Macroscopic Quantum Mechanics in a Classical Spacetime

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We apply the many-particle Schrödinger-Newton equation, which describes the coevolution of a many-particle quantum wave function and a classical space-time geometry, to macroscopic mechanical objects. By averaging over motions of the objects' internal degrees of freedom, we obtain an effective Schrödinger-Newton equation for their centers of mass, which can be monitored and manipulated at quantum levels by state-of-the-art optomechanics experiments. For a single macroscopic object moving quantum mechanically within a harmonic potential well, its quantum uncertainty is found to evolve at a frequency different from its classical eigenfrequency—with a difference that depends on the internal structure of the object—and can be observable using current technology. For several objects, the Schrödinger-Newton equation predicts semiclassical motions just like Newtonian physics, yet quantum uncertainty cannot be transferred from one object to another.

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Introduction and summary.—Testing nonrelativistic macroscopic objects has been a minor approach towards the search for effects of quantum gravity. Apart from the standard formulation of linearized quantum gravity [1], which seems rather implausible to test in the lab, several signatures have been conjectured: (i) gravity decoherence [2–12], where gravity introduces decoherence to macroscopic quantum superpositions; (ii) modifications to canonical quantization motivated by the existence of a minimum length scale [13–15]; and (iii) semiclassical gravity [16–18], which will be the subject of this paper. As originally suggested by Moller [16] and Rosenfeld [17], spacetime structure might still remain classical even if it is sourced by matters of quantum nature, if we impose ($G = c = 1$)

$$G_{\mu\nu} = 8\pi\langle\psi|\hat{T}_{\mu\nu}|\psi\rangle. \tag{1}$$

Here $G_{\mu\nu}$ is the Einstein tensor of a (3 + 1)-dimensional classical spacetime, $\hat{T}_{\mu\nu}$ is the operator for the energy-stress tensor, and $|\psi(t)\rangle$ is the wave function of all matters that evolve within this classical spacetime.

Many arguments exist against semiclassical gravity. Some rely on the conviction that a classical system cannot properly integrate with a quantum system without creating contradictions. Others are based on "intrinsic" mathematical inconsistencies, the most famous one between Eq. (1), state collapse, and $\nabla^\nu G_{\mu\nu} = 0$ [19]. Towards the former argument, it is the aim of this paper to explicitly work out the effects of classical gravity on the quantum mechanics of macroscopic objects; although we will find them counterintuitive, they do not seem dismissible right away. In fact, we shall find these effects "right on the horizon of testability" by current experimental technology. Towards the latter argument, we shall remain open-minded regarding the possibility of getting rid of quantum state reduction while at the same time avoiding the many-world interpretation of quantum mechanics [20,21] (also see Supplemental Material [22]).

The nonrelativistic version of Eq. (1), the so-called Schrödinger-Newton (SN) equation, has been extensively studied for single particles [23–29]. In this paper, we consider instead a macroscopic object consisting of many particles and will show that within certain parameter regimes the center-of-mass (c.m.) wave function approximately satisfies the following SN equation:

$$i\hbar\frac{\partial\Psi}{\partial t} = \left[-\frac{\hbar^2\nabla^2}{2M} + \frac{1}{2}M\omega_{\mathrm{c.m.}}^2 x^2 + \frac{1}{2}\mathcal{C}(x - \langle x\rangle)^2\right]\Psi. \tag{2}$$

Here $\langle x\rangle \equiv \langle\Psi|\hat{x}|\Psi\rangle$ is the expectation value of the c.m. position; $\omega_{\mathrm{c.m.}}$ is the eigenfrequency in the absence of gravity, determined by how the c.m. is confined; $\mathcal{C}$ is the SN coupling constant, from which we introduce $\omega_{\mathrm{SN}} \equiv \sqrt{\mathcal{C}/M}$. For Si crystal at 10 K, we estimate $\omega_{\mathrm{SN}} \sim 0.036\ \mathrm{s}^{-1}$, much larger than the naively expected $\sqrt{G\rho_0}$ from the object's mean density ρ_0, due to the high concentration of mass near lattice points.

For a single macroscopic object prepared in a squeezed Gaussian state, Eq. (2) leads to different evolutions of expectation values and quantum uncertainties, as illustrated in Fig. 1. Such a deviation can be tested by optomechanical devices in the quantum regime [30–34]. For two macroscopic objects interacting through gravity, we show further, using the two-body counterpart of Eq. (2), that classical gravity cannot be used to transfer quantum uncertainties—experimental demonstration of this effect will be much more difficult than demonstrating modifications in single-object dynamics.

