Macroscopicity of Mechanical Quantum Superposition States

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We propose an experimentally accessible, objective measure for the macroscopicity of superposition states in mechanical quantum systems. Based on the observable consequences of a minimal, macrorealist extension of quantum mechanics, it allows one to quantify the degree of macroscopicity achieved in different experiments.

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Introduction.—Experiments probing the quantum superposition principle at the borderline to classical mechanics are a driving force of modern physics. This includes the demonstration of superposition states of counterrunning currents involving 10^{14} electrons [1,2], of Bose-Einstein condensed atoms [3], and complex molecules [4].

Various measures have been suggested for the size of superposition states involving macroscopically distinct properties of complex quantum systems [5–11]. Most of them refer to specific types or representations of quantum states, or count the operational resources required to analyze them. While most proposals seem to be grounded in a common information-theoretic framework [12], we still lack a method of attributing a definite and unbiased measure to all experimental tests of the quantum superposition principle.

The task to define a macroscopicity measure "within" quantum theory is confounded by a fundamental problem: We are free to decompose a many particle Hilbert space into different tensor products, such that a complicated single-particle representation of a wave function may look mundane after a change of variables to collective degrees of freedom. This highlights the problems of an *ad hoc* selection of distinguished observables.

In view of this, we propose to call a quantum state of a mechanical system the more macroscopic the better its experimental demonstration allows one to rule out even a minimal modification of quantum mechanics, which would predict a failure of the superposition principle on the macroscale. Turning this characterization into a definite measure requires one to specify the minimal modification.

Fortunately, it is not necessary to worry about the details of possible nonlinear or stochastic additions to the Schrödinger equation, which might embody the coarse-grained effects of a deeper theory, say, incorporating gravitation or a granular space-time [13–15], or might represent a fundamental stochasticity [16–18]. All that matters empirically are their observable consequences, described by the dynamics of the many-body density operator. We argue that basic consistency, symmetry, and scaling arguments lead to an explicit, parametrizable characterization of the impact of a minimally intrusive modification. The fact that no evidence of such physics beyond the Schrödinger equation is seen in a quantum experiment rules out a certain parameter region. For a superposition state in a different experiment to be more macroscopic, its demonstration must exclude a larger parameter region, implying that possible modifications must be even weaker. Diverse experiments can thus be compared without prejudice.

Minimal modification of quantum mechanics.—The modification must serve to "classicalize" the state evolution in the sense that superpositions of macroscopically distinct mechanical states are turned rapidly into mixtures. The operational description of quantum theory, based on the state operator ρ, its completely positive and trace-preserving time evolution, and a consistent rule of assigning probabilities to measurements [19], allows one to treat (nonrelativistic) quantum and classical mechanics in a common general formalism. It is therefore natural to account for an objective modification of the quantum time evolution in the framework of dynamical semigroups [20]. That is, the effect of the modification can be expressed as a generator $\mathcal{L}_N$ added to the von Neumann equation for the state of motion ρ_N of an arbitrary system of N particles, $\partial_t \rho_N = [\mathsf{H}, \rho_N]/i\hbar + \mathcal{L}_N \rho_N$.

In addition, we require the modification (i) to be invariant under Galilean transformations, avoiding a distinguished frame of reference, (ii) to leave the exchange symmetry of identical particles unaffected, (iii) to respect the "innocent bystander" condition that adding an uncorrelated system leaves the reduced state unchanged, and (iv) to display scale invariance with respect to the center-of-mass of a compound system. We will see that these requirements essentially determine the form of a possible minimal modification.

Let us first consider an elementary particle of mass m using the formalism of quantum dynamical semigroups. A theorem by Holevo [21] states that any Galilean invariant addition to the von Neumann equation for the state of motion ρ must have the form $\partial_t \rho = [\mathsf{H}, \rho]/i\hbar + \mathcal{L}_1 \rho$, with

$$\mathcal{L}_1\rho = \frac{1}{\tau}\left[\int d^3s d^3q g(s,q)\mathsf{W}(\boldsymbol{s},\boldsymbol{q})\rho\mathsf{W}^\dagger(\boldsymbol{s},\boldsymbol{q}) - \rho\right], \quad (1)$$

if one disregards unbounded diffusion terms, which would yield a substantially more drastic modification. The operators

$$\mathsf{W}(\boldsymbol{s}, m\boldsymbol{v}) = \exp\left[\frac{i}{\hbar}(\mathbf{P}\cdot\boldsymbol{s} - m\boldsymbol{v}\cdot\mathbf{X})\right] \quad (2)$$

effect a translation $\boldsymbol{s}$ and a velocity boost $\boldsymbol{v}$ of the elementary particle, while $g(s,q)$ is a positive, isotropic, and normalized phase-space distribution, whose standard deviations for the position and the momentum variable will be denoted by σ_s and σ_q. The von Neumann equation is reobtained for $\sigma_s = \sigma_q = 0$.

