Connecting Dirac and Majorana Neutrino Mass Matrices in the Minimal Left-Right Symmetric Model

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Probing the origin of neutrino mass by disentangling the seesaw mechanism is one of the central issues of particle physics. We address it in the minimal left-right symmetric model and show how the knowledge of light and heavy neutrino masses and mixings suffices to determine their Dirac Yukawa couplings. This in turn allows one to make predictions for a number of high and low energy phenomena, such as decays of heavy neutrinos, neutrinoless double beta decay, electric dipole moments of charged leptons, and neutrino transition moments. We also discuss a way of reconstructing the neutrino Dirac Yukawa couplings at colliders such as the LHC.

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Introduction.—In the standard model (SM), all particles get their masses from the vacuum. This profound mechanism can be verified through the decays of the Higgs-Weinberg boson [1,2], apparently found by CMS and ATLAS [3]. In particular, to every charged fermion of mass m_f corresponds a unique (Dirac) Yukawa coupling, which implies the following branching ratio

$$\Gamma(h \to f\bar{f}) \propto m_f^2. \tag{1}$$

What about neutrinos? Being neutral, they could be described by real Majorana spinors of mass m_ν [4]. This happens naturally in the seesaw mechanism when one adds heavy right-handed (RH) neutrinos of mass m_N to the SM [5]. However, even if one were able to measure both light and heavy neutrino masses and the light neutrino mixing matrix V_L, the Dirac couplings still could not be unambiguously determined [6,7]. This is evident from the expression for the neutrino Dirac Yukawa couplings:

$$M_D = i\sqrt{m_N}O\sqrt{m_\nu}V_L^\dagger, \tag{2}$$

where O is an arbitrary orthogonal complex matrix. Thus, no prediction analogous to (1) can be made for neutrinos. The portion of parameter space where the imaginary components of the Euler angles parametrizing O are large leads to large $\nu - N$ mixing, and the origin of neutrino mass is hidden from the processes that could probe it.

The question is what happens in a more fundamental theory, such as the left-right (LR) symmetric model, introduced in order to understand the origin of parity violation [8]. Historically, this model led to neutrino masses long before the experiment and also to the seesaw mechanism [9,10].

We show that, once the mass matrix of heavy neutrinos is measured, the relation between heavy and light neutrinos can be made definite in the usual manner: One first measures the particle masses and mixing before predicting Yukawa couplings. The Keung-Senjanović (KS) production process [11] of heavy neutrinos allows one to measure their masses and flavor composition and to determine their Majorana nature [12]. The theory then predicts the Dirac Yukawa couplings which can in principle be measured at the LHC. This amounts to probing the origin of neutrino mass, in complete analogy with the Higgs-Weinberg program for the charged fermions and gauge bosons. Moreover, it sheds light on neutrinoless double beta decay and lepton dipole moments.

The minimal LR model.—The minimal left-right symmetric model (LRSM) is based on the gauge group $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$, augmented by a LR symmetry which implies equality of gauge couplings $g_L = g_R \equiv g$. Fermions come in LR symmetric doublet representations $Q_{L,R} = (u, d)_{L,R}$ and $L_{L,R} = (\nu, \ell)_{L,R}$, and the relevant charged gauge interactions are

$$\mathcal{L}_{\text{gauge}} = \frac{g}{\sqrt{2}}(\bar{\nu}_L V_L^\dagger \not{W}_L e_L + \bar{N}_R V_R^\dagger \not{W}_R e_R) + \text{H.c.} \tag{3}$$

The Higgs sector consists [9] of a complex bidoublet $\Phi(2,2,0)$ and two triplets $\Delta_L(3,1,2)$ and $\Delta_R(1,3,2)$ with quantum numbers referring to the LR gauge group.

In the seesaw picture, the Majorana neutrino mass matrix is given by [13]

$$M_\nu = M_L - M_D^T \frac{1}{M_N} M_D, \tag{4}$$

where M_D is the neutrino Dirac mass matrix, while on the other hand, $M_L \propto M_{W_L}^2/M_{W_R}$ and $M_N \propto M_{W_R}$ are the symmetric Majorana mass matrices of the left- and right-handed neutrinos, respectively. The above formula connects the smallness of neutrino mass to the scale of parity restoration at high energies.

