] or verification [14] of the WF law, respectively. Therefore, there is a strong need to investigate the validity of the WF law in further quantum critical HF metals.

In this Letter, we describe electrical resistivity and thermal conductivity measurements performed on the HF metal YbAgGe to investigate the quantum critical behavior and validity of the WF law. Yb ions in YbAgGe with hexagonal ZrNiAl structure form a distorted kagome

lattice, where the magnetic interactions between the first
nearest neighbors are fully frustrated. In zero magnetic
field, the specific heat displays a broad maximum at 1 K
and subsequent sharp antiferromagnetic (AF) phase tran-
sition at 0.7 K [18]. The highly complex phase diagram in
magnetic field, as shown in Fig. 1 [19], implies the impor-
tance of magnetic frustration. Interestingly, the isostruc-
tural CePdAl exhibits an unusual partial ordering, with 2/3
of Ce-4$f$ moments ordered and the remaining 1/3 in a
paramagnetic state, due to frustrated interactions [20]. For
YbAgGe, neutron-scattering studies have revealed that the
$a$ and $c$ phases are ordered with a commensurate wave
vector (1/3, 0, 1/3) whereas the $b$ phase displays an
incommensurate ordering vector (0, 0, 0.325) [21,22].
The microscopic nature of all the other phases is still
unknown.

At the critical field separating the $c$ and $d$ phases, $H_c =$
4.5 T, the thermal and the magnetic Grüneisen ratios di-
verge as the temperature is lowered, indicating quantum
criticality [19,23]. The positions of a crossover feature in
the Hall effect and a sign change of thermal expansion
($\alpha = 0$) also extrapolate for $T \to 0$ to 4.5 T, at which field a
metamagnetic-like signature of the magnetization has been
found [24]. These recent observations led to a speculation
that a new type of quantum critical end point (QCEP) is
caused by a spin-flop transition of local moments [19,23].
This is different to the case of itinerant moment metamag-
netism as observed in $Sr_3Ru_2O_7$ and $CeRu_2Si_2$ [25,26]. A
further field-induced QCP in YbAgGe has been proposed
at $H \simeq 7.2$ T, where both the specific heat $C/T$ and ther-
mopower $S/T$ display a logarithmic divergence towards

low temperatures [19]. Our measurements, discussed
below, indicate that the WF law is satisfied in this NFL
region $e$ of the phase diagram. Remarkably, however, we
observe a marked suppression of Lorenz ratio $L(T)/L_0$ at
$H = 4.5$ T. Extrapolation to $T \to 0$ suggests a moderate
violation of the WF law near this critical field.

The heat and charge transport of YbAgGe were measured
parallel to the hexagonal $c$ axis at various magnetic fields
applied along the $ab$ plane. Details are provided in the
Supplemental Material [27]. Figure 2 displays the tempe-
rature dependence of the electrical resistivity down to
30 mK at selected magnetic fields (for further data
see the Supplemental Material [27]). The sample used
in the present work exhibits a residual resistivity $\rho_0 =$
43 $\mu\Omega$ cm at zero magnetic field. However, this relatively
high value is not solely related to structural disorder, since
the residual resistivity is strongly reduced by magnetic field
and amounts to $\rho_0 = 19$ $\mu\Omega$ cm, corresponding to a resid-
ual resistivity ratio of RRR = 5.9, at 14 T [27]. In zero field,
the rapid drop of $\rho(T)$ at 0.65 K confirms the first-order
transition that leads to a clear hysteresis in resistivity and
magnetization measurements [24,28]. In low fields, this
transition is suppressed to lower temperatures and finally
disappears at $H = 2$ T. At 4.5 T, a weak shoulderlike
anomaly is found around 0.4 K. At higher fields, the elec-
trical resistivity can be described by $\rho(T) = \rho_0 + AT^n$ with
the exponent $n \simeq 1.60$ and 1.58 for $H = 7.2$ and 9 T, respec-
tively. The NFL region thus largely exceeds the proposed
QCP at 7.2 T in agreement with the phase diagram con-
structed by earlier resistivity measurements [29]. This may
be related to the negative low-temperature magnetoresis-
tance $d\rho/dH < 0$ in this field regime, which saturates only
beyond 10 T (see Supplemental Material [27]).

