Stochastic Acceleration by Multi-Island Contraction during Turbulent Magnetic Reconnection

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The acceleration of charged particles in magnetized plasmas is considered during turbulent multi-island magnetic reconnection. The particle acceleration model is constructed for an ensemble of islands which produce adiabatic compression of the particles. The model takes into account the statistical fluctuations in the compression rate experienced by the particles during their transport in the acceleration region. The evolution of the particle distribution function is described as a simultaneous first- and second-order Fermi acceleration process. While the efficiency of the first-order process is controlled by the average rate of compression, the second-order process involves the variance in the compression rate. Moreover, the acceleration efficiency associated with the second-order process involves both the Eulerian properties of the compression field and the Lagrangian properties of the particles. The stochastic contribution to the acceleration is nonresonant and can dominate the systematic part in the case of a large variance in the compression rate. The model addresses the role of the second-order process, how the latter can be related to the large-scale turbulent transport of particles, and explains some features of the numerical simulations of particle acceleration by multi-island contraction during magnetic reconnection.

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Introduction.—The production of nonthermal particles in magnetized plasmas is a ubiquitous complex phenomenon which is believed to also involve magnetic reconnection. Magnetic reconnection is the process that controls the conversion of magnetic energy into kinetic energy [1]; it is the driver of impulsive phenomena such as solar flares, substorms in the Earth's magnetosphere, and disruptions in laboratory fusion devices. The relation between magnetic reconnection and particle acceleration has been extensively discussed in the terrestrial magnetosphere based on *in situ* observations [2–4]. Moreover, x-ray observations and studies of the energy budget during solar flares indicate that a significant fraction of the magnetic energy released in a flare is carried by the accelerated 10–100 keV nonthermal electrons [5]. However, how particles can be accelerated in large numbers to high energies as the magnetic field lines reconnect remains an outstanding problem.

Numerical particle-in-cell (PIC) simulations aiming to address the problem of particle acceleration during magnetic reconnection in a self-consistent manner, have confirmed that particles are efficiently accelerated in the vicinity of the X line by reconnection electric fields [6,7]. However, an important limitation of X-type acceleration mechanisms is that they hardly explain alone the large number of accelerated particles, in particular during a solar flare, because the volume occupied by a current sheet where the strong electric field capable of particle acceleration is present, is quite small.

Hence, intensive efforts have been made to understand the role played by the region inside the separatrices for particle acceleration, leaning toward the idea of O-type acceleration mechanisms which take advantage of the closed geometry of the field within magnetic islands. Drake *et al.* [8] have developed a model of particle acceleration which is based on the dynamical motion of the islands. They show that particles trapped in the contracting magnetic field of the islands are adiabatically compressed and therefore can be efficiently accelerated through a first-order Fermi process. In addition, many studies have revealed the importance of magnetohydrodynamic turbulence [9–11] and plasmoid dynamics [12,13] with regards to particle acceleration in a reconnecting plasma. Current sheets are naturally prone to tearing and their fragmentation leads to the formation of magnetic islands having a complex multiscale and intermittent dynamical behavior [14,15]. Our goal in this Letter is to study the effect on particle acceleration of the ensemble of contracting islands and develop a simple model of particle acceleration during turbulent magnetic reconnection.

When a magnetic island changes its length L at the velocity $V = dL/dt$, particles that are trapped within the island change their speed v according to $dv/dt = -(\alpha V/L)v \equiv Wv$. This relation, derived in Ref. [8], is a consequence of the conservation of the longitudinal action for particles trapped within an island and the coefficient α represents the relative magnitude of the reconnecting magnetic field; i.e., $\alpha = (\delta B/B_0)^2$. As a result, magnetic islands that are contracting at a speed of the order of the Alfvén speed $V \sim V_A$ accelerate the trapped particles through adiabatic compression, provided $v \gg V_A$. The acceleration rate associated with this first-order Fermi process is given by $\alpha V_a/L_0$, where L_0 is the typical length of the islands. If, on the contrary the islands are expanding, then first-order adiabatic deceleration of the particles will result at the same energy independent rate. When contraction is magnetically favorable and when the energy gained from the magnetic field by the particles is balanced with energy losses, including transport losses and/or backreaction of the accelerated particles, power-law distribution in particle energy may be obtained, which is determined by standard techniques, as was originally done in Ref. [8].

