

## Explanation for the Low Flux of High-Energy Astrophysical Muon Neutrinos

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There has been some concern about the unexpected paucity of cosmic high-energy muon neutrinos in detectors probing the energy region beyond 1 PeV. As a possible solution we consider the possibility that some exotic neutrino property is responsible for reducing the muon neutrino flux at high energies from distant sources; specifically, we consider (i) neutrino decay and (ii) neutrinos being pseudo-Dirac-particles. This would provide a mechanism for the reduction of high-energy muon events in the IceCube detector, for example.

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The most recent data from the IceCube Collaboration [1] place stringent limits on the muon neutrino flux at high energies from astrophysical sources. The new limits appear to put severe bounds on models of neutrino production in gamma-ray bursts (GRBs) and active galactic nuclei (AGNs) [2]. Similarly, other experiments probing the ultrahigh-energy regime, such as ANITA [3] and AUGER [4] have not seen any evidence of long anticipated cosmic neutrinos. It should be noted that very recently there have been reevaluations of the expected neutrino fluxes from GRBs, especially following the stringent upper limits from the IceCube detector [1].

It has been pointed out [5,6] that the IceCube [1] calculation of the Waxman-Bahcall neutrino flux from the observed gamma ray flux may have been an overestimation by as much as a factor of 5. So the discrepancy may not be that dire, yet; but the possibility remains that as the bounds get tighter with future observations, the Waxman-Bahcall models [2] will be challenged. In such an eventuality, we would like to offer in this Letter the possibility of other causes for the smallness of the muon neutrino flux, which arise from neutrino properties. We note that there are alternative astrophysical models (see Refs. [7,8] and references therein) which predict a lower neutrino flux compared to the Waxman-Bahcall models [2].

In this note we would like to raise the possibility that these severe bounds are illusory because the small flux may be due to depletion of muon neutrinos which in turn is caused by neutrino properties. We consider two possible scenarios. One is that neutrino decay is responsible for depletion of muon neutrinos and the other is that neutrinos are pseudo-Dirac-particles and there is leakage into the sterile components of the pseudo-Dirac-particles. Both of these were considered almost ten years ago [9,10], but the focus then was on the modification of the flavor mix from the canonical 1:1:1 as expected from conventional flavor oscillations with the known neutrino mixings [11].

In the following, we describe both possibilities. To be definite, we are considering neutrino energies in the

vicinity of order of a PeV, and the distances from the sources of order of hundreds of megaparsecs. In principle, when the distances become large enough, the cosmological redshift becomes important, and the travel distance  $L$  is limited; these effects were discussed some time ago in Refs. [10,12] and more recently in Refs. [13,14].

Of course, because of the uncertainty in predicting fluxes, we do not know precisely what amount of depletion is needed. But the scenarios we suggest below can provide a wide range of suppression ranging from none to an order of magnitude.

*Neutrino decay.*—We consider here scenarios with three light neutrinos and assume that the source distances are large enough so that two of the three mass eigenstates, specifically  $\nu_2$  and  $\nu_3$ , have decayed away completely. If the neutrino masses are quasidegenerate, that is the masses of  $\nu_2$  and  $\nu_3$  are close to that of  $\nu_1$ , then the daughter neutrino  $\nu_1$  carries most of the energy of the parent, and so contributes to the flux at that energy; in this case even though the final state is pure  $\nu_1$ , there is not much depletion. So for our purpose here, the preferred mass spectrum is quasihierarchical, namely  $m_2$  and  $m_3$  much larger than  $m_1$ . In this case the daughter neutrino energy is much lower than the parent and the final  $\nu_1$  does not contribute to the flux at that energy and can be counted out. This is discussed in detail in several papers, especially clearly in Ref. [15]. This means that the exponential decay factor  $\exp(-L/\gamma c\tau)$  is negligibly small for them. Since distances to GRBs are of the order of hundreds of megaparsecs, for energies in the PeV range,  $\frac{L}{\gamma c\tau} = \frac{L}{E} \left(\frac{mc^2}{c\tau}\right) \gg 1$  corresponds to  $\tau/m < 10^3$  s/eV where  $\tau$  is the rest frame lifetime. A lower bound on the lifetime follows from the big bang nucleosynthesis (BBN). If the standard picture is to remain intact then all three neutrinos must be present and in equilibrium in the BBN era so that the crucial  $n/p$  ratio and the nuclear abundances as obtained in standard picture remains unaffected. This puts a lower bound of  $\frac{\tau}{m} E > 1$  s on the neutrino lifetime with  $E \sim$  MeV. These considerations restrict the allowed window of lifetime in the range

