Parametric Phase Locking in an Electron rf Oscillator

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We propose a way to achieve phase locking of output rf radiation produced by an oscillator driven by a cw electron beam. The locking mechanism is provided by fast periodic modulation of electromagnetic properties of the operating cavity. Ohmic loss and/or the eigenmode frequency are modulated using induced photoconductivity of a semiconductor insert, as affected by a laser with a pulse repetition rate equal to twice the rf frequency.

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Powerful electron-beam based oscillators with a controllable phase have been the object of numerous studies over the past several decades [1–5]. Such sources are needed for particle accelerators, where electron bunches born at a photo cathode illuminated by a high repetition rate laser are assumed to be strictly in phase with the accelerating fields provided by a high-power rf source [6–8]. However, the techniques used to satisfy this synchronization condition are not free of phase errors caused by timing jitter in the sequence of laser pulses. In this Letter, we study a possibility to provide phase locking using a technique that is less costly than would be an amplifier based on similar principles, and is evidently able to deliver higher power. Phase locking can be obtained using parametric modulation in a property of the operating cavity (i.e., Ohmic loss or operating mode eigenfrequency). Such modulation can be provided by a photoconducting insert in the cavity that is irradiated by a pulse-periodic laser whose wavelength is near the band gap of the semiconductor [9]. The concept is illustrated in Fig. 1. Depending upon on the concentration of electrons in the conductivity band caused by absorption of laser photons, the modulation could cause either additional rf absorption or displacement of fields away from the semiconductor insert that would lead in turn to a change in eigenfrequency.

As in a classical parametric oscillator, the modulation frequency ω_p (the laser pulse repetition rate) should be approximately twice the eigenfrequency of the operating mode ω_0, i.e., $\omega_p \approx 2\omega_0$. If the phase deviation between laser pulses is small enough, a subharmonic of ω_p can also be used. The scenario where modulation of Ohmic loss is a locking factor is illustrated in Fig. 1(b)). Mode "1", having a near-zero electric field at the times when photoconductivity losses are highest, has a higher Q factor as compared to mode "2" whose phase is shifted by $\pi/2$ relative to mode 1.

This proposed means of phase control has two important benefits: it is not sensitive to the stability of laser power from pulse to pulse; and one can control the phase as well as the frequency during an rf pulse by changing slowly the laser repetition rate (in the frequency band, which is inversely proportional to the rf cavity Q factor). The practical use of this locking mechanism for rf accelerators possible for rf frequencies as high as several GHz. A possible semiconductor insert for these purposes could be either GaAs ($\varepsilon = 11$–13, $\tan\delta \sim 10^{-4}$, relaxation time $\sim$300 ps) to be irradiated by 800–850 nm light at rf frequencies up to 1 GHz [10], or CVD diamond films ($\varepsilon = 5.7$, $\tan\delta \sim 10^{-5}$, relaxation time 0.1–50 ps) irradiated by a uv laser with $\lambda = 260$ nm or193 nm, allow operating rf frequencies up to about 10 GHz [11,12]. Existing lasers having pulse durations from femtoseconds to picoseconds, repetition rates at a gigahertz level, and energies as high as 10^1–10^5 nJ per pulse are able to provide a peak loss tangent up to $\sim$1 for samples with sizes comparable with a wavelength. Thus, they can be used for phase locking based on modulation in Ohmic losses.

Let us consider field dynamics in the operating cavity of an electron rf oscillator. Asymptotic equations of electron motion [13] are obtained from the equation for the electron Lorentz factor, $\gamma = (1 - \mathbf{v}^2/c^2)^{-1/2}$, namely,

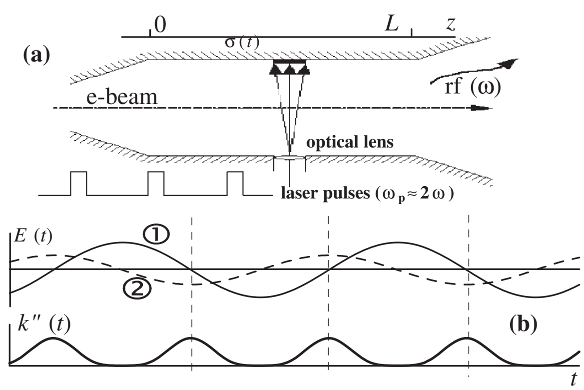

FIG. 1. (a) Schematic of a phase-locked electron maser. (b) Electric fields of high-Q (1) and low-Q (2) eigenmodes, $E(t)$, in the case of modulated Ohmic losses of the cavity, $k''(t)$.