We emphasize that it is *not* our aim to use the SN equation to explain the collapse of quantum states, or to provide a pointer basis for gravity decoherence, as has been

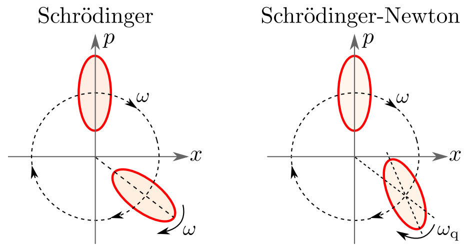

FIG. 1 (color online). Left: according to standard quantum mechanics, both the vector $(\langle x\rangle, \langle p\rangle)$ and the uncertainty ellipse of a Gaussian state for the c.m. of a macroscopic object rotate clockwise in phase space, at the same frequency $\omega = \omega_{\rm c.m.}$. Right: according to the c.m. Schrödinger-Newton equation (2), $(\langle x\rangle, \langle p\rangle)$ still rotates at $\omega_{\rm c.m.}$, but the uncertainty ellipse rotates at $\omega_{\rm q} \equiv (\omega_{\rm c.m.}^2 + \omega_{\rm SN}^2)^{1/2} > \omega_{\rm c.m.}$.

attempted in the literature [23–29]. We will take a conservative strategy, avoiding experimental regimes with exotic wave functions [10–12] and constrain ourselves to Gaussian states whose evolutions deviate little from predictions of standard quantum mechanics: just enough to be picked up by precision measurements. In this way, solutions to the SN equation we consider are much less complex than those in previous literature [23–29].

Many-particle SN equation.—For n nonrelativistic particles, if we denote their joint wave function as $\varphi(t, \mathbf{X})$ with $3n$-D vector $\mathbf{X} \equiv (\mathbf{x}_1, \ldots, \mathbf{x}_n)$ and $\mathbf{x}_k$ the 3D spatial coordinate of kth particle, then the many-particle SN equation, obtained by Penrose and Diósi [5,23], is

$$i\hbar\partial_t\varphi = \sum_k\left[-\frac{\hbar^2\nabla_k^2}{2m_k} + \frac{m_k U(t, \mathbf{x}_k)}{2}\right]\varphi + V(\mathbf{X})\varphi, \quad (3)$$

where $V(\mathbf{X})$ is the potential energy for nongravitational interactions, while the Newtonian potential U is given by

$$\nabla^2 U(t, \mathbf{x}) = 4\pi\sum_j \int d^{3n}\mathbf{X}|\varphi(t, \mathbf{X})|^2 m_j \delta(\mathbf{x} - \mathbf{x}_j). \quad (4)$$

Center of mass and separation of scales.—Equations (3) and (4) are still not suitable for experimental studies, because we cannot separately access each particle in a macroscopic object. In optomechanical devices, a light beam often probes the average displacements of atoms within the first few layers of the reflective coating of a mirror-endowed mass. Motion of this effective surface can often be well approximated by the c.m. motion of the entire object (see Refs. [31,32,35]); the error of this approximation is referred to as the "internal thermal noise" and has been shown to be suppressible below the free-mass standard quantum limit (SQL) [36], a quantum level of c.m. motion defined by the object's total mass and the measurement time scale [37–39]. This suppression is possible because (i) we tend to measure c.m. motion by averaging over a large number of atoms at the surface of the object, and (ii) we measure c.m. motion over a time scale much longer than ones at which atoms oscillate due to thermal or zero-point fluctuations. Obtaining the SN equation for the c.m. is therefore central to the experimental test of this model. Before doing so, let us consider the separation of temporal and spatial scales in the motion of a macroscopic piece of crystal.

The scales of c.m. motion are determined externally by how we confine the object during measurement, and by how we measure it. Here we consider motions with $\omega_{\rm c.m.}/(2\pi)$ from Hz to kHz scale. If the thermal noise level is below the free-mass SQL [36], then one can either use optical or feedback trapping to create mechanical oscillators with coherence time $\tau_{\rm c.m.}$ longer than $1/\omega_{\rm c.m.}$ [40,41]. Although not yet achieved, research towards sub-SQL devices in the Hz–kHz regime is being actively pursued [35,42,43]. In this regime, we have $\Delta x_{\rm c.m.} \sim \sqrt{\hbar/(M\omega_{\rm c.m.})}$; for $1\ {\rm g} < M < 10\ {\rm kg}$, $\Delta x_{\rm c.m.} \sim 10^{-19}$–$10^{-17}$ m.

