Characterization of Dynamical Phase Transitions in Quantum Jump Trajectories Beyond the Properties of the Stationary State

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We describe how to characterize dynamical phase transitions in open quantum systems from a purely dynamical perspective, namely, through the statistical behavior of quantum jump trajectories. This approach goes beyond considering only properties of the steady state. While in small quantum systems dynamical transitions can only occur trivially at limiting values of the controlling parameters, in many-body systems they arise as collective phenomena and within this perspective they are reminiscent of thermodynamic phase transitions. We illustrate this in open models of increasing complexity: a three-level system, the micromaser, and a dissipative version of the quantum Ising model. In these examples dynamical transitions are accompanied by clear changes in static behavior. This is however not always the case, and, in general, dynamical phases need to be uncovered by observables which are strictly dynamical, e.g., dynamical counting fields. We demonstrate this via the example of a class of models of dissipative quantum glasses, whose dynamics can vary widely despite having identical (and trivial) stationary states.

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Recent experimental progress in quantum optics and cold atomic physics has stimulated great interest in the study of open many-body quantum systems [1–7]. Currently, much effort is dedicated to the understanding and classification of dynamical phases and transitions among them, for example, in the Dicke model [7–9], in lattice bosons subject to engineered dissipation [1], and in spin systems [3–5,10]. Typically, dynamical phase transitions are detected and analyzed through changes in static order parameters, such as superfluid density or spin polarization, calculated in the system's stationary state.

The aim of this work is to characterize dynamical phases exclusively through time correlations within quantum jump trajectories, and not through static order parameters. We do so by building on an elegant connection between open quantum systems—described by a Lindblad master equation—and matrix product states (MPS) [11] put forward in Ref. [12]. We pursue the usual dual description of open system dynamics. On the one hand, any individual realization of the dynamics is stochastic. In the case of an open quantum system this is represented by a stochastic wave function corresponding to a specific sequence of quantum jumps, with the whole ensemble of these stochastic trajectories being encoded in a MPS. On the other hand, the evolution of probabilities is deterministic and is derived from the evolution of the density matrix under the action of a quantum master operator (QMO). The dynamical phase structure of an open system is given by the low lying spectrum of the QMO [5]. More specifically, dynamical phases are characterized by a nonanalytic behavior of the spectrum with respect to changes in physical parameters [3–5]. However, the QMO spectrum of many-body systems is typically computationally intractable, and difficult to measure experimentally. In contrast, the quantum jump trajectories are experimentally accessible and the associated MPS carries complete information about the QMO spectrum [13]. Our aim is thus to characterize dynamical phase transitions in terms of correlations of quantum jump trajectories and their (nonanalytic) response to biasing induced by counting fields [14,15].

To illustrate our ideas we proceed as follows: After a presentation of the formalism we apply it to three open quantum systems each exhibiting dynamical phase transitions which are also accompanied by clear changes in the statics: a "blinking" three-level system [15,16], the simplest example of a quantum system displaying metastability, the micromaser, which has dynamical transitions of both first- and second-order kind [17,18], and a dissipative quantum Ising model displaying a dynamical first-order transition [3,4]. These examples span the range from simple single-body to complex many-body systems—where dynamical transitions are a consequence of genuine collective effects—and appear to exhaust all possible generic cases. We consider, however, a fourth class of problems, illustrated via a model of dissipative quantum glasses [10], whose dynamical behavior can change drastically while their statics remain invariant throughout. This highlights the need for a dynamical approach like the one presented here for characterizing complex open systems where static order parameters are nonexistent or difficult to identify.

The density matrix ρ of the open quantum systems we consider evolves under the master equation $\partial_t \rho = \mathcal{W}(\rho)$ with the QMO given by $\mathcal{W}(\bullet) \equiv \mathcal{H}(\bullet) + \mathcal{D}(\bullet)$. Here, the superoperators $\mathcal{H}(\bullet) = -i[H, \bullet]$ and $\mathcal{D}(\bullet) = \sum_{m=1}^{N} L_m \bullet L_m^\dagger - \frac{1}{2}\sum_{m=1}^{N}\{L_m^\dagger L_m, \bullet\}$ govern the coherent and dissipative dynamics, respectively. They depend on the Hamiltonian H and the set $\{L_m, m = 1, \ldots, N\}$ of jump operators. The specific form of the latter depends on the coupling of the system to the bath. It is well established that an open system dynamics generates a MPS on the bath degrees of freedom [12]. We use this connection to link the analysis of the emission sequence of the bath quanta, i.e., the quantum jump trajectories, to the more familiar picture of static phases of the ground state of a one-dimensional spin system. The prescription of Ref. [12] is clearest for the evolution of the density matrix ρ over short but finite time intervals δt, represented, for instance, by the Kraus map,

$$T_{\delta t}(\rho) \equiv e^{\mathcal{H}\delta t} e^{\mathcal{D}\delta t}(\rho) = K_0 \rho K_0^\dagger + \sum_{m=1}^{N} K_m \rho K_m^\dagger, \quad (1)$$

with Kraus operators $K_0 = e^{-i\delta t H}\sqrt{1 - \delta t \sum_{k=1}^N L_k^\dagger L_k}$ and $K_m = e^{-i\delta t H}\sqrt{\delta t} L_m$. Such discrete (but nonunique) representations of dissipative evolutions have been employed recently in experiment to simulate the dynamics of open many-body systems [19].