$$10^{-6} \text{ s/eV} \leq \frac{\tau}{m} \leq 10^3 \text{ s/eV}. \quad (1)$$

As for the neutrino decay modes, we know the following. The radiative decays such as  $\nu_i \rightarrow \nu_j + \gamma$  are severely constrained by their contribution to  $\nu + e \rightarrow e + \nu'$  and from the current bounds on such contributions the radiative decay lifetime must satisfy [16],

$$\tau_i/m_i > 10^{17} \text{ s/eV}. \quad (2)$$

The three-body invisible decay mode

$$\nu_i \rightarrow \nu_j + \nu\bar{\nu} \quad (3)$$

is constrained by BBN and the deviation of the invisible width of  $Z$  from the expected value (with three neutrinos) in the standard model [17]; and the lifetime is given by

$$\tau_i/m_i > 10^{28} \text{ s/eV}. \quad (4)$$

The kinds of decay models possible are quite restricted. Models where the coupling is chirality conserving (e.g., into a light vector boson or into a scalar boson with a derivative coupling), would by  $SU(2)_L \times U(1)$  symmetry lead to flavor changing decays of charged leptons at the same strength. The severe bounds on flavor changing decays of  $\mu$  and  $\tau$  into invisible two-body modes lead to limits on lifetimes of  $\nu_2$  and  $\nu_3$  of the order of  $\tau/m > 10^{20} \text{ s/eV}$  [18], and so such decays are ruled out. Hence, the only neutrino decay modes which can be relevant for the short lifetimes of interest here are helicity changing decays into a neutrino and a light boson, as discussed in Refs. [9,16]. The current limits on the lifetimes of the three mass eigenstates are as follows. The most stringent is on that of  $\nu_1$ , from the observation of neutrinos from SN1987A as being about  $\tau_1/m_1 > 10^5 \text{ s/eV}$  [19]. The limits on the other two mass eigenstates are  $\tau_2/m_2 > 10^{-4} \text{ s/eV}$  from the solar neutrino observations [15,20] and  $\tau_3/m_3 > 10^{-10} \text{ s/eV}$  from the atmospheric neutrino observations [21]. Obviously, the limits on the lifetimes of  $\nu_2$  and  $\nu_3$  are quite weak.

In the picture adopted here, all the neutrinos originating from GRBs reach the Earth as pure  $\nu_1$  whose flavor content is  $\nu_e:\nu_\mu:\nu_\tau = |U_{e1}|^2:|U_{\mu1}|^2:|U_{\tau1}|^2$  as observed long ago [22]. If we insert the current best fit values [23] for the Maki-Nakagawa-Sakata-Pontecorvo (MNSP) [24] neutrino mixing matrix elements, we find that  $|U_{\mu1}|^2$  ranges between 0.1 and 0.3 with a central value of about 0.16. (The unknown value of the  $CP$ -violating phase  $\delta$  in the MNSP mixing matrix determines the precise value.) This is a suppression beyond the factor of 2 due to the standard flavor oscillations. Thus, a suppression of the muon neutrino flux by an order of magnitude is easily achieved. Since the value of  $|U_{e1}|^2$  is between 0.65 and 0.72, the  $\nu_e$  flux is not affected much by the decays of  $\nu_2$  and  $\nu_3$ . We note that the flux ratio of  $\nu_e$  to  $\nu_\mu$  is between 2.5 and 8 with a central value of about 4, depending on the value of the phase  $\delta$ . We have discussed the most favorable scenario for  $\nu_\mu$  flux

reduction by assuming (i) normal hierarchy, because in the inverted hierarchy the decay of  $\nu_1$  has strong limits from SN1987a so only  $\nu_2$  can decay into  $\nu_3$  but in that case we do not achieve any suppression of  $\nu_\mu$  and (ii) hierarchial masses, namely  $m_2, m_3 \gg m_1$ ; otherwise, if the masses are degenerate, the energy of the decaying and daughter neutrino are the same and even though the flavor ratio  $\nu_e/\nu_\mu$  is large there is not much suppression of  $\nu_\mu$  flux because of enhancement of the  $\nu_1$  flux from the decay.

The invisible decays  $\nu_{2,3} \rightarrow \nu_1 + J$  arise naturally in Majoron models with  $J$  identified with the massless Majoron arising from the spontaneous breaking of total lepton number or some combination of  $L_i$ ,  $i = e, \mu, \tau$ . These models fall in two main categories: triplet Majoron models [25] with a low scale lepton number violation and singlet models [26] with lepton number typically broken at high scale. The former class of models give a large contribution to the invisible decay width of the  $Z$  boson and are ruled out. The singlet Majoron models are consistent with the  $Z$  decay width but mixing of Majoron with the doublet Higgs boson in this case lead to rapid energy loss from stars through Majoron emission. This can be prevented if lepton number breaking occurs at a high scale (typically  $> 10^7 \text{ GeV}$ ). It is, however, possible to consider hybrid models in which Majoron is a combination of the  $SU(2)_L$  doublet, triplet and singlet. Such models allow low lepton number breaking scale and can be made consistent with the existing experimental constraints [27].