By contrast, the internal motions of atoms are due to excitation of phonons [44], with a total variance of [45]

$$\langle x^2\rangle \equiv \frac{B^2}{8\pi^2} = \frac{\hbar^2}{mk_BT}\int_0^{+\infty}\frac{g(\nu)}{\xi}\left(\frac{1}{2} + \frac{1}{e^\xi - 1}\right)d\nu, \quad (5)$$

where B is also known as the "B factor" in x-ray diffraction, $\xi = h\nu/k_BT$, and $g(\nu)$ is the phonon density of states; the first term in the bracket gives rise to zero-point uncertainty $\Delta x_{\rm zp}^2$, while the second gives rise to thermal uncertainty $\Delta x_{\rm th}^2$. These have been studied experimentally by x-ray diffraction, through measurements of the Debye-Waller factor [46], and modeled precisely (for Si crystal, see Ref. [47]). Much below the Debye temperature, one can reach $\Delta x_{\rm th} \ll \Delta x_{\rm zp}$, with most atomic motion due to zero-point fluctuations near the Debye frequency $\omega_{\rm D}$. For Si crystal, $\omega_{\rm D} \sim 10^{14}\ {\rm s}^{-1}$, $\Delta x_{\rm zp} = 4.86 \times 10^{-12}$ m, and $\Delta x_{\rm th}(293\ {\rm K}) = 5.78 \times 10^{-12}$ m [47]. At lower temperatures, $\Delta x_{\rm th} \propto T$; therefore, on the scale of $\sim$10 K, at which our proposed experiment operates, we have $\Delta x_{\rm zp} \gg \Delta x_{\rm th} \gg \Delta x_{\rm c.m.}$.

SN equation for the c.m.—For a crystal with n atoms, the c.m. is at $\mathbf{x}_{\rm c.m.} = (1/n)\sum_k \mathbf{x}_k$, and motion of the kth atom in the c.m. frame is $\mathbf{y}_k \equiv \mathbf{x}_k - \mathbf{x}_{\rm c.m.}$. In standard quantum mechanics, for interatom interaction that only depends on the separation of atoms, the c.m. and internal DOFs are separable: $\varphi(t, \mathbf{X}) = \Psi_{\rm c.m.}(t, \mathbf{x})\Psi_{\rm int}(t, \mathbf{Y})$, with $3(n-1)$-D vector $\mathbf{Y} \equiv (\mathbf{y}_1, \ldots, \mathbf{y}_{n-1})$. The two wave functions evolve independently:

$$i\hbar\partial_t\Psi_{\rm c.m.}(t, \mathbf{x}) = H_{\rm c.m.}\Psi_{\rm c.m.}(t, \mathbf{x}), \quad (6)$$

$$i\hbar\partial_t\Psi_{\rm int}(t, \mathbf{Y}) = H_{\rm int}\Psi_{\rm int}(t, \mathbf{Y}). \quad (7)$$

For classical gravity, let us first still assume separability, $\varphi = \Psi_{\rm c.m.}\Psi_{\rm int}$, and we will show this remains true (with negligible error) under evolution. Specifically, the sum of SN terms in Eq. (3) becomes

$$V_{\mathrm{SN}}(\mathbf{x},\mathbf{Y}) = \sum_k m_k U(\mathbf{x}_k)/2 = \sum_k \int \varepsilon[\mathbf{x}-\mathbf{z}+\mathbf{y}_k]\Psi^2_{\mathrm{c.m.}}(\mathbf{z})d^3\mathbf{z}. \quad (8)$$

Here we have suppressed dependence on time and defined

$$\varepsilon(\mathbf{z}) = -\frac{Gm}{2}\int \frac{\tilde{\rho}_{\mathrm{int}}(\mathbf{y})}{|\mathbf{z}-\mathbf{y}|}d^3\mathbf{y} \quad (9)$$

as *half* the gravitational potential energy of a mass m at location $\mathbf{z}$ (in a c.m. frame), due to the entire lattice, and

$$\tilde{\rho}_{\mathrm{int}}(\mathbf{y}) = m\sum_{j=1}^{n}\int \delta(\mathbf{y}-\mathbf{y}'_j)|\Psi_{\mathrm{int}}(\mathbf{Y}')|^2 d^{3n-3}\mathbf{Y}' \quad (10)$$

is the c.m. frame mass density. (Note that $\mathbf{y}_n \equiv -\sum_{j=1}^{n-1}\mathbf{y}_j$.) We will now show that V_{SN} approximately separates into a sum of terms that either only depend on $\mathbf{Y}$ or only on $\mathbf{x}$. Taylor expansion of V_{SN} in $\mathbf{x}$ and $\mathbf{z}$ leads to (for one direction)