$$mc^2 \frac{d\gamma}{dz} = -e\frac{\mathbf{v}\cdot\mathbf{E}}{v_z}, \tag{1}$$

where $\mathbf{v}$ is the electron velocity, $\mathbf{E} \propto \mathrm{Re}\hat{a}F_z \exp(i\omega_0 t)$ is the rf-wave electric field, $\hat{a}(t)$ is the slow amplitude, and $F_z(z) = \sin(k_z z)$ describes the axial wave structure. In both cyclotron masers and free-electron masers the velocity has an oscillatory component, $v_\sim \propto \exp(i\int \omega_e dt)$. Then, the motion equations are represented as follows

$$\frac{d\gamma}{d\zeta} = -\chi \,\mathrm{Re} aF_z \exp(i\theta), \quad \text{and} \quad \frac{d\theta}{d\zeta} = \frac{\omega_0 - \omega_e}{k_0 v_z}, \tag{2}$$

where $\zeta = k_0 z$, $k_0 = \omega_0/c$, χ is the electron-wave coupling factor, and $\theta = \omega_0 t - \int \omega_e dt$ is the phase. Similar equations can be obtained for the Smith-Purcell (Cherenkov) maser, in which case, $\omega_e = k_{\mathrm{cor}} v_z$ corresponds to corrugation of the waveguide wall.

In the case of a near-cutoff operating wave (typically the gyrotron [14]), $k_z \approx \pi/L \ll k_0$, so the electron-wave resonance condition becomes $\omega_0 \approx \omega_e$, and the phase θ changes slowly. In the case of a far-from-cutoff wave, the resonance condition is $\omega_0 \approx \omega_e \pm k_z v_z$, and equations of motion with slowly varying phase $\theta \mp k_z\zeta/k_0$ are obtained.

If the electron-wave coupling causes a small change in the energy, then Eq. (2) is represented as [13]

$$\frac{d\theta}{d\zeta} = -b(\gamma - \gamma_0) - D, \tag{3}$$

where $D = [\omega_e(\gamma_0) - \omega_0]/k_0 v_{z0}$ is the normalized mismatch in electron-wave resonance, and $b = \partial/\partial\gamma[(\omega_e - \omega_0)/k_0 v_z]$ is the electron bunching factor. By neglecting the dependence of χ and b on electron energy, one obtains equations for $u = b(\gamma_0 - \gamma)$ and $a = \chi b \hat{a}$ to be

$$\frac{du}{d\zeta} = \mathrm{Re} aF_z \exp(i\theta), \qquad \frac{d\theta}{d\zeta} = u - D. \tag{4}$$

If there is no parametric modulation, then the equation of rf wave excitation has the form [13,14]

$$\frac{\partial a}{\partial \tau} + a = J, \quad \text{where } J = G\int_0^Z F_z \rho \, d\zeta. \tag{5}$$

And where $\tau = \omega_0 t/2Q$, $\rho = \langle \exp(-i\theta)\rangle$, Q is the diffraction quality factor characterizing losses caused by the rf wave output, G is the excitation factor proportional to the electron current, $\mathrm{Z} = k_0 L$ is the normalized length, and $\langle\cdots\rangle$ denotes averaging over all initial electron phases with $\theta_0 \in [0, 2\pi]$. Equations (4) and (5) lead to the energy balance equation

$$\frac{1}{2G}\frac{\partial |a|^2}{\partial \tau} + p = \langle u \rangle, \tag{6}$$

with the normalized output rf power $p = |a|^2/G$, so that in the stationary regime $p = \langle u\rangle$.

