Asymptotic Freedom in Strong Magnetic Fields

M. A. Andreichikov, V. D. Orlovsky,* and Yu. A. Simonov

Institute of Theoretical and Experimental Physics, B. Cheremushkinskaya 25, 117118 Moscow, Russia

(Received 3 December 2012; published 18 April 2013)

Perturbative gluon exchange interaction between quark and antiquark, or in a $3q$ system, is enhanced in a magnetic field and may cause vanishing of the total $q\bar{q}$ or $3q$ mass, and even unlimited decrease of it—recently called the magnetic collapse of QCD. The analysis of the one-loop correction below shows a considerable softening of this phenomenon due to $q\bar{q}$ loop contribution, similar to the Coulomb case of QED, leading to approximately logarithmic damping of gluon exchange interaction ($\langle V\rangle \approx \mathcal{O}(1/\ln|eB|)$) at large magnetic field.

DOI: 10.1103/PhysRevLett.110.162002 PACS numbers: 12.38.Aw, 13.40.Ks

Analysis of the hydrogen atom or positronium in a strong magnetic field shows a considerable enhancement of the Coulomb interaction, leading to the increase of binding energy [1–4]. This fact is due to reduction of the system size in the plane perpendicular to the direction of the magnetic field (MF) **B**, making it closer to the one-dimensional Coulomb system. As was shown in Refs. [2,5], the binding energy in the leading order in α grows as $\ln^2(B/me^3)$. It was shown later that the one-loop corrections to the one-photon exchange seriously change the situation: in the hydrogen atom the binding energy tends to the finite limit [6,7], while it shows an unbounded growth in positronium [8]. One should note that the absolute value of binding energy in both cases is not large and the upper limit of binding energy in hydrogen atom is 1.74 keV [7,9], while in positronium the collapse (vanishing) of the total mass occurs at very strong fields: $B_{cr} \sim 10^{40}$ Gauss [8].

Recently the dynamics of the $q\bar{q}$ system in a strong magnetic field was studied in the framework of the relativistic Hamiltonian, derived from the path integral for the corresponding Green's function [10]. The relevant technique in the case of no MF was extensively developed in Ref. [11]. This formalism essentially exploits the background perturbation theory [12], where in our QCD case, the role of background is played by vacuum nonperturbative configurations ensuring confinement, while the perturbative series better converges due to the presence of infrared (IR) regulators and lack of IR renormalons, as compared to standard perturbation theory. It was shown in Ref. [10] that the one-gluon-exchange (OGE) interaction, or color Coulomb, becomes increasingly important for large MF when OGE is taken in the leading (no quark loop) approximation. In particular, the mass of the $(q\bar{q})$ meson vanishes at $\sqrt{|e_qB|} \sim O(1\ \text{GeV})$, i.e., for $B \approx 10^{19}$–10^{20} Gauss. This fact would imply a radical reconstruction of the vacuum, a proposal made in a different context in Refs. [13,14].

A similar situation occurs in the case of baryons in strong MF: the baryon (e.g., the neutron) mass vanishes at approximately the same B_{crit} as for mesons [15].

It is therefore very important to check whether the quark loop corrections may stabilize the hadron mass at high MF, similar to the case of the hydrogen atom. As for gluon loop corrections, ensuring asymptotic freedom (AF), they are neutral to MF, and AF only decreases the growth of binding energy (b.e.) [10] (b.e. grows as $\ln\ln(eB/\sigma)$ instead of $\ln^2 eB$ in atoms) but does not prevent the collapse. But those are fermion loop contributions which stabilized the hydrogen atom, and below we shall study the quark-antiquark loops in the case of the $q\bar{q}$ mesons, taking into account both confinement and OGE interaction.

One should stress at this point that in our problem of the strongly squeezed $q\bar{q}$ system, similar to the case of hydrogen atom, the role of the scale parameter is played by $Q^2 \simeq eB$, with $\alpha_s(Q^2) \ll 1$ for $eB \gg \sigma$, Λ^2_{QCD}, and one can expect a good convergence of perturbative loop corrections. For the hydrogen atom this convergence was proved in Ref. [9], and we expect the same in our case.