$$V_{\mathrm{SN}} = \sum_k \varepsilon(\mathbf{y}_k) + (x_{\mathrm{c.m.}} - \langle x_{\mathrm{c.m.}}\rangle)\sum_k \varepsilon'(\mathbf{y}_k) + \frac{x^2_{\mathrm{c.m.}} - 2x_{\mathrm{c.m.}}\langle x_{\mathrm{c.m.}}\rangle + \langle x^2_{\mathrm{c.m.}}\rangle}{2}\sum_k \varepsilon''(\mathbf{y}_k), \quad (11)$$

while higher orders fall as powers of $\Delta x_{\mathrm{c.m.}}/\Delta x_{\mathrm{zp}} \ll 1$. Here in V_{SN}, the first term describes the leading SN correction to internal motion and can be absorbed into H_{int}. The second term describes the interaction between the c.m. motion and each individual atom—it can be shown to have negligible effects, because internal motions of different atoms are largely independent, and at much faster time scales. The third term is largely a correction to the c.m. motion; its main effect is captured if we replace it by its ensemble average over internal motion (again allowed by approximate independence between atoms; see Supplemental Material [22]): $\sum_k \varepsilon''(\mathbf{y}_k) \to \mathcal{C} \equiv \langle \sum_k \varepsilon''(\mathbf{y}_k)\rangle$, with

$$\mathcal{C} = -\frac{1}{2}\frac{\partial^2}{\partial z^2}\left[\int \frac{G\tilde{\rho}_{\mathrm{int}}(\mathbf{y})\tilde{\rho}_{\mathrm{int}}(\mathbf{y}')}{|\mathbf{z}+\mathbf{y}-\mathbf{y}'|}d\mathbf{y}d\mathbf{y}'\right]_{\mathbf{z}=0}, \quad (12)$$

which is half the double spatial derivative of the "self-gravitational energy" of the lattice as it is being translated. As this is independent from the internal motion $\mathbf{Y}$, we therefore obtain the leading correction to $H_{\mathrm{c.m.}}$, which justifies Eq. (2) introduced at the beginning.

Estimates for ω_{SN}.—Let us now estimate the magnitude of ω_{SN} from Eq. (12). Naively assuming a homogeneous mass distribution with constant density ρ_0 leads to

$$\mathcal{C}^{\mathrm{hom}} \approx GM\rho_0, \qquad \omega^{\mathrm{hom}}_{\mathrm{SN}} \approx \sqrt{G\rho_0}, \quad (13)$$

up to a geometric factor that depends on the shape of the object. This is a typical estimate for the gravity-decoherence time scale for a homogeneous object prepared in a nearly Gaussian quantum state with position uncertainty much less than its size [12]. Using the mean density of Si crystal, this is roughly $4 \times 10^{-4}\ \mathrm{s}^{-1}$. However, mass in a lattice is highly concentrated near lattice sites; the realistic $\tilde{\rho}_{\mathrm{int}}$ at low temperatures contains a total mass of m around each lattice point, Gaussian distributed with uncertainty of Δx_{zp} in each direction. This gives, through Eq. (12),

$$\omega^{\mathrm{crystal}}_{\mathrm{SN}} = \sqrt{Gm/(12\sqrt{\pi}\Delta x^3_{\mathrm{zp}})}. \quad (14)$$

For $\Delta x_{\mathrm{zp}} \approx 4.86 \times 10^{-12}$ m, we obtain $\omega^{\mathrm{Si}}_{\mathrm{SN}} \approx 0.036\ \mathrm{s}^{-1}$, nearly 100 times $\omega^{\mathrm{hom}}_{\mathrm{SN}}$. If we define

$$\Lambda = (\omega^{\mathrm{crystal}}_{\mathrm{SN}}/\omega^{\mathrm{hom}}_{\mathrm{SN}})^2 = m/(12\sqrt{\pi}\rho_0\Delta x^3_{\mathrm{zp}}), \quad (15)$$

then $\Lambda = 8.3 \times 10^3$ for Si crystal.