In the case of a first-order Fermi process, the rate of energy gained by the particles is proportional to the mean compression $\langle W\rangle$, where the brackets $\langle\rangle$ denote an average over the ensemble of islands in the system, possibly weighted by the relative number of islands that are undergoing contraction [8,16]. The mere existence of this average, or the range of possible contraction rates, suggests that we consider also the effect of the finite variance in the adiabatic compression experienced by the particles in the sea of islands. Indeed, the contraction rate changes in time due to the fire hose condition [16], so an assembly of contracting islands will have nonzero variance. Further, PIC simulations [12] emphasize the bouncing motion of merged islands, so that a contracting motion of an island is followed by an expanding motion.

For an ensemble of multiple contracting islands, the presence of the nonzero average $\langle W\rangle$ and nonzero $\langle (W-\langle W\rangle)^2\rangle$ leads to both first and second order accelerations. The mean controls the first-order Fermi acceleration and additional statistical acceleration occurs at a rate proportional to the variance of the compression, also when the mean compression rate $\langle W\rangle$ is nonzero. A continuity equation can be written for the omnidirectional particle distribution function $F(p,t) = 4\pi p^2 f(p,t)$ [8,16],

$$\frac{\partial F(p,t)}{\partial t} + \frac{\partial}{\partial p}\left[\left(\frac{dp}{dt}\right)F(p,t)\right] = 0, \tag{1}$$

where p is the particle momentum with the time rate of change in momentum given by

$$\frac{dp}{dt} = -\frac{\alpha V}{L}p \equiv Wp. \tag{2}$$

In Refs. [8,16], a term modeling the effect of escape of particles out of the acceleration region is also included in Eq. (1).

Let us consider the case where the compression rate is small and assume first that it is a function of time only with zero average, i.e., the bouncing motion of the islands

$$\langle W(t)\rangle = 0, \tag{3}$$

and its correlation function decays exponentially,

$$C(t) = \langle W^2(t)\rangle \exp(-t/\tau_c). \tag{4}$$

In these cases, even when on average the islands are neither contracting nor expanding, i.e., $\langle W\rangle = 0$, there is a stochastic acceleration effect that remains operative. Although the particles do not experience any systematic change in their energy, the average particle energy could still grow with the acceleration efficiency associated with a second-order Fermi process is proportional to the variance of the compression $\langle W^2\rangle$. Therefore, we obtain that the mean omnidirectional distribution function $F_0(p,t)$ obeys the diffusion equation

$$\frac{\partial F_0(p,t)}{\partial t} = D\frac{\partial}{\partial p}p\frac{\partial}{\partial p}pF_0(p,t), \tag{5}$$

with the diffusion coefficient in momentum space given by

$$D = \int_0^\infty dt C(t) = \tau_c\langle W^2(t)\rangle. \tag{6}$$

The mean distribution function $F_0(p,t)$ solution of Eq. (5) is the normal distribution with respect to the variable $u = \ln(p/p_0)$. Indeed, the particle dynamics is described by the Langevin equation $du/dt = W(t)$ with $\langle W(t)\rangle = 0$. Therefore, $F_0(u,t)$ satisfies the standard diffusion equation

$$\frac{\partial F_0(u,t)}{\partial t} = D\frac{\partial^2 F_0(u,t)}{\partial u^2}, \tag{7}$$

which also confirms that fluctuations in the compression rate are responsible for the growth of the variance in the momentum distribution function.