Equation (5) is followed from the wave equation,

$$\frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} - \frac{\partial^2 \mathbf{A}}{\partial z^2} - \Delta_\perp \mathbf{A} = \frac{4\pi}{\mathrm{c}}\mathbf{j}.$$

If the vector-potential $\mathbf{A}$ has the form $\mathbf{A} = \mathbf{F}_\perp(\mathbf{r}_\perp)F_z(z)A_t(t)$ with $\Delta_\perp \mathbf{F}_\perp = -k_\perp^2 \mathbf{F}_\perp$, one obtains

$$\frac{1}{c^2}\frac{\partial^2 A_t}{\partial t^2} + k^2 A_t = J_t \quad \text{with } J_t = \frac{4\pi}{cN}\int \mathbf{j}\mathbf{F}_\perp F_z d^2\mathbf{r}_\perp dz. \tag{7}$$

Here, $k^2 = k_\perp^2 + k_z^2$ and N is the wave normalization factor. Let us introduce a slow complex amplitude $A_t = \mathrm{Re}a(t) \times \exp(i\omega t)$, where $\omega = \omega_0 - i\omega''$ with the "cold" eigenfrequency ω_0 and an imaginary part describing the diffraction losses, $Q = \omega_0/\omega''$. Then $k = k_0 + ik''$ (here k'' describes Ohmic losses), and one obtains the following from Eq. (7):

$$\frac{\partial a}{\partial \tau} + a + 2Q\frac{k''}{k_0}a = J, \quad \text{where } J = 2Q\langle J_t \exp(-i\tau)\rangle_\tau. \tag{8}$$

Let us generalize Eq. (5) to the case of the parametric modulation in the properties of the cavity, when $k = k_0 + k_1(\tau) + ik''(\tau)$. Instead of Eq. (8) one then obtains

$$\frac{\partial a}{\partial \tau} + a + F = J, \tag{9}$$

where $F = 2Q\langle \exp(-i\tau)((k''(\tau) - ik_1(\tau))/k_0)\mathrm{Re}a\exp(i\tau)\rangle_\tau$, and $\langle\cdots\rangle_\tau$ denotes averaging in time. Small oscillations of the eigenfrequency are described by oscillations of the real part of the wave number, $k_1/k_0 = (-s_1/2Q)\cos\omega_p t$. As for Ohmic losses, they oscillate around some averaged value, so that they are always positive, $k''/k_0 = (s/2Q)(1 + \cos\omega_p t)$. In this case, Eq. (9) reduces to

$$\frac{\partial a}{\partial \tau} + a + sa + \frac{s + is_1}{2}a^* \exp(2i\Delta\tau) = J, \tag{10}$$

where $\Delta = (\omega_p/2 - \omega_0)/(2Q\omega_0)$ is the parametric half-frequency mismatch. Here, the term (a) describes diffraction losses, the term (sa) corresponds to averaged Ohmic losses, and the terms $\sim(sa^*)$ and $\sim(s_1 a^*)$ describe fast modulation in the Ohmic losses and of the eigenfrequency, respectively.

In analysis of electron oscillators, it is convenient to use a "klystron-like" model of the electron-wave interaction, where the rf field structure is a sum of two delta functions; i.e., $F_z = \delta(\zeta) + \delta(\zeta - Z)$, where $\delta(\zeta)$ describes modulation of electron energies at the beginning, whereas $\delta(\zeta - Z)$ models the interaction of the rf field with a bunched electron beam. Then, $J = iaG\exp(i\Psi)J_1(Z|a|)/|a|$ with $\Psi = DZ$. Having approximated the Bessel function as $J_1(x) \approx x/2 - x^3/16$, one obtains

$$J = iaI\exp(i\Psi)(1 - \beta|a|^2), \tag{11}$$

where $I = GZ/2$ and $\beta = Z^2/8$.

In the small-signal approximation, $|a| \ll 1$, we neglect parametric modulations and obtain from Eqs. (10) and (11) the maximum increment for the phase $\Psi = 3\pi/2$:

$$\Gamma = I - 1 - s. \tag{12}$$

Thus, the starting threshold is $I_{st} = 1 + s \approx 1$.

Now, let us study stationary operation, when $a = a_0 = |a_0| \exp(i\Omega\tau + i\varphi)$. By introducing $s + is_1 = S\exp(i\varphi_s)$, in the case of $\Psi = 3\pi/2$ one obtains

$$\Omega = \Delta = \frac{S}{2}\sin(2\varphi - \varphi_s), \tag{13}$$

and

$$1 + s + (S/2)\cos(2\varphi - \varphi_s) = I(1 - \beta|a_0|^2). \tag{14}$$

If $S = 0$, there exists the only solution with the normalized "hot" frequency shift $\Omega_0 = 0$, the wave amplitude $\beta|a_0|^2 = 1 - I^{-1}$, and an arbitrary wave phase φ.