Evolutions of Gaussian states and experimental tests.—As one can easily prove, Gaussian states remain Gaussian under Eq. (2); the self-contained evolution equations for first and second moments of $\hat{x}$ and $\hat{p}$, which completely determine the evolving Gaussian state, are given by

$$\langle \dot{\hat{x}}\rangle = \langle \hat{p}\rangle/M, \qquad \langle \dot{\hat{p}}\rangle = -M\omega^2_{\mathrm{c.m.}}\langle \hat{x}\rangle, \quad (16)$$

$$\dot{V}_{xx} = 2V_{xp}/M, \qquad \dot{V}_{pp} = -2M(\omega^2_{\mathrm{c.m.}} + \omega^2_{\mathrm{SN}})V_{xp}, \quad (17)$$

$$\dot{V}_{xp} = V_{pp}/M - M(\omega^2_{\mathrm{c.m.}} + \omega^2_{\mathrm{SN}})V_{xx}. \quad (18)$$

For covariance we have defined $V_{AB} \equiv \langle \hat{A}\hat{B} + \hat{B}\hat{A}\rangle/2 - \langle \hat{A}\rangle\langle \hat{B}\rangle$. Equation (16) indicates that $\langle \hat{x}\rangle$ and $\langle \hat{p}\rangle$ evolve the same way as a harmonic oscillator with angular frequency $\omega_{\mathrm{c.m.}}$—any semiclassical measurement of on $\langle \hat{x}\rangle$ and $\langle \hat{p}\rangle$ will confirm classical physics. On the other hand, evolution of second moments (which represent quantum uncertainty), is modified to that of a harmonic oscillator with a different frequency (see Fig. 1):

$$\omega_{\mathrm{q}} \equiv \sqrt{\omega^2_{\mathrm{c.m.}} + \omega^2_{\mathrm{SN}}}. \quad (19)$$

Equations (16)–(18) for Gaussian states can also be reproduced by a set of effective Heisenberg equations that contain expectation values:

$$\dot{\hat{x}} = \hat{p}/M, \qquad \dot{\hat{p}} = -M\omega^2_{\mathrm{c.m.}}\hat{x} - \mathcal{C}(\hat{x} - \langle \hat{x}\rangle). \quad (20)$$

Classical gravity introduces a $\mathcal{C}$-dependent term to Eq. (20), in a way that only affects quantum uncertainty.

The most obvious test for the SN effect is to prepare a mechanical oscillator into a squeezed initial state, let it evolve for a duration τ, and carry out state tomography. We need to detect an extra phase $\Delta\theta = \omega_{\mathrm{c.m.}}\tau(\omega^2_{\mathrm{SN}}/\omega^2_{\mathrm{c.m.}})$ in the rotation of the quantum uncertainty ellipse. This seems rather difficult because $\omega_{\mathrm{SN}}/\omega_{\mathrm{c.m.}}$ is often a very small number, yet $\omega_{\mathrm{c.m.}}\tau$ is often not large either.

However, we have not taken advantage of the fact that $\Delta\theta$ is deterministic and repeatable. One way of doing so is to carry out a frequency-domain experiment. Suppose we use light (at ω_0) to continuously probe a mechanical object's position, with quantum backaction noise (in the form of radiation-pressure noise) comparable in level to thermal noise, as has been achieved by Purdy *et al.* [34]. The effective Heisenberg equations (valid for Gaussian states) for such an optomechanical device is given by [48,49]

$$\dot{\hat{x}} = \hat{p}/M, \tag{21}$$

$$\dot{\hat{p}} = -M\omega_{\rm c.m.}^2\hat{x} - 2\gamma_m\hat{p} - \mathcal{C}(\hat{x} - \langle\hat{x}\rangle) + \hat{F}_{\rm BA} + F_{\rm th}, \tag{22}$$

$$\hat{b}_2 = \hat{a}_2 + n_x + (\alpha/\hbar)\hat{x}, \qquad \hat{b}_1 = \hat{a}_1. \tag{23}$$

Here γ_m is the damping rate, α the optomechanical coupling constant, $\hat{F}_{\rm BA} \equiv \alpha\hat{a}_1$ the quantum backaction, and $F_{\rm th}$ the classical driving force (e.g., due to thermal noise). $\hat{a}_{1,2}$ represent quadratures of the ingoing optical field and $\hat{b}_{1,2}$ those of the outgoing field. (They correspond to amplitude and phase modulations of the carrier field at ω_0.) We have used n_x to denote sensing noise. As we show in the Supplemental Material [22], the outgoing quadrature $\hat{b}_2$ contains two prominent frequency contents, peaked at $\omega_{\rm c.m.}$ (due to classical motion driven by thermal forces) and at $\omega_{\rm q}$ (due to quantum motion driven by quantum fluctuation of light), respectively. Both have the same width (γ_m), and height (if thermal and backaction noises are comparable). In order to distinguish them, we require

$$S_{F_{\rm th}} \approx S_{F_{\rm BA}}, \qquad Q \gtrsim (\omega_{\rm c.m.}/\omega_{\rm SN})^2. \tag{24}$$

This indicates a SN-induced shift of $\Delta\theta \approx 2\pi/Q$ per cycle can be picked up by the frequency domain experiment, even in the presence of classical thermal noise $F_{\rm th}$.