Using the Hamiltonian, derived in Ref. [16], we obtain the following expression for the mass of charged meson in magnetic field B (the lowest spin-1 state, corresponding to ρ^+) in the case of zero quark masses,

$$M(B,\omega,\gamma) = \frac{1}{2\omega}\left(\sqrt{e^2B^2 + \frac{8\sigma\omega}{\gamma}} + \frac{1}{2}\sqrt{\frac{2\sigma\omega}{\gamma}}\right) + \frac{\gamma\sigma}{2} + \omega - \frac{eB}{2\omega} + \Delta M_{\text{Coul}} + \Delta M_{\text{SE}} + \Delta M_{\text{SS}}, \quad (1)$$

where σ is the string tension and ω and γ are einbein parameters, minimizing the meson mass. The last three terms are the Coulomb correction ΔM_{Coul} as the average of perturbative gluon exchange potential $\langle V(Q)\rangle_{\text{mes}}$, the nonperturbative self-energy term ΔM_{SE}, and the contribution from spin-spin interaction ΔM_{SS} (see Ref. [10] for explicit definitions). Note that without the last three corrections (1) tends to finite limit at large MF. As we said before, at large B the Coulomb term decreases unboundedly in the leading approximation without quark loops. We will concentrate now only on the Coulomb correction and see how the quark loop contribution can change the situation.

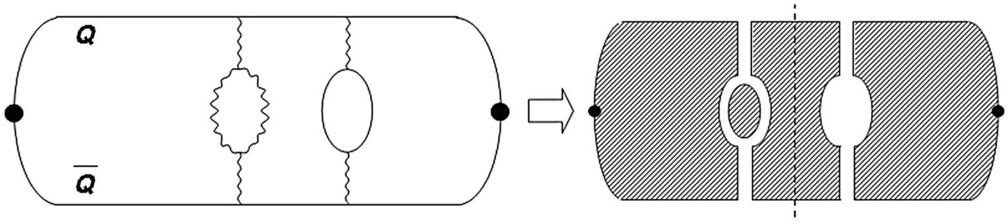

FIG. 1. Gluon and $q\bar{q}$ loop insertions in the gluon exchange between quark Q and antiquark $\bar{Q}$ in the meson $(Q\bar{Q})$.

We start with the standard one-loop expression for the gluon self-energy part, which contributes to the gluon propagator as [17]

$$D(q) = \frac{4\pi}{q^2 - \frac{g^2(\mu_0^2)}{16\pi^2}\tilde{\Pi}(q)}, \tag{2}$$

where $\tilde{\Pi}(q)$ contains the sum of gluon and quark loop terms,

$$\tilde{\Pi}(q) = \tilde{\Pi}_{gl}(q) - \tilde{\Pi}_{q\bar{q}}(q). \tag{3}$$

In the absence of MF and neglecting strong interaction between gluons, one has

$$\tilde{\Pi}_{gl}(q) = -\frac{11}{3}N_c q^2 \ln\frac{|q^2|}{\mu_0^2},$$
$$\tilde{\Pi}_{q\bar{q}}(q) = -\frac{2}{3}n_f q^2 \ln\frac{|q^2|}{\mu_0^2}, \tag{4}$$

leading to the standard AF expression for the OGE potential ($q^2 = -\mathbf{Q}^2 = -(q_\perp^2 + q_3^2)$, $\alpha_s^{(0)} = (g^2(\mu_0^2))/4\pi$),

$$V(Q) = -\frac{4}{3}\frac{\alpha_s^{(0)}4\pi}{Q^2(1 + \frac{\alpha_s^{(0)}}{4\pi}\beta_0 \ln\frac{Q^2}{\mu_0^2})} = -\frac{16\pi}{3Q^2}\alpha_s(Q),$$
$$\alpha_s(Q) = \frac{4\pi}{\beta_0 \ln\frac{Q^2}{\Lambda^2}}, \tag{5}$$

where $\beta_0 = (11/3)N_c - (2/3)n_f$.

In the case of strong MF one can retain in $\tilde{\Pi}_{q\bar{q}}(q)$ the contribution of the lowest Landau levels, which couples only to $(q_0, 0, 0, q_3)$ polarizations, and obtain the expression, known for a long time [18] for the $(e^+ \ e^-)$ case, which is rewritten in our case by the replacement $\alpha_{\rm QED} \to \alpha_s^{(0)}(n_f/2)$,

$$\frac{\alpha_s^{(0)}}{4\pi}\tilde{\pi}_{q\bar{q}}(q) = -\frac{\alpha_s^{(0)} n_f|e_qB|}{\pi}\exp\left(-\frac{q_\perp^2}{2|e_qB|}\right)T\left(\frac{q_3^2}{4m^2}\right), \tag{6}$$

where

$$T(z) = -\frac{\ln(\sqrt{1+z}+\sqrt{z})}{\sqrt{z(z+1)}} + 1 = \begin{cases} \frac{2}{3}z, & z \ll 1 \\ 1, & z \gg 1 \end{cases}.$$

A convenient approximation with accuracy better than 10% is $T(z) = (2z/(3+2z))$ [7].