It is crucial that there be new physical phenomena that allow us to directly probe the Majorana nature of RH neutrinos and determine their masses and mixings from experiment [11], as discussed in the following section.

We opt for charge conjugation $\mathcal{C}$ as LR symmetry, with the fields transforming as $f_L \leftrightarrow (f_R)^c$, $\Phi \to \Phi^T$, and $\Delta_L \leftrightarrow \Delta_R^*$ (the case of parity will be discussed elsewhere). The mass matrices then satisfy

$$M_L = \frac{v_L}{v_R} M_N, \tag{5}$$

$$M_D = M_D^T, \tag{6}$$

where $v_R \equiv \langle \Delta_R^0 \rangle$ sets the large scale (e.g., $M_{W_R} = g v_R$) and $v_L \equiv \langle \Delta_L^0 \rangle$ is naturally suppressed by the large scale, and it can be shown that $v_L \leq \mathcal{O}(10 \text{ GeV})$ [15]. For the complex issues related to determining v_L, we refer the reader to Ref. [16].

In the case of $\mathcal{C}$, there is a theoretical lower bound on the LR scale $M_{W_R} \gtrsim 2.5$ TeV [17,18], coming essentially from $K - \bar{K}$ mixing. It is noteworthy that direct searches for W_R at the LHC are now probing this scale [19,20].

From Majorana to Dirac.—The above seesaw formula seemingly obfuscates the connection between heavy and light neutrinos, and common lore was that this connection cannot be unraveled [6]. However, since the Dirac mass matrix must be symmetric, it can be obtained directly from (4)

$$M_D = M_N \sqrt{\frac{v_L}{v_R} - \frac{1}{M_N} M_\nu}, \tag{7}$$

and thereby one can determine the mixing between light and heavy neutrinos. The square root of an n-dimensional matrix always has 2^n discrete solutions which can be found in Ref. [21]. (Ambiguities might arise in singular points of the parameter space.)

The above expression offers a unified picture of the low energy phenomena, such as lepton flavor violation, lepton number violation through the neutrinoless double beta decay, electric dipole moments of charged leptons, neutrino transition moments, neutrino oscillations, and neutrino cosmology. Some examples are discussed below, while the rest will be dealt with in a forthcoming publication.

It should be mentioned that the determination of the RH neutrino mass matrix as a function of the Dirac Yukawa coupling was studied before in Refs. [22,23]. This approach requires additional theoretical structure such as quark lepton symmetry and $SO(10)$ unified theories [23].

Here, we wish to show, on the contrary, that, without any new assumption, the LRSM is a complete theory of neutrino masses and mixings, in the sense that the measurements of the heavy sector at colliders can determine and interconnect the low energy phenomena, including those which proceed via Dirac Yukawa couplings. Thus, our program is in the same spirit as the SM: to predict the couplings with the Higgs-Weinberg boson as a function of the basic fermion properties such as masses and gauge mixings. It may take a long time before these Dirac Yukawa couplings are measured; the essential point is the capacity of the theory to relate them to the basic measurable quantities.

On the absence of ambiguity of M_D: As expressed in (2), in the conventional seesaw mechanism, M_D is undetermined. On the other hand, in this case [equivalent to setting $v_L = 0$ in (7)], one gets

$$M_D = i M_N \sqrt{M_N^{-1} M_\nu}. \tag{8}$$

The crucial point here is that M_D is symmetric, and from this requirement the matrix O can be shown to be fixed in terms of physical parameters m_ν, m_N, V_L, and V_R [Unlike in the case of the seesaw in the SM, V_R is a physical parameter, as defined in (3).]

$$O = \sqrt{m_N} \sqrt{m_N^{-1} V_R^\dagger V_L^* m_\nu V_L^\dagger V_R^*} V_R^T V_L \sqrt{m_\nu^{-1}}. \tag{9}$$

As can be seen from above, the elements of O take at most values of order one. Moreover, this parametrization offers an alternative method of computing M_D which will be discussed elsewhere.