For Si oscillators with $\omega_{\rm SN} \approx 0.036\ {\rm s}^{-1}$, if $\omega_{\rm c.m.} \approx 2\pi \times 10$ Hz, Eq. (24) requires $Q \gtrsim 3 \times 10^6$, which is challenging but possible [35]. If a lower-frequency oscillator, e.g., a torsional pendulum with $\omega_{\rm c.m.} \approx 2\pi \times 0.1$ Hz [50] can be probed with backaction noise above thermal noise, then we only require $Q \gtrsim 3 \times 10^2$.

SN equation for two macroscopic objects.—Now suppose we have two objects confined within potential wells frequencies $\omega_{1,2}$, and moving along the same direction as the separation vector $\mathbf{L}$ connecting their equilibrium positions (from 1 to 2). The standard approach for describing this interaction is to add a potential

$$V_g = \mathcal{E}_{12}'[x_{\rm c.m.}^{(1)} - x_{\rm c.m.}^{(2)}] + (\mathcal{C}_{12}/2)[x_{\rm c.m.}^{(1)} - x_{\rm c.m.}^{(2)}]^2 \tag{25}$$

into the Schrödinger equation, with

$$\mathcal{E}_{12} \equiv -\int d^3\mathbf{x}d^3\mathbf{y}\frac{G\tilde{\rho}_{\rm tot}^{(1)}(\mathbf{x})\tilde{\rho}_{\rm tot}^{(2)}(\mathbf{y})}{|\mathbf{L}+\mathbf{y}-\mathbf{x}|}, \qquad \mathcal{C}_{12} \equiv \frac{\partial^2\mathcal{E}_{12}}{\partial L^2}, \tag{26}$$

with $\tilde{\rho}_{\rm tot}^{(1)}$ and $\tilde{\rho}_{\rm tot}^{(2)}$ the mass densities of objects 1 and 2, respectively. As has been argued by Feynman, this way of including gravity tacitly assumes that gravity is quantum. Although quantum operators have not been assigned for the gravitational field, they can be viewed as have been adiabatically eliminated due to their fast response: quantum information can transfer between these objects via gravity. Suppose $\omega_1 = \omega_2 = \omega$, then V_g modifies the frequency of the two objects' differential mode—allowing a quantum state to slosh between them, at a frequency of $\Delta = |\omega_+ - \omega_-| = \mathcal{C}_{12}/(2M\omega)$.

Suppose we instead use the SN equation for the two macroscopic objects. In addition to modifying each object's own motion, we add a mutual term of

$$V_{\rm SN} = \mathcal{E}_{12}'[x_{\rm c.m.}^{(1)} - x_{\rm c.m.}^{(2)}] + \frac{\mathcal{C}_{12}}{2}[(x_{\rm c.m.}^{(1)} - \langle x_{\rm c.m.}^{(2)}\rangle)^2 + (x_{\rm c.m.}^{(2)} - \langle x_{\rm c.m.}^{(1)}\rangle)^2]. \tag{27}$$

This $V_{\rm SN}$ makes sure that only $\langle x_{\rm c.m.}\rangle$ gets transferred between the two objects the same way as in classical physics: quantum uncertainty does not transfer from one object to the other. To see this more explicitly for Gaussian states, we can write down the full set of effective Heisenberg equations governing these two c.m.'s:

$$\begin{aligned}\dot{\hat{x}}_j &= \hat{p}_j/M_j,\\ \dot{\hat{p}}_j &= -M_j\omega_{\rm c.m.}^2\hat{x}_j - \sum_{k,j}[\mathcal{E}_{kj}' + \mathcal{C}_{kj}(\hat{x}_j - \langle\hat{x}_k\rangle)].\end{aligned} \tag{28}$$

It is clear that expectation values follow classical physics, and quantum uncertainties are confined within each object—and evolve with a shifted frequency. Although we have shown theoretically that the inability of transferring quantum uncertainty and the shift between $\omega_{\rm c.m.}$ and $\omega_{\rm q}$ share the same origin, in practice, observing the frequency shift for a single object will be much easier, because $\mathcal{C}_{12} \sim GM^2/L^3 \lesssim GM\rho_0 \ll \mathcal{C}_{11}, \mathcal{C}_{22}$, due to the lack of the amplification factor Λ in $\mathcal{C}_{12}$ [cf. Eq. (15)].