The modification (1) serves its purpose of classicalizing the motion of a single particle: It effects a decay of the position and the momentum off-diagonal matrix elements of ρ. The parameter τ provides the corresponding time scale for those matrix elements which are more than the critical length scale $\hbar/\sigma_q$ off the diagonal in position, or more than $\hbar/\sigma_s$ off in momentum, whereas smaller-scale coherences may survive much longer. Delocalized superposition states thus get localized in phase space as time evolves, ultimately rendering the phase-space representation of ρ indistinguishable from an equivalent classical distribution. At the same time, the modification (1) induces a position and momentum diffusion, implying that any bound particle gradually gains energy. For harmonic binding potentials with frequency ω, the energy increases as $\sigma_q^2/2m + m\omega^2\sigma_s^2/2$ per unit of time τ.

Many-particle description.—The requirement of Galilean invariance in a general mechanical system of N particles implies that the phase-space translation operators must effect a net shift of the center-of-mass coordinates by $\boldsymbol{s}$ and $\boldsymbol{v}$. On the other hand, the scale invariance conditions with respect to an innocent bystander (iii) and to the center-of-mass (iv) require that the equation for the N-particle state reduces to the single-particle form whether one traces over the other $N-1$ particles or over the relative coordinates in a compound object of rigidly bound constituents; in the latter case the single mass should be replaced by the total mass $M = \sum_n m_n$. This is achieved by composing the N-particle operators as the weighted sum of the single-particle operators (2),

$$\mathsf{W}_N(\boldsymbol{s},\boldsymbol{q}) = \sum_{n=1}^{N}\frac{m_n}{m_e}\exp\left[\frac{i}{\hbar}\left(\mathbf{P}_n\cdot\frac{m_e}{m_n}\boldsymbol{s} - \boldsymbol{q}\cdot\mathbf{X}_n\right)\right], \quad (3)$$

where m_e is an arbitrary reference mass; see Ref. [22] for details.

We note that the operators (3) conserve the exchange symmetry of a quantum state. The corresponding N-particle equation,

$$\mathcal{L}_N\rho_N = \frac{1}{\tau_e}\int d^3s d^3q g_e(s,q)\Big[\mathsf{W}_N(\boldsymbol{s},\boldsymbol{q})\rho_N\mathsf{W}_N^\dagger(\boldsymbol{s},\boldsymbol{q}) - \frac{1}{2}\{\mathsf{W}_N^\dagger(\boldsymbol{s},\boldsymbol{q})\mathsf{W}_N(\boldsymbol{s},\boldsymbol{q}),\rho_N\}\Big], \quad (4)$$

thus leaves boson and fermion statistics invariant (ii). The equation is completely determined once we specify the mass m_e, the coherence time parameter τ_e, and the normalized distribution function $g_e(s,q)$ for the reference particle. The innocent bystander condition (iii) guarantees that no correlations are introduced between different (possibly uncorrelated or even far apart) subsets of particles. Property (iv), on the other hand, admits the single-particle description (1) not only for elementary point particles (e.g., electrons) but also for compound objects such as atoms, molecules, or even solids.

The classicalization of the center-of-mass motion of an extended compound object of total mass M can be approximated by the single-particle form (1), if the relative motion of the constituents around their rigidly bound equilibrium positions can be neglected. The Fourier transform $\tilde{\varrho}(\boldsymbol{q}) = \int d^3x\varrho(\boldsymbol{x})e^{-i\boldsymbol{q}\cdot\boldsymbol{x}/\hbar}$ of the mass density $\varrho(\boldsymbol{x})$ of the compound modifies the rate and the phase-space distribution of the effective center-of-mass classicalization,

$$\frac{1}{\tau} = \frac{1}{\tau_e}\frac{1}{m_e^2}\int d^3s d^3q g_e(s,q)|\tilde{\varrho}(\boldsymbol{q})|^2, \quad (5)$$

$$g(s,\boldsymbol{q}) = \frac{\tau M^3}{\tau_e m_e^5} g_e\left(\frac{M}{m_e}s, q\right)|\tilde{\varrho}(\boldsymbol{q})|^2. \quad (6)$$