The case with nonzero v_L is completely analogous (see Ref. [24]), and, similarly, the matrix O is a function of physical observables only.

M_N from the LHC: The mass matrix of light neutrinos

$$M_\nu = V_L^* m_\nu V_L^\dagger \tag{10}$$

is being probed by low energy experiments, while the one of heavy neutrinos (The mass matrix of charged leptons, being symmetric, can be taken as diagonal without a loss of generality.)

$$M_N = V_R m_N V_R^T, \tag{11}$$

on the other hand, can be determined at high energy colliders through the KS reaction [11]. This amounts to producing W_R at the usual Drell-Yan resonance, with a reach of about $\sim$6 TeV for W_R mass and 10 GeV $\lesssim m_N \lesssim$ 3.5 TeV for the N mass at the LHC [25,26]. One can also verify the chirality of the new charged gauge boson [25,27]. Unlike in the case of W_L, where neutrinos act as missing energy, here the decays of heavy RH neutrinos lead to a lepton number violating final state of two same-sign leptons and two jets. Moreover, one can directly probe the Majorana nature of RH neutrinos through their equal branching ratios into charged leptons and antileptons [11]. Due to the absence of missing energy in the final state, one can fully reconstruct the heavy neutrino masses m_N from the invariant mass of one of the leptons and two jets in the final state [17,19], together with mixings V_R by tagging the flavor of the final state leptons [28]. (In the case that RH neutrinos are too light to be probed at the LHC, one may still indirectly determine their masses and mixings, as in the case when the lightest one is the warm dark matter [14].)

While waiting for the LHC to provide this information, the reader may find it useful to have a simple working example

$$V_R = V_L^*. \tag{12}$$

Although, in general, (7) may require some computational tedium, for this example, one gets

$$M_D = V_L^* m_N \sqrt{\frac{v_L}{v_R} - \frac{m_\nu}{m_N}} V_L^\dagger. \tag{13}$$

It is easy to see from the generalization of (9) that $O = 1$ in this case.

Phenomenological implications.—The low scale LRSM contains a host of experimentally accessible phenomena related to lepton number and flavor violation [29], both at high and low energies, which we discuss in this section.

Probing M_D at the LHC: We start with the high energy probe of M_D at the LHC. The crucial thing is that N, besides decaying through virtual W_R as discussed above, decays also into the left-handed charged lepton through M_D/M_N. In a physically interesting case, when N is heavier than W_L, which facilitates its search through the KS process, the decay into left-handed leptons proceeds through the on-shell production of W_L. For the sake of illustration, we choose again the example of (12), in which case and one can compute the ratio of N decays into the corresponding charged lepton via the W_L and W_R channels

$$\frac{\Gamma_{N\to\ell_L jj}}{\Gamma_{N\to\ell_R jj}} \simeq 10^3 \frac{M_{W_R}^4}{M_{W_L}^2 m_N^2}\left|\frac{v_L}{v_R} - \frac{m_\nu}{m_N}\right|. \tag{14}$$

The branching ratios into the Higgs-Weinberg and SM gauge bosons are shown in Fig. 1 (The SM bosons W_L, Z, and h can decay into a lighter N, but the small couplings make the corresponding branching ratios too tiny to matter at this point.)

The issue here is how to observe these rare channels. Ideally, one should measure the chirality of the outgoing charged lepton [25,27] or establish the kinematics of the two jets associated with the on-shell production of W_L. This may be a long shot but could still be feasible for the LHC with a luminosity in the hundreds of fb^{-1}. The bottom line is that this probes in principle all the matrix elements of M_D, once the heavy neutrinos are identified through their dominant W_R mediated decays. This offers a clear program that brings the issue of the origin of neutrino mass to the same level of charged fermions masses in the SM.