The Majoron couplings to neutrinos is flavor diagonal in the simplest triplet model [25] and are nearly so in singlet Majoron [26] models. Both these do not allow short neutrino lifetime as required in Eq. (1) but such lifetimes can be achieved by allowing Majoron to be associated with a spontaneous breaking of some combination of lepton numbers [28] and may also need extension of the simplest model. Denote the coupling relevant to decay as

$$g_{1a} \frac{m_a}{f} \bar{\nu}_1 \gamma_5 \nu_a J, \quad (5)$$

where  $m_a$ ,  $a = 2, 3$  is the mass of the decaying neutrino,  $f$  is the symmetry breaking scale, and  $g_{1a}$  is a model dependent overall coupling. The allowed window for the lifetime as given in Eq. (1) constrains the symmetry breaking scale to lie in the range

$$8 \text{ eV} \leq \frac{f}{g_{1a}} \leq 0.15 \text{ MeV} \quad (6)$$

for normal hierarchy with  $m_a \sim 0.05 \text{ eV}$ . This in particular rules out models with lepton number broken at very high scale but hybrid models [27,28] with a low  $f$  are still allowed.

It has been pointed out [29] that neutrino interactions with a light scalar can make the neutrino fluid tightly coupled at the time of photon decoupling (when  $T_\gamma = 0.256 \text{ eV}$ ). If neutrinos do not free stream after photons decouple then they can be a source for photon perturbations

which would be observable in cosmic microwave background (CMB) anisotropy data. If all neutrinos are assumed to be free steaming during decoupling, then neutrino-Majoron coupling and hence the scale  $f$  is  $\frac{f}{g_{1a}} > 10^{11} m_a \sim 5$  GeV which would rule out decay of PeV neutrinos over cosmological distances of 100 Mpc. However, it has been shown in Ref. [30] that the CMB data do not preclude one or even two neutrino species from being strongly coupled ( $g_{1a} m_a / f > 10^{-7}$ ) and this still keeps the possibility of ultrahigh energy (UHE) neutrino decay viable. The recent Planck data may be able to put stronger constraints on the number of tightly coupled neutrinos at decoupling and rule out the possibility of UHE neutrino invisible decays [31].

Among other consequences, the neutrino counting in the early Universe is modified from a count of 3 to  $3 + 4/7$  due to the extra bosonic degree of freedom. This is consistent with most recent cosmological bounds [32].

The bottom line is that if neutrinos decay, substantial reduction in  $\nu_\mu$  fluxes is possible, and consistent with  $\nu_1$  being the lightest mass eigenstate.

*Pseudo-Dirac-neutrinos.*—If each of the three neutrino mass eigenstates is actually a doublet with very small mass difference (smaller than  $10^{-6}$  eV), then there are no current experiments that could have detected this. Such a possibility was raised long ago [33]. It turns out that the only way to detect such small mass differences in the range  $10^{-12}$  eV<sup>2</sup>  $>$   $\delta m^2 >$   $10^{-18}$  eV<sup>2</sup> is by measuring flavor mixes of the high-energy neutrinos from cosmic sources.

Let  $(\nu_1^+, \nu_2^+, \nu_3^+; \nu_1^-, \nu_2^-, \nu_3^-)$  denote the six mass eigenstates where  $\nu^+$  and  $\nu^-$  are a nearly degenerate pair. A  $6 \times 6$  mixing matrix rotates the mass basis into the flavor basis  $(\nu_e, \nu_\mu, \nu_\tau; \nu'_e, \nu'_\mu, \nu'_\tau)$ . In general, for six Majorana neutrino, there would be fifteen rotation angles and fifteen phases. However, for pseudo-Dirac-neutrinos, Kobayashi and Lim [34] have given an elegant proof that the  $6 \times 6$  matrix  $V_{\text{KL}}$  takes the very simple form

$$V_{\text{KL}} = \begin{pmatrix} U & 0 \\ 0 & U_R \end{pmatrix} \begin{pmatrix} V_1 & iV_1 \\ V_2 & -iV_2 \end{pmatrix}, \quad (7)$$

where the  $3 \times 3$  matrix  $U$  is just the usual mixing matrix; the  $3 \times 3$  matrix  $U_R$  is an unknown unitary matrix, and  $V_1$  and  $V_2$  are the diagonal matrices  $V_1 = \text{diag}(1, 1, 1)/\sqrt{2}$ , and  $V_2 = \text{diag}(e^{-i\phi_1}, e^{-i\phi_2}, e^{-i\phi_3})/\sqrt{2}$  with the  $\phi_i$  being arbitrary phases. A very similar mass spectrum can be produced in the mirror model [35].