The effective coherence time τ depends on the relation between the size of the compound and the critical length scale $\hbar/\sigma_q$ of the reference distribution g_e; the effective distribution g remains normalized. The description fails as soon as the relative motion of the constituents must be taken into account. The inner structure of nuclei becomes relevant for femtometer-scale distribution functions, $\hbar/\sigma_q \lesssim 10$ fm $\lesssim (m/m_e)\sigma_s$, with $m \approx 1$ amu the nucleon mass. Here ends the domain of nonrelativistic quantum mechanics, and with it the validity of our approach. We thus restrict its parameters to about $\sigma_s \lesssim 20$ pm and $\hbar/\sigma_q \gtrsim 10$ fm, noting that the macroscopicities will not change if these bounds are varied by a few orders of magnitude.

The restriction to a single reference distribution $g_e(s,q)$ in (4), as opposed to individual distributions for different types of particles, yields a universal single-particle description (1). The choice of the reference mass m_e is arbitrary, since the coherence time parameter and the distribution rescale to $\tau = \tau_e(m_e/m)^2$ and $g(s,q) = (m/m_e)^3 g_e(ms/m_e, q)$ for a point particle of different mass m, as follows from (3). This renders the translation $\boldsymbol{s}$ negligible for heavy objects, $m \gg m_e$.

In the following we will use the electron as the reference particle fixing both τ_e and g_e. Moreover, we take g_e to be a Gaussian distribution in s and q, fully specified by the standard deviations σ_s and σ_q. The latter determine the main behavior of the classicalization effect; a more involved description with additional parameters would complicate matters without significantly modifying the generic behavior.

It is remarkable that a special form of Eq. (4), which we arrived at using the assumptions (i)–(iv), describes the observable consequences of the theory of continuous spontaneous localization (CSL) [16,23] if one takes $\sigma_s = 0$ [24]. This shows that one can set up explicit theories which modify the dynamics on the level of the Schrödinger equation and whose observable consequences fit into the present framework [25–28]. The stochastic Schrödinger equation in Refs. [16,23] may thus be seen as one example, but not the most general form, of a theory which yields a minimal modification in the sense described above.

Assessing superposition states.—The experimental demonstration of quantum coherence in a mechanical degree of freedom rules out a certain parameter region of the classicalizing modification; i.e., it provides a lower bound of the time parameter τ_e for any fixed value of σ_s and σ_q. For a superposition state in a different experiment to be more macroscopic, its demonstration must exclude a larger set of τ_e, implying that the modification must be even weaker.

Figure 1 shows the greatest excluded τ_e for a number of different setups, as a function of the critical length scale $\hbar/\sigma_q$ and at fixed $\sigma_s = 20$ pm. The solid and the dotted curve correspond to exemplary modern matter-wave experiments: the interference of cesium atoms in free fall over hundreds of milliseconds (solid line) [29], and the

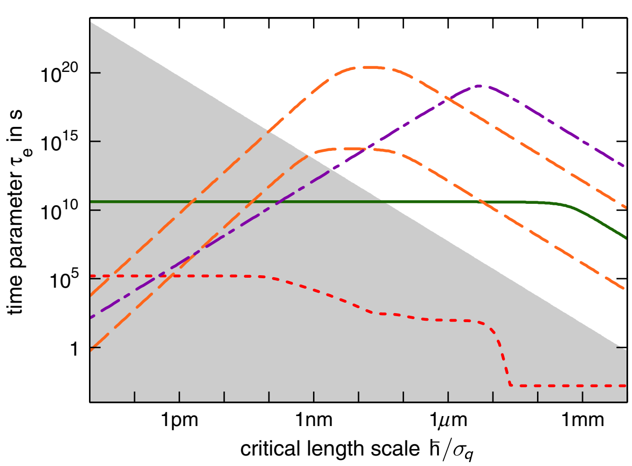

FIG. 1 (color online). Lower bounds on the time parameter τ_e, as set by various experiments. The calculations are done for the relevant range of critical length scales $\hbar/\sigma_q$, and at $\sigma_s = 20$ pm. The solid line corresponds to the atom interferometer of Ref. [29]; it rules out all time parameters τ_e below the curve. Future experiments may exclude a larger set, e.g., by interference of 10^5–10^7 amu gold clusters [30] (dashed lines) or of micromirror motion [31] (dash-dotted line). The dotted line corresponds to demonstrated persistent current superpostions in a SQUID loop [1]. The shaded region represents the excluded τ_e by a conceivable classical measurement of less than 1 μK/s temperature increase in a Rb gas.

superposition of counterpropagating currents of 10^{14} superconducting electrons in a Josephson ring (dotted line) [1]. The dashed and dash-dotted lines illustrate what would be achieved in proposed superposition experiments with nanoclusters [30] or micromirrors [31]. Whereas the value of σ_s matters for the SQUID experiment, it is not important for the other cases due to the large masses involved. Our results then resemble the predictions of a CSL model with varying localization length $\hbar/\sigma_q$. Detailed results on each experiment are reported in Ref. [22].