The thermal conductivity $\kappa(T)/T$ of YbAgGe as a func-
tion of temperature is displayed in Fig. 3, at temperatures
$T \leq 0.3$ K and different magnetic fields up to 9 T. Further
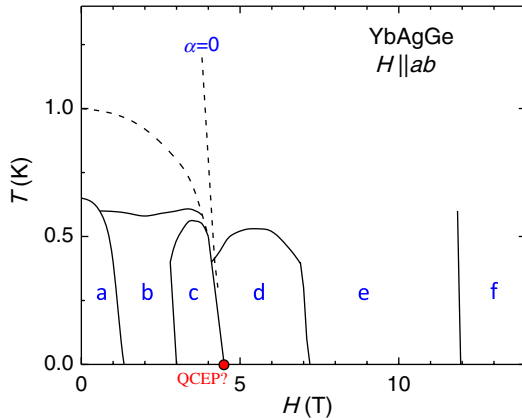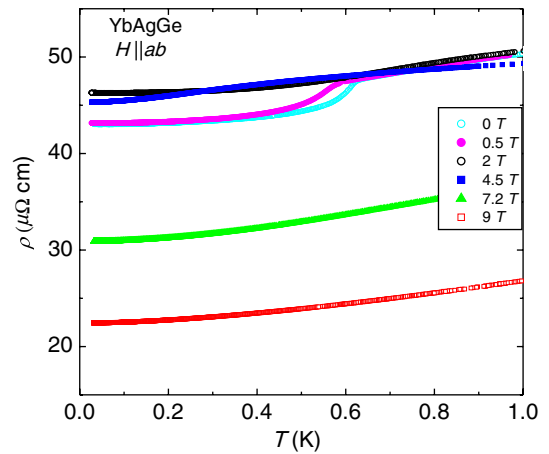data are shown in the Supplemental Material [27].



FIG. 1 (color online). Schematic $T$-$H$ phase diagram of
YbAgGe for $H \parallel ab$, summarizing earlier measurements [19].
Below 4.5 T, commensurate AF order forms in phase $a$, then
changes to incommensurate AF order in phase $b$, and finally
returns back to commensurate AF order in $c$ [21,22]. A sign
change of the thermal expansion coefficient ($\alpha$) appears near the
border between the regions $c$ and $d$. Electrical resistivity $\rho(T)$
varies as $T^n$ with $n \simeq 1$ in region $d$, $1 < n < 2$ in region $e$, and $n = 2$ in
region $f$ [18]. The red circle indicates the position of the
presumed quantum critical end point at $H_c = 4.5$ T.



FIG. 2 (color online). Temperature dependence of the electri-
cal resistivity $\rho(T)$ of YbAgGe along the $c$ direction at various
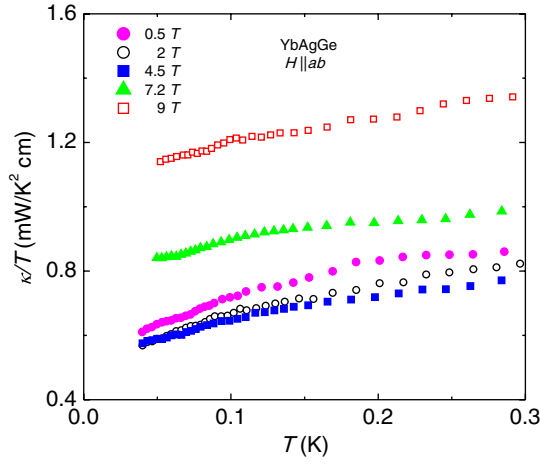magnetic fields $H \parallel ab$.

FIG. 3 (color online).   Temperature dependence of the thermal conductivity divided by temperature $\kappa(T)/T$ of YbAgGe at different magnetic fields.

In general, heavy quasiparticles, phonons, and magnons could transport energy in YbAgGe. We plot $\kappa(T)/T$, since bosonic contributions from phonons and magnons die out at $T \rightarrow 0$, giving rise to a finite intercept due to electrons. Upon cooling to low temperatures, $\kappa(T)/T$ steadily declines in all fields and no indication of saturation is observed. Unlike in YbRh$_2$Si$_2$ [13,14] and CeCoIn$_5$ [16], where the total thermal conductivity is dominated by the electronic transport and thus $\kappa(T)/T$ increases with decreasing temperature, the bosonic contributions in YbAgGe cannot be neglected. The relatively high residual resistivity in YbAgGe strongly reduces the electronic thermal conductivity. This is also reflected in the convex shape of $\kappa(T)/T$ in particular at low fields, which makes it unfeasible to estimate the phonon contribution to thermal conductivity by power-law fitting with reasonable exponent. Therefore, it is impossible to analyze the temperature dependence of the electronic thermal resistivity, as done for CeCoIn$_5$ and YbRh$_2$Si$_2$. By contrast, we focus on the total thermal conductivity and the Lorenz ratio at very low temperatures. The overall behavior presented in Fig. 3, indicates that $\kappa(T)/T$ is gradually suppressed in low fields and reaches a minimum at $H = 4.5$ T. At higher fields, for $H = 7.2$ and 9 T, $\kappa(T)/T$ becomes strongly enhanced, which corresponds to the pronounced reduction of the electrical resistivity in this field regime.