As a result, the three active neutrino states are described in terms of the six mass eigenstates as

$$\nu_{\alpha L} = U_{\alpha j} \frac{1}{\sqrt{2}} (\nu_j^+ + i\nu_j^-). \quad (8)$$

The nontrivial matrices  $U_R$  and  $V_2$  are not accessible to active flavor measurements. The flavor conversion probability can thus be expressed as

$$P_{\alpha\beta} = \frac{1}{4} \left| \sum_{j=1}^3 U_{\alpha j} \{ e^{i(m_j^+)^2 l/2E} + e^{i(m_j^-)^2 l/2E} \} U_{\beta j}^* \right|^2. \quad (9)$$

In the description of the three active neutrinos, the only new parameters are the three pseudo-Dirac-neutrino mass differences,  $\delta m_j^2 = (m_j^+)^2 - (m_j^-)^2$ . In the limit that they are negligible, the oscillation formulas reduce to the standard ones and there is no way to discern the pseudo-Dirac nature of the neutrinos.

Incidentally, the effective mass for neutrinoless double beta decay is given by

$$\langle m \rangle_{\text{eff}} = \frac{1}{2} \sum_j U_{ej}^2 (m_j^+ - m_j^-) = \frac{1}{2} \sum_j U_{ej}^2 \frac{\delta m_j^2}{2m_j}. \quad (10)$$

The value of this effective mass is smaller than  $10^{-4}$  eV for inverted hierarchy and smaller for normal hierarchy and renders neutrinoless double beta decay unobservable.

When  $L/E$  becomes large enough, flavor fluxes will deviate from the canonical value of  $1/3$  by [10]

$$\delta P_\beta = \frac{1}{3} [ |U_{\beta 1}|^2 \chi_1 + |U_{\beta 2}|^2 \chi_2 + |U_{\beta 3}|^2 \chi_3 ], \quad (11)$$

where  $\chi_i = \sin^2(\delta m_i^2 L/4E)$ .

We assume that for the neutrinos from distant sources arriving in the IceCube detector,  $\chi_1 \approx 0$  but  $\chi_2 = \chi_3 \approx 1/2$ ; i.e.,  $\delta m_1^2 \ll \delta m_2^2$  and  $\delta m_3^2$ . For example, if  $\delta m_1^2 \ll 10^{-17}$  eV<sup>2</sup> and  $\delta m_2^2, \delta m_3^2 \gtrsim 10^{-15}$  eV<sup>2</sup> then the condition for  $\chi_1 \approx 0$  and  $\chi_2 = \chi_3 \approx \frac{1}{2}$  for GRB neutrinos is satisfied.

The deviation from  $1/3$  for  $\nu'_\mu s$  is given by

$$\delta P_\mu = -\frac{1}{3} \left[ \frac{1}{2} (|U_{\mu 2}|^2 + |U_{\mu 3}|^2) \right]. \quad (12)$$

Using the current best values for the mixing parameters [23], this can be very close  $1/6$ , thus giving an extra reduction by a factor of 2 for the flux of  $\nu'_\mu s$ . In a model for pseudo-Dirac-neutrinos via mirror-world, a further suppression by a factor  $1/2$  results in a net suppression by a factor of  $1/4$  [36]. Furthermore, the shift in  $P_e$  from the value  $1/3$  is about 0.8, and so the ratio  $\nu_e/\nu_\mu$  is about 3.

This is a very different physics possibility from the decay case but also gives rise to low fluxes of  $\nu_\mu s$  consistent with the lack of observation in the IceCube detector.

To summarize, we raise two rather different possibilities of neutrino properties which can account for the low fluxes of  $\nu'_\mu s$  at high energies, and give rather large values for the ratio of  $\nu_e$  to  $\nu_\mu$  fluxes. The two can be distinguished in several ways. The decay changes the primordial neutrino counting from 3 to  $3 + 4/7$ , and the pseudo-Dirac-neutrinos make the neutrinoless double beta decay unobservable. The flavor ratios  $\nu_e/\nu_\mu$  is another clear indicator of the mechanism responsible; in the decay case it may vary between 2.5 and 8, whereas is 3 for the pseudo-Dirac case. Only further experimental data can confirm or rule out these speculations. Since the scenarios considered here

do not suppress the electron neutrino flux, we have no problem with the PeV shower events reported by the IceCube Collaboration [37].

If  $\nu_\mu$  events in the PeV energy range are seen in the IceCube detector, the drastic explanation offered here becomes unnecessary. In that case, the observed flavor ratios can be used to constrain parameters of models such as the ones discussed here as has been discussed before [38].

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