One observes a common feature of all quantum curves in Fig. 1: they saturate or assume a local maximum. This is because the classicalizing modification (4) is bounded in the operator norm, and any given position or momentum superposition state of a total mass M thus survives at least for a time $\tau_e(m_e/M)^2$.

The interferometer results (solid and dashed lines) reach the maximum where the critical length scale $\hbar/\sigma_q$ is comparable to the interference path separation. This is where the solid line saturates, in accordance with the CSL results in Ref. [27]. The dashed line drops at length scales smaller than the size of the interfering object, when only a fraction of its mass contributes to its center-of-mass coherence time, as given by Eq. (5). The latter also holds if the object is larger than the path separation (dash-dotted curve). For smaller values of $\hbar/\sigma_q$ the slope is mainly determined by the mass density $\varrho(\mathbf{x})$ of the interfering object, while in the diffusive limit of large values it solely depends on σ_q.

The superposition of persistent currents probed in the SQUID experiment [1] can be described by displaced Fermi spheres of Cooper-paired electrons [32]. The classicalization gradually redistributes and dephases electrons between the Fermi spheres, thereby undermining the quantum coherence [22]. At large momentum spreads σ_q the effect is governed by the redistribution of electrons, as would be the case in spontaneous localization [33]. The dotted curve assumes its maximum at a value σ_q where the redistribution covers all electrons in the Fermi sphere. This effect vanishes for smaller σ_q once the superconducting energy gap can no longer be overcome; in this limit the classicalization effect is governed by the dephasing that is induced by the position diffusion with spread σ_s.

Because of the diffusion effect inherent in the classicalizing modification there are also classical experiments not explicitly demonstrating quantum behavior that can be used to narrow down the range of plausible coherence time parameters. This is indicated by the shaded area in Fig. 1 which represents the values of τ_e excluded by an anticipated precision measurement of the temperature increase of a dilute gas of Rb atoms, $\sigma_q^2/2m_{\mathrm{Rb}}\tau_{\mathrm{Rb}} < 1.5k_B \times 1\ \mu\mathrm{K/s}$.

Macroscopicity measure.—In view of the parameter bounds displayed by Fig. 1, we suggest to quantify the macroscopicity of a superposition state realized in an experiment by the greatest excluded time parameter τ_e of the modification (4). In addition, one must comply with the above restriction to the nonrelativistic domain, $\sigma_s \lesssim$ 20 pm and $\hbar/\sigma_q \gtrsim 10$ fm. To set a scale, we take the logarithm of τ_e in units of seconds as the measure of macroscopicity,

$$\mu = \log_{10}\left(\frac{\tau_e}{1\ \mathrm{s}}\right). \tag{7}$$

That is, a positive value for μ is obtained if one demonstrates an electron to behave like a wave for more than 1 s, or a proton for about a microsecond.

A simple approximate expression for the macroscopicity μ is obtained for interference experiments with point particles or with compound objects of total mass M, whose size is much smaller than the path separation. The single-particle modification (1) predicts an exponential decay of coherence with time scale $\tau = \tau_e(m_e/M)^2$, a mass dependence also obtained in the CSL case [25–27]. This is to be compared with the period t during which coherence is maintained in the experiment. Measuring with confidence a fraction $f < 1$ of the expected interference visibility, one gets

$$\mu = \log_{10}\left[\left|\frac{1}{\ln f}\right|\left(\frac{M}{m_e}\right)^2 \frac{t}{1\ \mathrm{s}}\right]. \tag{8}$$

That is to say, if one measures 30% contrast in an interference experiment where the visibility is predicted to be, say, 60%, then $f = 0.5$ must be used in the above expression.

Macroscopicity of specific experiments.—In Fig. 2 we present the macroscopicities attained in a selection of quantum experiments versus their publication date. They include tests of the superposition principle with neutrons, electrons, individual and Bose-condensed atoms, and molecules. Details on the calculations for specific experiments can be found in Ref. [22].