Using the electrical resistivity $\rho(T)$ and thermal conductivity $\kappa(T)$ we can calculate $L(T) = \kappa/\sigma T$. The temperature dependence of the normalized Lorenz ratio $L(T)/L_0 = (\kappa/T)\rho/L_0$ is displayed in Fig. 4. We are interested in the zero-temperature limit. For this purpose it is sufficient to calculate $L(T)$ without subtraction of the unknown additional phonon and magnon contributions, since both will disappear as $T \rightarrow 0$. We turn our attention to very low temperatures ($T \leq 0.15$ K), at which the Bosonic contributions gradually die out. In our accessible
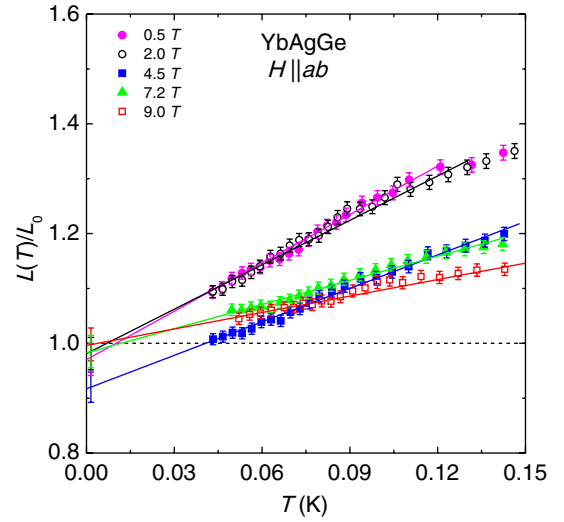


FIG. 4 (color online).   Temperature dependence of the normalized Lorenz ratio $L(T)/L_0 = \kappa\rho/(L_0 T)$ for various different magnetic fields. The solid lines indicate linear fits to the data. Error bars at data points represent the statistical scattering whereas those at the lines near absolute zero temperature indicate the systematic error for the extrapolated Lorenz ratio arising from the finite width of the contacts.

temperature range above 40 mK, the Lorenz ratio $L(T)/L_0 > 1$. Again this behavior, which is caused by the relatively high $\rho_0$ in YbAgGe, is in sharp contrast to the observations in the very clean HF systems CeCoIn$_5$ [15,16] and YbRh$_2$Si$_2$ [13,14] for which $L(T)/L_0 < 1$ at finite temperatures below 1 K. Empirically, we observe a linear temperature dependence of $L(T)/L_0$ for all investigated fields with no trace of a downturn to lowest temperatures (see the Supplemental Material for further data at additional fields and towards higher temperatures [27]). Although we cannot justify this linear temperature dependence, which arises from the inclusion of bosonic contributions to $\kappa(T)$, it nevertheless allows us to obtain the 0 K values by simple linear extrapolation. At $H \leq 3.5$ T, the Lorenz ratio is weakly field dependent, while we observe a drastic reduction of the low-temperature value at the critical field $H_c = 4.5$ T. Upon further increasing the field, it increases again.

While the depression of $L(50 \text{ mK})$ clearly indicates anomalous behavior near the QCP, we now turn to the extrapolation of the linear $L(T)$ dependence found at all different fields (see also Supplemental Material [27]) towards $T \rightarrow 0$. It reveals a value significantly below 1 only at the critical field with $L(T \rightarrow 0)/L_0 = 0.92 \pm 0.03$. The observed reduction of $L(T)$ at $H_c$ therefore suggests a violation of the WF law. On the other hand, for $H = 7.2$ and 9 T, there is a clear change of the slope of $L(T)/L_0$ and the value of $L(T \rightarrow 0)/L_0$ extrapolates to $1.00 \pm 0.03$, proving that the WF law holds in this extended NFL region of the phase diagram.

The electrical resistivity $\rho(T)$ of YbAgGe is almost temperature independent below 0.1 K in all fields.