The presence of parametric modulations ($S \neq 0$) may result in stationary operation with a fixed frequency shift, Ω, which coincides with the shift of the modulation half-frequency, Δ. Such stationary operation is possible if the modulation frequency shift is small enough, i.e., $-S/2 < \Delta < S/2$. The phase of the rf wave is determined by Eq. (13), $\sin(2\varphi - \varphi_s) = 2D/S$. Since the rf phase appears in Eqs. (13) and (14) as 2φ, it is fixed with specificity $\varphi \pm \pi$.

According to Eqs. (13) and (14), there exist two stationary states with the same $\sin(2\varphi - \varphi_s)$, but with opposite $\cos(2\varphi - \varphi_s)$ (Fig. 2). Thus, $\cos(\ldots) < 0$ corresponds to a mode with a higher Q factor and a greater rf wave amplitude, whereas $\cos(\ldots) > 0$ corresponds to a mode with a lower Q factor and a smaller amplitude. Thereby,

$$\beta|a_0|^2 = 1 - (1 + s)I^{-1} - (S/2I)\cos(2\varphi - \varphi_s). \tag{15}$$

Let us consider the case of fast modulation in Ohmic losses ($S = s$ and $\varphi_s = 0$). If the parametric half-frequency coincides with the eigenfrequency, $\Delta = \Omega_0 = 0$, then Eq. (13) yields two phases, $\varphi = 0$ and $\varphi = \pi/2$. Solution $\varphi = \pi/2$ with $\cos(2\varphi) < 0$ corresponds to lower Ohmic losses; the electric field "zeroes" of such a wave coincide with the maxima of Ohmic losses [Fig. 1(b), mode 1]. For this mode, $a = -a^*$, and the Ohmic losses are described by the term $(s/2)a$ in Eq. (10). Solution $\varphi = 0$ with $\cos(2\varphi) > 0$ corresponds to the wave with the field maxima coinciding with the maxima of Ohmic losses [Fig. 1(b), mode 2]. For this mode, $a = a^*$ and losses are higher as compared to the mode 1; they are described by the term $(3s/2)a$ in Eq. (10).

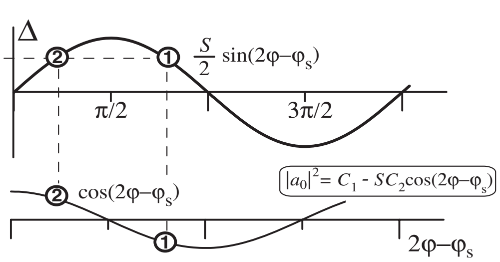

FIG. 2. At a fixed shift of the modulation half-frequency Δ, there exist two stationary states: (1) a stable state with negative cosine, and (2) nonstable state with positive cosine.

Analysis of the stability of these two steady-state modes shows instability of mode 2. If one represents the solution of Eqs. (10) and (11) in the form $a = a_0 + a_1$ [where $a_1 \propto \exp(\lambda\tau)$ is a small perturbation], then one obtains the only possible solution with a positive λ:

$$\lambda = S\cos(2\varphi - \varphi_s). \tag{16}$$

Thus, mode 2 with positive cosine is always unstable, whereas mode 1 is stable (Fig. 2).

All results predicted from this analytical approach have been confirmed by numerical simulations of Eqs. (4) and (10). We study an oscillator operating at a close-to-cutoff wave, with $F_z(\zeta) = \sin(\pi\zeta/Z)$. The normalized cavity length $Z = 2\pi L/\lambda$, corresponds to $L = 10\lambda$. Figure 3 illustrates the stationary states of the electron maser with no parametric modulation. The oscillator starts at $G \approx 0.014$. The output power increases with an increase of G and saturates at $G \approx 0.03$–0.05. The maximum power is achieved at the mismatch phase $\Psi \approx 2\pi$. A characteristic shift of the rf-wave frequency $\Omega \sim 1$ is within the frequency band of the cavity, $\delta\omega/\omega_0 \sim 1/2Q$.