Discussion.—The lack of experimental tests on the quantum coherence of dynamical gravity makes us believe that semiclassical gravity is still worth testing [18]. Our calculations have shown that signatures of classical gravity in macroscopic quantum mechanics, although extremely weak, can be detectable with current technology. In particular, the classical self-gravity of a single macroscopic object causes a much stronger signature than the classical mutual gravity between two separate objects: simply because the mass of a cold crystal is concentrated near lattice sites. We also speculate that the rate of gravity decoherence should also be expedited by $\Lambda^{1/2} \sim 100$—if it is indeed determined by gravitational self-energy [5,6]. However, due to the lack of a widely accepted microscopic model for gravity decoherence, this only makes it more hopeful for experimental attempts but would not enforce a powerful bound if decoherence were not to be found.

Finally, since classical gravity requires the existence of a global wave function of the Universe that does not collapse, (the unlikely case of) a positive experimental result will open up new opportunities of investigating the nature of quantum measurement.

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[1] P. M. Bonifacio, C. H.-T. Wang, J. T. Mendonça, and R. Bingham, Classical Quantum Gravity **26**, 145013 (2009).
[2] L. Diósi, Phys. Lett. A **120**, 377 (1987).
[3] L. Diósi, Phys. Rev. A **40**, 1165 (1989).
[4] L. Diósi and J. J. Halliwell, Phys. Rev. Lett. **81**, 2846 (1998).
[5] R. Penrose, Gen. Relativ. Gravit. **28**, 581 (1996)
[6] R. Penrose, *The Road to Reality: A Complete Guide to the Laws of the Universe* (Alfred A. Knopf Inc., New York, 2005).
[7] W. Marshall, C. Simon, R. Penrose, and D. Bouwmeester, Phys. Rev. Lett. **91**, 130401 (2003).
[8] T. Hong *et al.*, arXiv:1110.3349.
[9] J. van Wezel and T. H. Oosterkamp, Proc. R. Soc. A **468**, 35 (2012).
[10] R. Kaltenbaek, G. Hechenblaikner, N. Kiesel, O. Romero-Isart, K. C. Schwab, U. Johann, and M. Aspelmeyer, Exp. Astron. **34**, 123 (2012).
[11] O. Romero-Isart, A. C. Pflanzer, F. Blaser, R. Kaltenbaek, N. Kiesel, M. Aspelmeyer, and J. I. Cirac, Phys. Rev. Lett. **107**, 020405 (2011).
[12] O. Romero-Isart, Phys. Rev. A **84**, 052121 (2011).
[13] M. Maggiore, Phys. Lett. B **319**, 83 (1993).
[14] S. Das and E. C. Vagenas, Phys. Rev. Lett. **101**, 221301 (2008).
[15] I. Pikovski, M. R. Vanner, M. Aspelmeyer, M. S. Kim, and Č. Brukner, Nat. Phys. **8**, 393 (2012).
[16] C. Moller, *Les Theories Relativistes de la Gravitation* Colloques Internationaux CNRX 91 edited by A Lichnerowicz and M.-A. Tonnelat (Paris: CNRS) (1962).
[17] L. Rosenfeld, Nucl. Phys. **40**, 353 (1963).
[18] S. Carlip, Classical Quantum Gravity **25**, 154010 (2008).
[19] R. M. Wald, *General Relativity* (University of Chicago Press, Chicago, 1984).
[20] H. Everett III, Rev. Mod. Phys. **29**, 454 (1957).
[21] D. N. Page and C. D. Geilker, Phys. Rev. Lett. **47**, 979 (1981).
[22] See section C of the Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.110.170401.
[23] L. Diósi, Phys. Lett. **105A**, 199 (1984).
[24] I. M. Moroz, R. Penrose, and P. Tod, Classical Quantum Gravity **15**, 2733 (1999).
[25] R. Harrison, I. Moroz, and K. P. Tod, Nonlinearity **16**, 101 (2002).
[26] P. J. Salzman and S. Carlip, arXiv:gr-qc/0606120.
[27] S. L. Adler, J. Phys. A **40**, 755 (2007).
[28] F. S. Guzmán and L. A. Urena-López, Phys. Rev. D **69**, 124033 (2004).