An account for the effect on the stochastic acceleration of the spatial transport of particles in the pulsation field of the islands may be given on the following basis. Let us shrink the volume of each island into a point, this point being characterized by its compression $W(\mathbf{x},t)$, with $\mathbf{x}$ being the position of the center of the islands. Moreover, we envisage a situation where the large scale spatial transport of particles in the volume filled by the islands is turbulent and diffusive. Therefore, the particle dynamics is modeled by the following Langevin equations:

$$\frac{d\mathbf{x}}{dt} = \boldsymbol{\zeta}(t); \qquad \frac{du}{dt} = W(\mathbf{x},t), \tag{8}$$

with $\langle\boldsymbol{\zeta}(t)\rangle = 0$ and $\langle\zeta_i(t)\zeta_j(t')\rangle = 2\delta_{ij}\kappa_T\delta(t-t')$, $\langle W(\mathbf{x},t)\rangle = 0$ and $\langle W(0,0)W(\mathbf{x},t)\rangle = C(\mathbf{x},t)$, κ_T is the spatial diffusion coefficient. Here, $C(\mathbf{x},t)$ is the Eulerian correlation function associated with the compression or expansion field $W(\mathbf{x},t)$ of the islands, which is supposed to be homogeneous and stationary. The Eulerian correlation function depends on three parameters that characterize the statistics of the (isotropic) compression or expansion field—the variance $\langle W^2(\mathbf{x},t)\rangle = C(0,0)$, the correlation time τ_c, which is the decay time of the Eulerian correlation, and the correlation length λ_c, which is the decay length. So the particles have the probability to stay within the island or escape. As noted [8], the gyration radius of the particle increases near the separatrix, which in turn increases the probability of a particle to escape the island. The Langevin equations (8) are doubly stochastic in the sense that both the position $\mathbf{x}(t)$ of the particles and the compression field $W(\mathbf{x},t)$ are stochastic processes.

With the spatiotemporal statistics of the compression being specified via $C(\mathbf{x}, t)$, the problem is to calculate the diffusion coefficient in momentum space (when the latter exists) and to determine the form of the distribution function. The diffusion coefficient D is related to the time integral of the Lagrangian correlation function [17], viz.

$$D = \int_0^\infty dt C_L(t), \tag{9}$$

where the Lagrangian correlation function $C_L(t)$ is defined via

$$C_L(t) = \langle W(0,0) W(\mathbf{x}(t), t)\rangle, \tag{10}$$

where $\mathbf{x}(t)$ is a solution of Eqs. (8). The exact result (9) is a simple consequence of the definition $D = (1/2)d\langle u^2\rangle/dt$, combined with the second equation in (8). Indeed, $\langle u^2\rangle = \int_0^t dt' \int_0^t dt'' \langle W(t')W(t'')\rangle = 2\int_0^t dt' C_L(t')(t - t')$ and letting $t \to \infty$ (when the integral converges) gives Eq. (9). It also follows from Eq. (9) that the diffusion coefficient in momentum-space can be expressed as

$$D = \tau_L \langle W^2(\mathbf{x}, t)\rangle, \tag{11}$$

where τ_L is the Lagrangian correlation time, i.e., the correlation time of the compression and expansion field which is experienced by the particles along their trajectory. The problem remains to connect the Lagrangian and Eulerian statistics; i.e., to determine the functional dependence of τ_L with τ_c and λ_c. To this purpose, let us write the Lagrangian correlation function (10) in the equivalent form

$$C_L(t) = \int d\mathbf{x} \langle W(0,0) W(\mathbf{x}, t) \delta(\mathbf{x} - \mathbf{x}(t))\rangle. \tag{12}$$