If a parametric modulation takes place, then in the stationary state the rf frequency shift coincides with the shift of the parametric half-frequency, and $\Omega = \Delta$. Figure 4 illustrates ranges of Δ where this phase-locked regime is stable in the cases of modulated Ohmic losses ($s \neq 0$) and modulated eigenfrequency ($s_1 \neq 0$). We study the case of a close-to-saturation excitation factor, $G = 0.03$, and the optimal resonance mismatch, $\Psi = 1.9\pi$ as shown in Fig. 3.

If the modulations are absent, the "hot" eigenfrequency of the oscillator is $\Omega_0 \approx -0.25$. According to the theory (see Fig. 2), if the modulation half-frequency is close to this eigenfrequency $\Delta \approx \Omega_0$, then the stable phase-locked operation with the phase $2\varphi - \varphi_s = \pi$ builds up. In the

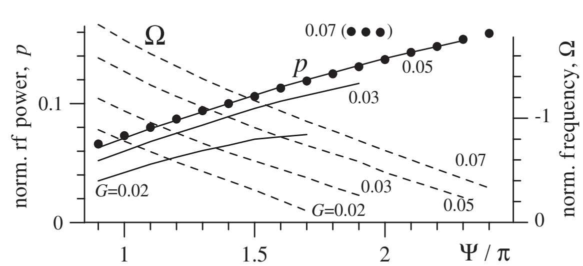

FIG. 3. Stationary states of the electron maser at various excitation factors, G. Normalized rf power p (solid curves and dots at $G = 0.07$) and the "hot" frequency shift Ω (dashed curves) versus the mismatch phase $\Psi = DZ$.

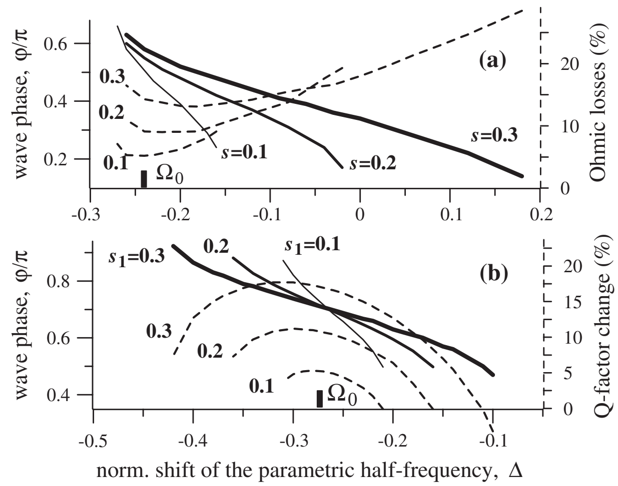

FIG. 4. Modulated Ohmic losses (a) and eigenfrequency (b) with various modulation factors, s and s_1. The wave phase (solid curves), Ohmic losses [(a), dashed curves], and change in the Q factor [(b), dashed curves] in the range of the parametric half-frequency Δ corresponding to stable phase-locked operation.

case of modulated Ohmic losses $\varphi_s = 0$ and $\varphi = \pi/2$ [see Fig. 4(a)]. In the case of a modulated eigenfrequency $\varphi_s = \pi/2$ and $\varphi = 3\pi/4$ [see Fig. 4(b)]. A change in $\Delta - \Omega_0$ leads to a change of the rf-wave phase φ. If $|\Delta - \Omega_0|$ becomes too great, then the stable regime with a fixed frequency ($\Omega = \Delta$) and locked phase is replaced with a regime with automodulations of the output rf power [see Fig. 5(a))]. However, if the phase-locked regime is stable, then at different phases of the initial rf noise [$a(\tau = 0) = a_{\rm in}\exp(i\varphi_0)$] one obtains the same final frequency $\Omega = \Delta$ and the rf-wave phase φ see [Fig. 5(b)].

In the case of modulated eigenfrequency [as in Fig. 4(b)], the range of the stable phase-locked operation is close to that predicted by the analytical theory, i.e., $\delta\Delta \approx S$. As for the case of modulated Ohmic losses, this range is wider [see Fig. 4(a)]. In the both cases, the effective Q factor of the excited wave is maximum, when the modulation half-frequency coincides with the "hot" eigenfrequency, $\Delta \approx \Omega_0$.