Electron electric dipole moment: One of the most sensitive probes of new physics beyond the SM is the T- and CP-violating electric dipole moment (EDM) of charged leptons. The SM contribution arises at four loops [30] and is around 11 orders of magnitude below the current experimental limit $d_e < 10^{-27}$ e cm [31]. In the LRSM, this process is significantly enhanced due to the mixing ξ_{LR} between W_L and W_R. The leading amplitude is present at one loop [32,33]

$$d_e = \frac{eG_F}{4\sqrt{2}\pi^2} \mathrm{Im}[\xi_{LR} V_R F(t) V_R^\dagger M_D]_{ee}, \tag{15}$$

where

$$F(t) = \frac{t^2 - 11t + 4}{2(t-1)^2} + \frac{3t^2 \log t}{(t-1)^3}, \qquad t = \frac{m_N^2}{M_{W_L}^2}. \tag{16}$$

There are strong limits on the ξ_{LR}, but, in any case, it is automatically small due to the suppression of the heavy gauge boson mass. It is bounded by

$$\frac{\alpha}{4\pi}\frac{m_t m_b}{M_{W_R}^2} \lesssim |\xi_{LR}| \lesssim \frac{M_{W_L}^2}{M_{W_R}^2}, \tag{17}$$

with a lower bound resulting from radiative electroweak corrections [34].

Taking the example in (12), the size of the EDM is shown in Fig. 2 as a function of the lightest neutrino mass for two different neutrino hierarchies. These values can be probed by future experiments [35]. In the case when the LR mixing is close to its lower bound, one has to go beyond the one loop approximation [36], but in that case the experimental outlook seems bleak and we do not pursue it here.

In the context of LRSM, EDM is a manifestly CP-odd process sensitive to Majorana and Dirac phases, complementary to Ref. [37]. This can easily be checked using the example of Eq. (13) in the EDM expression in Eq. (15) where the CP phases do not cancel out.

Neutrinoless double beta decay: The importance of this textbook example of lepton number violation was recognized in Ref. [38] soon after the seminal work of

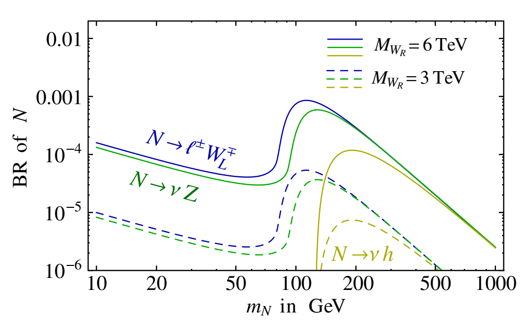

FIG. 1 (color online). Branching ratio (BR) for the decay of heavy N to the Higgs-Weinberg and SM gauge bosons, proceeding via Dirac couplings, exemplified for $v_L = 0$ and $V_R = V_L^*$. The solid (dashed) line corresponds to $M_{W_R} = 6(3)$ TeV.

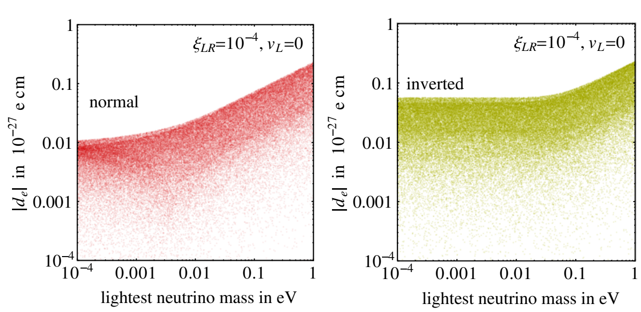

FIG. 2 (color online). Electron EDM size in the LRSM with Eq. (12), $v_L = 0$, and $m_{N_{1,2,3}} = 0.5$, 2, 2.5 TeV. The neutrino mixing angles are fixed at central values provided in Ref. [46], and the CP phases are scanned over.

Majorana [4]. The LRSM offers new sources for this process [13] that has been studied extensively over the years [39]. In particular, an in-depth analysis [40] (see also Ref. [41]) was recently performed on the connection between neutrinoless double beta decay (and lepton flavor violation) at low energies and the KS process [11] at colliders.

Although the standard source of this process in LRSM is due to the exchange of the heavy neutrinos, there is an additional contribution proportional to the Dirac mass matrix. We express it in the usual form of an effective mass term

$$m_{\nu N}^{ee} = \left(\xi_{LR} + \eta \frac{M_{W_L}^2}{M_{W_R}^2}\right) p (M_N^{-1} M_D)_{ee}, \qquad (18)$$

where $p \simeq 100$ MeV [40] and $\eta \simeq 10^{-2}$ [42] are determined by nuclear physics considerations.

