Comment on "Testing Three-Body Quantum Electrodynamics with Trapped Ti^{20+} Ions: Evidence for a Z-Dependent Divergence Between Experiment and Calculation"

The authors of the original Letter [1] report a measurement of the w-line in heliumlike titanium and combine this result with literature values of the same transition in different ions of the heliumlike isoelectronic sequence. Out of a statistical treatment of this combined data set, they claim evidence of an atomic number (Z) dependent divergence between the experimental body of data and the at present most sophisticated theoretical treatment of the helium isoelectronic sequence, reported in Ref. [2].

Although such a finding cannot be excluded in future experiments, the data set presently at hand over the whole Z range does not allow for such a conclusion and, unfortunately, the authors' claim of "evidence" in Ref. [1] has to be rejected. It seems that the authors of the original Letter [1] did not compile the full set of experimental data available (in May 2012). In particular, for $Z = 18$ they did not incorporate the data of Kubicek *et al.* [3] published in January 2012. This has severe consequences on the analysis since Ref. [3] provides the most accurate measurement of the w-line for the entire region $Z > 16$ with an standard error of 1.5 parts per million including an extensive discussion of the error budget.

In Fig. 1 of this comment, the data for $Z = 18$ of Ref. [1] are recompiled including Ref. [3]. As a result, the experimental accuracy for the averaged data point at $Z = 18$ increases significantly and gives a revised value for Fig. 2b in Ref. [1] of (-0.0016 ± 0.0049) eV. Figure 1 shows both, excellent agreement among the many individual experiments at $Z = 18$ as well as consistency with the prediction of Artemyev *et al.* [2]. The fit $(1 \times 10^{-5} \pm 1.97 \times 10^{-6})\ Z^3$ in Ref. [1] does not provide a reduced chi-squared close to unity anymore but gives about 10 instead. Now, the best fit of the form $f(Z) = d \times Z^3$ is given by $d = (1.4 \pm 0.8) \times 10^{-6}$ and a reduced chi-squared of 3.4, which excludes statistical evidence. The theory of Artemyev *et al.* [2] ($y = 0$) is still fully embedded in the one sigma confidence interval of that fit. In contrast, the experimental contribution in Ref. [1] to the data point at $Z = 22$ is not within the fit's two sigma confidence interval.

The fit of power laws to the data, in particular, in case of even higher exponents, potentially produces large discrepancies for high-Z ions with respect to theory. Those discrepancies are difficult to justify, given that for the two-electron interaction contribution to the ground state, the largest correction to the w-line (Dirac) energy, we find agreement between theory [2] and experiment [4] on the basis of few eV for uranium at $Z = 92$.

Moreover, the data set is also fairly fitted, with a reduced chi-squared of 3.7 (about the statistical quality of the w-line data in Fig. 1 of the original Letter [1]), by the function $y = (0.0017 \pm 0.0048)$ eV, which is in full agreement with the predictions of Artemyev *et al.* [2]. Almost half of the reduced chi-squared value of 3.7 is contributed by the compiled data point at $Z = 36$ alone and should provoke further experimental as well as theoretical investigations. It is understandable that a data set of more than four data points, which is already fairly fitted by a function $y =$ constant, could be easily fitted more accurately by a full polynomial of third order, but this is a consequence of mathematics. Without a justified physical model, such a fit does not provide any meaningful insight.

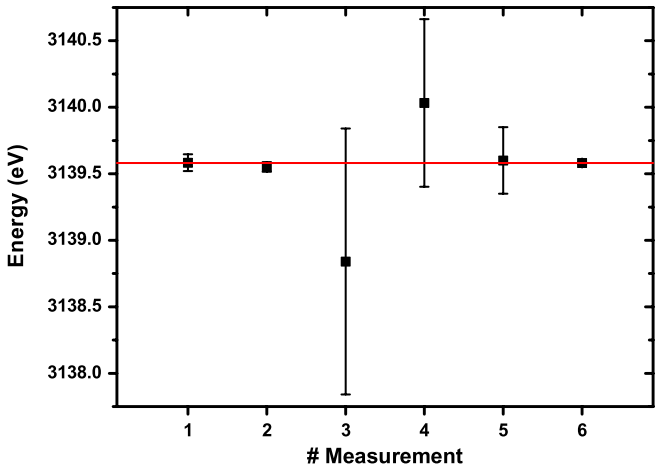

FIG. 1 (color online). Recompilation of the combined data point at $Z = 18$. Data points #1 to #5 are taken from Refs. [6–10] accordingly to the original Letter [1]. Data point #6 is taken from Ref. [3]. The red line is the fit to the data of the form $y = (3139.5805 \pm 0.0049)$ eV. The theoretical value from Ref. [2] is undistinguishable from the fitted red line on the given scale of the energy axis of the figure.

Looking at the data set of various atomic numbers Z presented in Ref. [1], we could conclude that the differences of experimental values minus theoretical [2] values are significantly more positive than negative, as noted already more than two decades ago in Ref. [5]. This might provoke speculations, but claiming an evidence for a Z-dependent divergence is potentially faulty and a certainly unjustified conclusion from the presently known body of experimental data.

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