At this point one should define the mass parameter m, which in the case of QED was (renormalized) electron mass [6–8]. In our case the gluon and quark loop contributions correspond to the graphs in Fig. 1, where we have denoted the gluon line as a double quark line to make clear the gauge interacting regions, and the confining regions are cross-hatched. One can see in Fig. 1 that q and $\bar{q}$ in the quark loop are not interacting by simple gluon exchange similarly to the $e^+ \ e^-$ loop in the lowest order, but in the $q\bar{q}$ case only the exchange of white objects (mesons or glueballs) can take place in higher orders.

Moreover, quarks are moving on the borders of the confining surfaces and hence should have the typical energies of quarks at the ends of the string—they are denoted as $\omega = \langle\sqrt{\mathbf{p}_q^2 + m_q^2}\rangle$ in the path-integral Hamiltonian [10,11] and are of the order of $\sqrt{\sigma}$, σ is string tension, and $\sigma = 0.18\ {\rm GeV}^2$. Thus one can replace $4m^2$ in (6) by 4σ.

Finally, one should take into account the nonperturbative (confining) interaction inside the gluon loops, as shown in Fig. 1. As shown in Ref. [19] this amounts to the replacement $\ln(Q^2/\mu_0^2) \to \ln((Q^2 + M_B^2)/\mu_0^2)$, where $M_B \approx 1$ GeV and is expressed solely through σ. As a result one obtains the following form of the OGE interaction, taking into account the gluon and quark loop effects,

$$V(Q) = -\frac{16\pi\alpha_s^{(0)}}{3\left[Q^2\left(1 + \frac{\alpha_s^{(0)}}{4\pi}\frac{11}{3}N_c \ln\frac{Q^2+M_B^2}{\mu_0^2}\right) + \frac{\alpha_s^{(0)} n_f|e_qB|}{\pi}\exp\left(\frac{-q_\perp^2}{2|e_qB|}\right)T\left(\frac{q_3^2}{4\sigma}\right)\right]}, \tag{7}$$

where $\alpha_s^{(0)} = 4\pi/((11/3)N_c \ln((\mu_0^2 + M_B^2)/\Lambda_V^2)$ and $Q^2 = q_\perp^2 + q_3^2$.

We can now estimate the average value of $V(Q)$ in the meson state with the wave function, which takes into account magnetic field and confinement, $V_{\rm conf} = \sigma\eta$, where $\boldsymbol{\eta}$ is the relative coordinate of two quarks. The latter is convenient to replace by the quadratic form $V_{\rm conf} \to \tilde{V}_{\rm conf} = \frac{\sigma}{2}((\eta^2/\gamma) + \gamma)$, with γ to be found from the

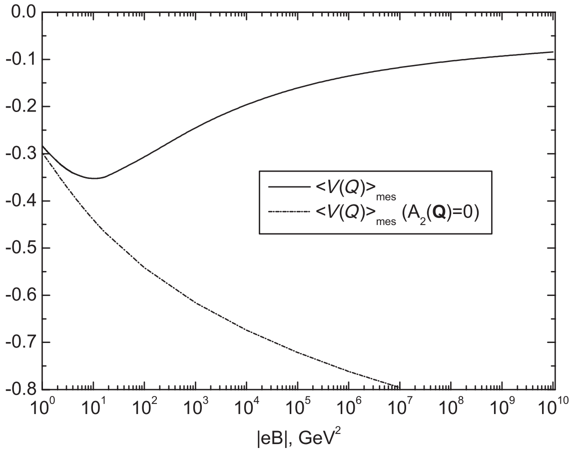

FIG. 2. Coulomb correction to the meson mass (10) in GeV as a function of the magnetic field with (solid line) and without (broken line) taking into account the quark loops contributions.