Therefore, the *electronic* thermal conductivity $\kappa_e/T$ should be constant, as observed, e.g., in $CeNi_2Ge_2$ [30]. According to the relation $\kappa/\sigma T = L(T)$, the slope of the Lorenz ratio reflects the magnitude of the bosonic thermal conductivity ($\kappa_{ph}$ and $\kappa_{mag}$) due to phonons and magnons. The remarkable depression of the slope in $L(T)/L_0$ at higher fields ($H = 7.2$ and 9 T) is probably linked to the loss of magnons due to the suppression of the AF order since phonons are unaffected by the magnetic field.

At $H_c = 4.5$ T, the extrapolated $T = 0$ value of $L(T)/L_0$ is less than 1, suggesting that the thermal transport ability is significantly depressed by additional scattering sources. Let us return our attention to the raw data of the electrical resistivity $\rho(T)$ and thermal conductivity $\kappa(T)/T$ at $H_c$ in more detail. Compared to the data at $H = 2$ T, $\rho(4.5$ T$)$ is significantly reduced below 0.2 K, leading to an enhancement of $\sigma$. However, the thermal conductivity $\kappa(4.5$ T$)/T$ does not show a corresponding increase. The disparate field response of $\sigma$ and $\kappa/T$ therefore leads to the distinct suppression of $L(T)/L_0$ at the critical field of 4.5 T compared to that of all other fields. This experimental observation is independent on the empirical $T \to 0$ extrapolation and indicates anomalous behavior near the QCP.

It is interesting to compare YbAgGe with the itinerant metamagnets $Sr_3Ru_2O_7$ [25] and $CeRu_2Si_2$ [31]. For both systems, thermodynamic indications for quantum criticality in accordance with the itinerant Hertz-Millis scenario have been found [26,32], although the approach of the QCEP is cut off by the formation of a nematic phase in the former and crossover to Landau Fermi liquid behavior in the latter case. Most interestingly, both materials obey the WF law within 5% error at their quantum critical regions [33,34]. The most significant difference between the QCEPs in YbAgGe and these two materials concerns the nature of the magnetic moments. In the above mentioned itinerant systems, the quasiparticles stay intact [26,35]. Accordingly, the WF law is obeyed. For YbAgGe, on the other hand, the metamagnetic behavior is more local in nature, resulting in strong critical fluctuations that may lead to a destruction of the quasiparticles at the entire Fermi surface. At $H \simeq 4.5$ T, previous transport measurements have found an abrupt jump in the field dependence of the Hall coefficient [36] and sign change of the isothermal thermopower $S(H)$ at low temperatures [37]. These observations indeed suggest a significant change of the Fermi surface at this field.

Another distinct difference of YbAgGe compared to $Sr_3Ru_2O_7$ and $CeRu_2Si_2$ is the geometrical frustration of $Yb^{3+}$ magnetic moments on the distorted kagome lattice. It is anticipated that the metamagnetic transition is associated with a weakening of the magnetic frustration. Once the metamagnetic transition is tuned towards zero temperature, the long-wavelength critical fluctuations of the order parameter start showing quantum mechanical character and

play an important role. The coexistence of magnetic frustration and quantum fluctuations may reinforce each other, giving rise to an increasing proportion of inelastic scattering of electrons. Close to the quantum critical region, strong inelastic scattering of electrons can then lead to inefficient heat transport compared to its counterpart charge transport, resulting in the violation of the WF law.

A further field-induced QCP in YbAgGe was proposed by Schmiedeshoff *et al.* near 7.2 T, evidenced by the onset of a divergence of the Grüneisen ratio [19]. This may account for the finite field region between 7 and 9 T, for which NFL behavior has been found. In the $T = 0$ K limit, however, the verification of WF law at $H = 7.2$ T implies that the quasiparticles still survive in this NFL region.

In summary, we have studied the low-temperature electrical resistivity and thermal conductivity of YbAgGe. The temperature dependence of the Lorenz ratio $L(T)/L_0$ indicates an anomalous suppression at the critical field $H_c = 4.5$ T of a metamagnetic quantum critical end point. Although we cannot neglect or subtract the phonon and magnon contributions to thermal conductivity, the observed linear temperature dependence of $L(T)/L_0$ for all different fields allows us to extrapolate towards $T \to 0$. Such extrapolation yields a value significantly below 1 ($L/L_0 = 0.92 \pm 0.03$) only very close to $H_c$. This suggests strong inelastic scattering even at absolute zero temperature due to the disintegration of heavy quasiparticles. We speculate that magnetic frustration and the more local magnetic character compared to that of itinerant metamagnets such as $Sr_3Ru_2O_7$ and $CeRu_2Si_2$ is responsible for this behavior.

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