A relation between the Lagrangian correlation $C_L(t)$ and the Eulerian correlation $C(\mathbf{x}, t)$ is obtained by invoking a procedure due to Corrsin [18,19] in which $\mathbf{x}(t)$ is replaced by its statistical average, so that we may replace $\delta(\mathbf{x} - \mathbf{x}(t))$ in Eq. (12) by $\delta(\mathbf{x} - \mathbf{x}(t))$. This leads to the factorization $C_L(t) = \int d\mathbf{x} \langle W(0,0) W(\mathbf{x}, t)\rangle \langle \delta(\mathbf{x} - \mathbf{x}(t))\rangle$. Hence, an expression for the diffusion coefficient in momentum space is found and given by

$$D = \int_0^\infty dt \int d\mathbf{x} C(\mathbf{x}, t) P(\mathbf{x}, t), \tag{13}$$

where $P(\mathbf{x}, t) \equiv \langle \delta(\mathbf{x} - \mathbf{x}(t))\rangle$ is the conditional probability for a particle to be under the influence of a magnetic island located at the position $\mathbf{x}$ at time t provided that this particle was at $\mathbf{x} = 0$ at $t = 0$. Equation (13) shows that D is the integral of the product of two quantities: $C(\mathbf{x}, t)$, the Eulerian correlation function, which characterizes the statistical properties of the compression field $W(\mathbf{x}, t)$, and the probability function $P(\mathbf{x}, t)$, describing the spatial transport of particles in the acceleration region. Here, $P(\mathbf{x}, t)$ is the solution of a standard diffusion equation with diffusion constant κ_T, i.e.,

$$P(\mathbf{x}, t) = \frac{1}{(4\pi\kappa_T t)^{3/2}} \exp(-|\mathbf{x}|^2/4\kappa_T t), \tag{14}$$

but the procedure can be generalized to more complex transport models.

Let us further take the illustrative example of an isotropic correlation function of the form given by

$$C(\mathbf{x}, t) = \langle W^2(\mathbf{x}, t)\rangle \exp(-|\mathbf{x}|^2/\lambda_c^2 - t/\tau_c). \tag{15}$$

From Eq. (13), we obtain that

$$D = \langle W^2(\mathbf{x}, t)\rangle \int_0^\infty dt \exp(-t/\tau_c)\left(1 + \frac{4\kappa_T \tau_c}{\lambda_c^2}\right)^{-3/2}. \tag{16}$$

Therefore, in the weak spatial diffusion limit $\kappa_T \ll \lambda_c^2/4\tau_c$, the momentum diffusion coefficient given by

$$D \sim \tau_c \langle W^2(\mathbf{x}, t)\rangle. \tag{17}$$

This is the case already given by Eq. (6), corresponding to Eq. (11) with $\tau_L \sim \tau_c$. However, in the opposite strong spatial diffusion limit, where $\kappa_T \gg \lambda_c^2/4\tau_c$, then

$$D \sim \frac{\lambda_c^2}{2\kappa_T} \langle W^2(\mathbf{x}, t)\rangle, \tag{18}$$

corresponding to the Lagrangian correlation time being of the order of the spatial transport time scale, i.e., $\tau_L \sim \lambda_c^2/\kappa_T$. In this strong spatial diffusion limit, the stochastic acceleration efficiency is governed both by the Eulerian properties of the compression field and the Lagrangian properties of the particles.

Let us notice that the diffusion coefficient in momentum space may also be expressed as $D = \iint d\mathbf{k} d\omega S(\mathbf{k}, \omega) \kappa_T k^2/[\omega^2 + (\kappa_T k^2)^2]$, where $S(\mathbf{k}, \omega)$ is the spectrum of $W(\mathbf{x}, t)$, i.e., the Fourier transform of the correlation function $C(\mathbf{x}, t)$. It can be clearly seen from this expression for D that the integral may diverge for scale-free power-law spectra such as $S(k, \omega) \propto k^{-q}\delta(\omega)$. This is the signal that the turbulent acceleration process cannot be described as a standard diffusion in u space as in Eq. (7). This situation has been dubbed Fermi acceleration of fractional order in Refs. [20,21]. Here, we focus on the second-order process with D finite.