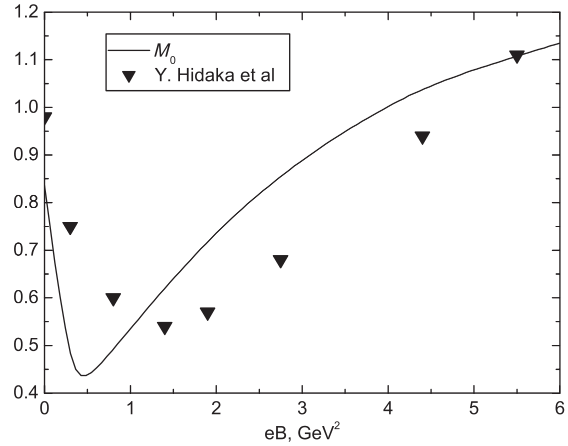

FIG. 3. The total meson mass in GeV as a function of magnetic field, taking into account the quark loop contributions in comparison with lattice data of Ref. [20].

stationary point condition, $\frac{\partial M_{\rm mes}}{\partial \gamma}|_{\gamma=\gamma_0} = 0$. This replacement has accuracy of the order of 5%, which is enough for our purposes. Then the lowest Landau levels wave functions can be easily written,

$$\psi(\eta_1, \eta_3) = \frac{1}{\sqrt{\pi^{3/2} r_\perp^2 r_3}} \exp\left(-\frac{\eta_\perp^2}{2r_\perp^2} - \frac{\eta_3^2}{2r_3^2}\right), \qquad (8)$$

where $r_\perp$ and r_3 are some functions of MF (see Ref. [10] for details), for large fields $r_\perp \approx \sqrt{2/eB}$, $r_3 \approx \sqrt{2/\sigma}$, and we can compute the OGE contribution to the meson mass $\langle V(Q)\rangle_{\rm mes}$,

$$\langle V(Q)\rangle_{\rm mes} = \int V(Q)\psi^2(q_\perp, q_3)\frac{d^2q_\perp dq_3}{(2\pi)^3}, \qquad (9)$$

where $\psi^2(q_\perp, q_3)$ is the Fourier transform of the squared wave function $\psi^2(\eta_1, \eta_3)$.

The insertion of (8) and (7) in (9) yields

$$\langle V(Q)\rangle_{\rm mes} = -C\int \frac{e^{-(q_\perp^2 r_\perp^2/4)-(q_3^2 r_3^2/4)} d^2q_\perp dq_3}{Q^2 A_1(q_\perp^2 + q_3^2) + A_2(q_\perp^2, q_3^2)}, \qquad (10)$$

where

$$A_1 = 1 + \frac{\alpha_s^{(0)}}{4\pi}\frac{11}{3}N_c \ln\left(\frac{q_\perp^2 + q_3^2 + M_B^2}{\mu_0^2}\right), \qquad (11)$$

$$A_2 = \frac{\alpha_s^{(0)} n_f |e_q B|}{\pi} e^{-(q_\perp^2/2|e_q B|)} T\left(\frac{q_3^2}{4\sigma}\right), \quad C = \frac{16\pi\alpha_s^{(0)}}{3(2\pi)^3}. \qquad (12)$$

Results of calculations for $\langle V(Q)\rangle_{\rm mes}$ as a function of MF are shown on Fig. 2 for asymptotically large fields. The total mass $M_0(B) = M(B, \omega_0, \gamma_0)$ is shown on Fig. 3 for relatively small fields. The values of parameters $\alpha_s^{(0)}$ and μ_0 are connected by the relation $\alpha_s^{(0)} = (4\pi/(11/3)N_c \ln((\mu_0^2 + M_B^2)/\Lambda_V^2))$, and we have chosen $n_f = 3$, $\mu_0 = 1.1$ GeV, $\Lambda_V = 0.385$ GeV, so $\alpha_s^{(0)} = 0.42$. As one can see from Fig. 2, accounting for quark loop contributions leads to the prevention of the so-called magnetic collapse of QCD—the resulting correction vanishes at large MF (roughly as $-\frac{1}{\ln|eB|}$), so the meson mass is always finite. The profile of the meson mass trajectory is in reasonable agreement with quenched lattice calculations [20].

The authors are grateful to B. O. Kerbikov for useful discussions. The financial support of the Dynasty Foundation to V. D. O. is gratefully acknowledged.

*orlovskii@itep.ru

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