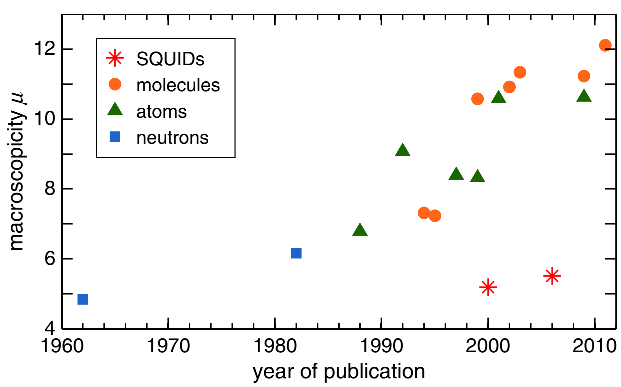

FIG. 2 (color online). Timeline of macroscopicities reached in quantum superposition experiments [22]. The squares, the triangles, and the dots represent interference experiments with neutrons [37,38], atoms [29,39–42] or atom Bose-Einstein condensates [3], and molecules [4,43–48], respectively. One notes that Bose-Einstein condensates do not substantially exceed the macroscopicities achieved with atom interferometers. This is due to the single-particle nature of the condensate wave function. The many-particle state is more involved in the case of superposition experiments with persistent supercurrent states in a large SQUID loop [1,2], as represented by the stars. However, despite the large number of Cooper pairs contributing to the current superpositions in SQUIDs, such experiments lag behind in macroscopicity due to the small coherence times observed.

State-of-the-art interferometers achieve macroscopicities of up to $\mu \approx 12$, and various ideas to surpass this value with future experiments have been suggested. As can be seen from Table I, the most promising proposals from the perspective of the macroscopicity measure employ oscillating micromirrors [31] and nanoclusters [30,36]. Their huge mass would trump a conceivable SQUID experiment with more than 10^{17} electrons or an atom interferometer hovering in free space with an interrogation time of 1 h [35].

Nevertheless, there are more than 30 orders of magnitude between experiments conceivable with present-day technology ($\mu = 12$–24) and something as manifestly macroscopic as an ordinary house cat ($\mu \sim 57$).

Conclusion.—Using the measure proposed in this Letter, any experiment testing the superposition principle in mechanical degrees of freedom can be quantified and compared. By definition it answers an empirically relevant question, namely, to what extent an observation serves to exclude minimally invasive modifications of quantum mechanics that produce classical behavior on the macroscale. As such, the measure follows directly from basic symmetry and consistency arguments and confers physical meaning on the abstract notion of macroscopicity of a quantum system.

The proposed measure does not depend on how a compound mechanical object is decomposed into elementary mass units. For instance, an interfering fullerene buckyball might be described in terms of 60 carbon atoms or equally of 1080 nucleons and electrons, and both descriptions should consistently lead to the same macroscopicity value for the overall state of the molecule. This issue, which was

TABLE I. Expected macroscopicities for various proposed and hypothetical quantum superposition experiments [22]. The oscillating micromembrane setup [34] will reach the stated μ value if coherence between the zero- and one-phonon state can be observed for over 1000 oscillation cycles. For the SQUID experiment we assume a loop length of 20 mm, a wire cross section of 100 μm^2, and 1 ms coherence time. In the gedanken experiment an idealized cat of 4 kg is kept in a spatial superposition of 10 cm distance for 1 s.

Conceivable experiments	μ
Oscillating micromembrane	11.5
Hypothetical large SQUID	14.5
Talbot-Lau interference [30] at 10^5 amu	14.5
Satellite atom (Cs) interferometer [35]	14.5
Oscillating micromirror [31]	19.0
Nanosphere interference [36]	20.5
Talbot-Lau interference [30] at 10^8 amu	23.3
Schrödinger gedanken experiment	~57

not addressed in previous studies, is explicitly taken into account in our approach. Moreover, we do not refer to specific classes of quantum states, or to preferred measurement operations or observables, rendering the measure of macroscopicity applicable to arbitrary mechanical systems.

The last 20 years have witnessed a remarkable rise in demonstrated macroscopicities. Yet, new experimental strategies for quantum tests, in particular, using nanoclusters and microresonators, may soon venture deeper into the macroworld. As more and more effort is put into this field, we may well experience an unprecedented leap towards the macroscopic domain.

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