It is important that in the case of modulated Ohmic losses [Fig. 4(a)], a higher Q factor corresponds to lower losses $\delta p_{\rm ohm} = (\langle u\rangle - p)/\langle u\rangle$, whereas the eigenfrequency modulation [Fig. 4(b)] leads to a change in the effective diffraction Q factor, $\delta Q = (p - \langle u\rangle)/\langle u\rangle$. Thus, in case "a", an increase in the modulation factor S leads to an increase of the Ohmic losses and thus to a decrease in the total Q factor, whereas in case "b" this leads to increase of the effective diffraction Q factor.

Let us estimate the modulation strength s. If the change in the dielectric permittivity, $\Delta\varepsilon = \Delta\varepsilon' + i\Delta\varepsilon''$ is small, then the complex shift of the eigenfrequency is approximated as $s_1 + is \approx \Delta\varepsilon Q(V_p/V)$, where V_p is the photoconductive layer volume and V is the cavity volume [15].

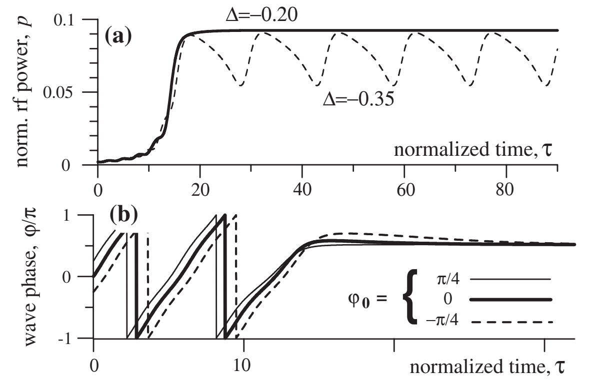

FIG. 5. Modulated Ohmic losses, $s = 0.3$. (a) Normalized output power in the case of the stable phase-locked operation (when the modulation half-frequency $\Delta = -0.2$ is close to the "hot" cavity eigenfrequency), as well as in the case of the nonstationary operation ($\Delta = -0.35$). (b) rf-wave phase at of various phases of the initial rf noise, φ_0, in the case of the phase-locked operation.

The concentration of free carriers $n_{\rm e}$ is assumed small, so that the rf wave frequency and the plasma frequency are much lower than the carrier collision frequency ν. Then, $\Delta\varepsilon'' = 4\pi e n_e \mu/\omega$ [9], where μ is the mobility of carriers. If each photon produces one free electron in the conduction band ($n_{\rm e} = N_{\rm ph}/V_p$, where $N_{\rm ph}$ is the number of photons in a single laser pulse), then $s = 4\pi Q e\mu N_{\rm ph}/(\omega V)$. For example, in order to obtain $s = 0.3$ in a 30 GHz TE_{11}-mode gyrotron ($L = 10\lambda$, $Q = 25(L/\lambda)^2 = 2500$) phase controlled by CVD diamond ($\mu = 2200$ cm^2/V s for electrons and 1600 cm^2/V s for holes, respectively), one needs a laser with a pulse energy $W_1 = N_{\rm ph}hc/\lambda_1 \approx 50$ nJ at the wavelength $\lambda_l = 193$ nm (here h is Planck's constant). Note that within the strong collision plasma model, the change of the real part of $\Delta\varepsilon$ is small, $\Delta\varepsilon' = \Delta\varepsilon''\omega/\nu$ [9]. Therefore, the magnitude of the modulation of the real part of the eigenfrequency is much less than ω_0/Q, so that an increased mode competition is not expected.

In conclusion, we have discussed a relationship between parametric phase locking of an electron rf oscillator and the "classical" parametric instability of a pendulum. In the latter case, modulation of the pendulum eigenfrequency results in a growing-in-time solution with the frequency and phase fixed by the parametric modulation. As for the rf oscillator, its operating cavity looks like a pendulum with significant losses (caused by the rf radiation output), so that the classical parametric excitation of this pendulum is impossible. However, this pendulum is excited by an external force (emission from an electron beam). Since this force is self-consistent, parametric modulation of the pendulum eigenfrequency leads to phase locking of this force. In this situation, approximately the same result takes place in the cases of parametric modulation of both the real part of the pendulum eigenfrequency and its imaginary part (losses).

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