As a consequence of (7), the contribution in (18) is subleading for N heavy enough to be visible at the LHC and for naturally small values of v_L (an extra contribution without W_R [43] is also suppressed), and the total decay rate is governed by the effective mass parameter

$$|m_{\nu+N}^{ee}|^2 = |V_{Lej}^2 m_{\nu_j}|^2 + \left| p^2 \frac{M_{W_L}^4}{M_{W_R}^4} \frac{V_{Rej}^2}{m_{N_j}} \right|^2. \qquad (19)$$

Since $|m_{\nu+N}^{ee}|$ and the size of the electron EDM both depend on the heavy neutrino mass, there is a correlation between the two processes, which is shown in Fig. 3. One should keep in mind, though, that in the case when RH neutrinos are not directly observable at the LHC, the dominant contribution to this process might proceed through M_D.

Neutrino transition moments: By defining

$$M = \xi_{LR}^* V_L^T m_\ell M_N^{-1} M_D V_L, \qquad (20)$$

we get the following matrix of neutrino magnetic transition moments

$$\mu = \frac{ieG_F}{\sqrt{2}\pi^2} \mathrm{Im}[M + M^\dagger]. \qquad (21)$$

This result was already derived in Ref. [44] and can be reproduced using Ref. [33]. Here, we neglect the contribution from light neutrino masses, which is roughly 9 orders of magnitude smaller than the current experimental limit $\mu < 2 \times 10^{-10} \mu_B$ [45]. One should keep in mind that the Majorana transition moments in the SM (with nonzero neutrino mass) are negligibly small: $\mu_{SM} \simeq 10^{-23} \mu_B$ [45].

It is easy to see that (21) gives roughly $\mu \simeq 10^{-19} \mu_B$ for generic values of M_D in (7), still a hopelessly small value. Therefore, an observation of neutrino transition moments would deal a serious blow to the LRSM.

Conclusions and outlook.—In the SM, the knowledge of charged fermion masses uniquely predicts Higgs decay branching ratios. As shown here, exactly the same happens in the LRSM for the masses of light and heavy neutrinos. The reason behind this is the LR symmetry itself, which allows one to compute the Dirac Yukawa couplings in the context of the seesaw mechanism.

The main result of our Letter is summarized in Eq. (7). Its phenomenological impact is exemplified both on the high energy frontier at the LHC and on the phenomena of neutrinoless double beta decay, dipole moments of charged leptons, and neutrino transition moments. This result was achieved at no expense of imposing additional *ad hoc* symmetries but by the structure of the theory itself. The bottom line is that one can predict and measure the Dirac neutrino Yukawa couplings in complete analogy with the SM situation for charged fermions.

It is interesting to compare our program to the one followed over the years in the quark sector of the LRSM. Here, we took the conventional path of predicting the Yukawa couplings from the knowledge of particle masses and mixings. In the quark sector, on the contrary, the symmetry of quark mass matrices was historically used to fix the flavor structure of the right-handed gauge interaction, which led to the strict bound on the LR scale. Now, with the advent of the LHC, the conventional route can be taken up again.

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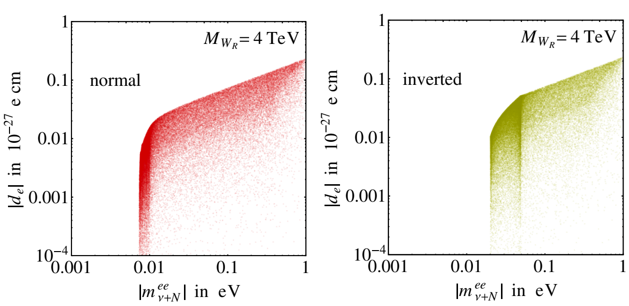

FIG. 3 (color online). Electron EDM size and the effective mass for the neutrinoless double beta computed with the same choice of parameters as in Fig. 2. The mass of W_R is fixed at 4 TeV and $p = 193$ MeV [40].

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