The statistical effect discussed above can be felt also in addition to the systematic energy change. Indeed, when both the mean and the variance of the compression are finite, the first and second-order Fermi processes operate together. In this case, $F_0(u, t)$ obeys an advection-diffusion equation in velocity space,

$$\frac{\partial F_0(u, t)}{\partial t} + a_1 \frac{\partial}{\partial u} F_0(u, t) = a_2 \frac{\partial^2}{\partial u^2} F_0(u, t), \tag{19}$$

where $u = \ln(p/p_0)$ and where the coefficients of systematic and stochastic acceleration are given by

$$a_1 = \langle W(\mathbf{x}, t)\rangle; \quad a_2 = \tau_L \langle [W(\mathbf{x}, t) - \langle W(\mathbf{x}, t)\rangle]^2\rangle, \tag{20}$$

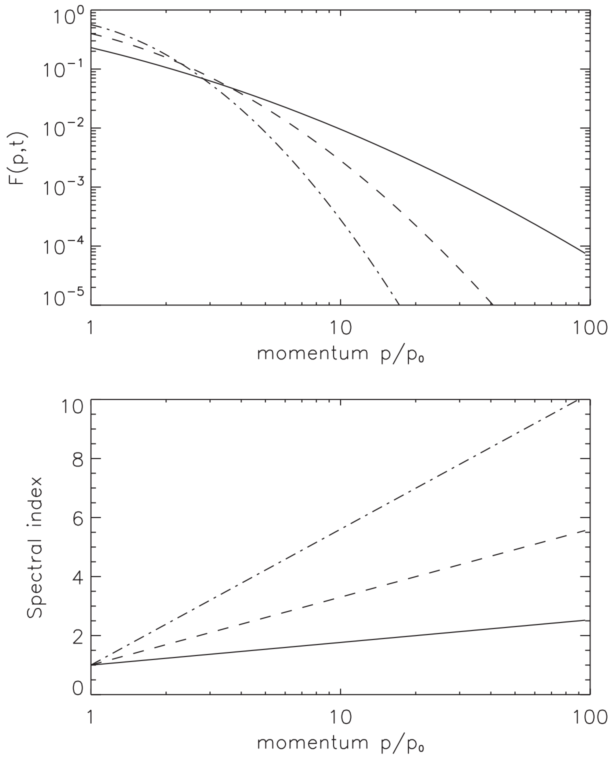

FIG. 1. Particle distribution function (top panel) and the spectral index of the distribution (bottom panel). The solutions of Eq. (7) for $Dt = 3$ (solid line), $Dt = 1$ (dashed line), and $Dt = 0.5$ (dash-dotted line). All distributions are normalized so that $\int F_0(p, t)dp = 1$.

respectively. When the islands contract on average, the distribution function $F_0(u, t)$ shifts toward large u at a rate given by a_1 while the variance of $F_0(u, t)$ grows at a rate given by a_2 and the stochastic component to the acceleration process dominates the systematic part when $a_2 \gg a_1$. The time-dependent solution $F_0(u, t)$ of the advection-diffusion equation (19) is the normal distribution in the variable $u - a_1 t$.

Although the time-dependent solution is not a power law, but only asymptotically at $t \to \infty$, the characteristic solutions and spectral indices

$$-\frac{d\ln F_0(p, t)}{d\ln p} = 1 + \frac{\ln p/p_0}{Dt}, \tag{21}$$

are found for a few values of Dt and are presented in Fig. 1. The values appear to be similar to those obtained in numerical simulations, e.g., Refs. [12,16] and closer to the observed values in solar flares [5] than, for example, in Ref. [11].

In summary, we show that both the first- and second-order Fermi acceleration process can operate together to increase the particle energy when the acceleration region consists of an ensemble of contracting islands. In the case when islands are both contracting and expanding with zero mean effect, only the second-order acceleration process operates. However, even when contraction is dominant, the second order effect can be substantial. The stochastic component to the acceleration corresponds to a nonresonant mechanism according to the classification scheme established in Ref. [21]. It involves the turbulent transport properties of the particles in the acceleration region and becomes more efficient for higher levels of variance in the compression rate.

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