[29] J. R. Meter, Classical Quantum Gravity **28**, 215013 (2011). In this article, the authors assumed that the c.m. degree of freedom satisfies the single-particle SN equation. This is significantly different from what we have obtained. In particular, in the limiting case when the c.m. wave function spread approaches zero, their SN term diverges due to divergence in the self-gravitational energy of a single particle, and the state changes dramtically. By contrast, our SN term also approaches zero in this case, as it arises from the change in many-particle gravitational energy induced by the c.m. motion, which decreases as the uncertainty in the c.m. motion decreases.
[30] See, e.g., a recent review article by M. Aspelmeyer, P. Meystre, and K. Schwab, Phys. Today **65**, 29 (2012).
[31] A. D. O'Connel *et al.*, Nature (London) **464**, 697 (2010).
[32] J. D. Teufel, T. Donner, D. Li, J. W. Harlow, M. S. Allman, K. Cicak, A. J. Sirois, J. D. Whittaker, K. W. Lehnert, and R. W. Simmonds, Nature (London) **475**, 359 (2011).
[33] J. Chan, T. P. Mayer Alegre, A. H. Safavi-Naeini, J. T. Hill, A. Krause, S. Gröblacher, M. Aspelmeyer, and O. Painter, Nature (London) **478**, 89 (2011).
[34] T. P. Purdy, R. W. Peterson, and C. A. Regal, arXiv:1209.6334.
[35] T. Corbitt, Y. Chen, E. Innerhofer, H. Müller-Ebhardt, D. Ottaway, H. Rehbein, D. Sigg, S. Whitcomb, C. Wipf, and N. Mavalvala, Phys. Rev. Lett. **98**, 150802 (2007); T. Corbitt, C. Wipf, T. Bodiya, D. Ottaway, D. Sigg, N. Smith, S. Whitcomb, and N. Mavalvala*ibid.***99**, 160801 (2007); T. Corbitt, Y. Chen, F. Khalili, D. Ottaway, S. Vyatchanin, S. Whitcomb, and N. Mavalvala, Phys. Rev. A **73**, 023801 (2006).
[36] V. B. Braginsky and F. Ya. Khalili, *Quantum Measurement* (Cambridge University Press, Cambridge, England, 1992).
[37] Y. Levin, Phys. Rev. D **57**, 659 (1998); G. Harry *et al.*, Appl. Opt. **45**, 1569 (2006); T. Hong *et al.*, arXiv:1207.6145; G. Cole *et al.*, arXiv:1302.6489.
[38] T. Westphal *et al.*, Appl. Phys. B **106**, 551 (2012); S. Gossler, Classical Quantum Gravity **27**, 084023 (2010).
[39] T. Uchiyama *et al.*, Phys. Rev. Lett. **108**, 141101 (2012).
[40] H. Müller-Ebhardt, H. Rehbein, C. Li, Y. Mino, K. Somiya, R. Schnabel, K. Danzmann, and Y. Chen, Phys. Rev. A **80**, 043802 (2009); H. Müller-Ebhardt, H. Rehbein, R. Schnabel, K. Danzmann, and Y. Chen, Phys. Rev. Lett. **100**, 013601 (2008); F. Khalili, S. Danilishin, H. Miao, H. Müller-Ebhardt, H. Yang, and Y. Chen, Phys. Rev. Lett. **105**, 070403 (2010).
[41] H. Miao, S. Danilishin, H. Müller-Ebhardt, H. Rehbein, K. Somiya, and Y. Chen, Phys. Rev. A **81**, 012114 (2010).
[42] B. Abbott *et al.*, New J. Phys. **11**, 073032 (2009); A. R. Neben, T. P. Bodiya, C. Wipf, E. Oelker, T. Corbitt, and N. Mavalvala, New J. Phys. **14**, 115008 (2012).
[43] H. Miao *et al.*, LIGO Report No. T1200008, 2012; S. L. Danilishin and F. Ya. Khalili, Living Rev. Relativity **15**, 5 (2012).
[44] C. Kittel, *Introduction to Solid State Physics* (Wiley, New York, 2004), 8th ed.
[45] M. Blackman and R. H. Fowler, Math. Proc. Cambridge Philos. Soc. **33**, 380 (1937).
[46] R. M. Housley and F. Hess, Phys. Rev. **146**, 517 (1966).
[47] C. Flensburg and R. F. Stewart, Phys. Rev. B **60**, 284 (1999).
[48] V. B. Braginsky, M. Gorodetsky, F. Khalil, A. Matsko, K. Thorne, and S. Vyatchanin, Phys. Rev. D **67**, 082001 (2003).
[49] F. Ya. Khalili, H. Miao, H. Yang, A. H. Safavi-Naeini, O. Painter, and Y. Chen, Phys. Rev. A **86**, 033840 (2012).
[50] K. Ishidoshiro, M. Ando, A. Takamori, H. Takahashi, K. Okada, N. Matsumoto, W. Kokuyama, N. Kanda, Y. Aso, and K. Tsubono, Phys. Rev. Lett. **106**, 161101 (2011).