Figure 1(a) sketches the action of the Kraus operators: K_0 corresponds to nonunitary (i.e., no-jump) evolution, while $K_{m>0}$ represents the effect of the quantum jump associated with L_m. The evolution of the density matrix over macroscopic times is generated by multiple applications of the map (1). This produces quantum jump trajectories as shown in the upper panel of Fig. 1(b). The probability for a certain trajectory $\{n_1, n_2, \ldots, n_M\}$ to occur after M time steps is given by $p_{n_1 n_2 \ldots n_M} = \sum_f |\langle f | K_{n_M} \ldots K_{n_2} K_{n_1} | i \rangle|^2$, where $|i\rangle$ is the initial state of the system. The sum runs over a basis of final system states $|f\rangle$, and each term is the probability for connecting the initial and final states via a certain sequence of quantum jumps.

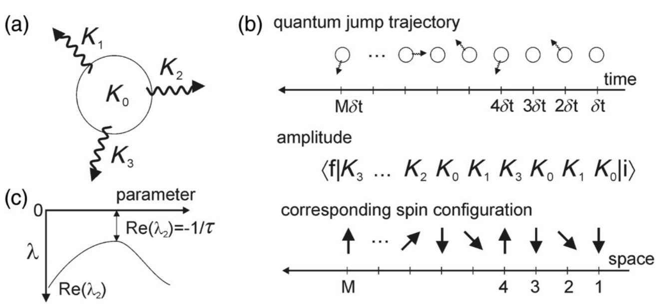

FIG. 1. (a) Kraus operators defining open dynamics: K_0 is the nonunitary (no-jump) evolution, while $K_{m>0}$ refers to the occurrence of a quantum jump of kind m. (b) The amplitude for a sequence of quantum jumps is obtained directly from the Kraus operators. This can be mapped to the state of a fictitious one-dimensional spin system, and all possible trajectories with their amplitudes can be gathered in a MPS interpreted as the quantum state of the spin chain. (c) The two-time correlation time of quantum jumps (or correlation length of the spin chain) is the inverse of the spectral gap of the QMO $\mathcal{W}$; dynamical transitions occur when this gap closes as a function of an external parameter.

These probabilities are encoded in a MPS which can be thought of as being generated by letting the system interact sequentially with a chain of $(N+1)$-dimensional spins initially prepared in a fixed pure state [12]. After M steps the quantum state of the system *and* the M spins with which it has interacted is $|\Psi\rangle = \sum_f |f\rangle \otimes |\psi(f)\rangle$, where $|\psi(f)\rangle$ is the (unnormalized) MPS,

$$|\psi(f)\rangle = \sum_{n_M, \ldots, n_1 = 0}^{N} \langle f | K_{n_M} \ldots K_{n_1} | i \rangle | n_1, \ldots, n_M \rangle, \quad (2)$$

with the sum running over all spin basis vectors. We can think of $|\psi(f)\rangle$ as the ground state of a fictitious one-dimensional spin system with specific boundary conditions [see Fig. 1(b)]. The state $|\Psi\rangle$ therefore encodes the whole *ensemble* of quantum trajectories: each basis state $|n_1, \ldots, n_M\rangle$ corresponds to a specific trajectory and its amplitude $\langle f | K_{n_M} \ldots K_{n_1} | i \rangle$ is related to the probability $p_{n_1 n_2 \ldots n_M}$ of it occurring dynamically. While this connection is only formal, it illustrates that the study of dynamical phases of open systems is not different from that of the static properties of a one-dimensional spin system, regardless of the actual spatial dimension of the open problem. Dynamical phase transitions will then become visible in the time correlations of the quantum jumps which correspond to spatial correlations in the spins.

The limit of very long times is the "thermodynamic limit" of the associated spin problem. In this regime the two-time (connected) correlations of observables at positions y and $y + x$ have the asymptotic form (see, e.g., Ref. [20]) $\langle \psi(f) | A^{(y)} B^{(y+x)} | \psi(f) \rangle_c \propto \mathrm{Re}\mu_2^x = e^{-x/\xi}\cos(x\phi_2)$. Here $\mu_2 = |\mu_2| \exp(\pm i\phi_2)$ denotes the eigenvalue(s) with the second largest absolute value, of the transfer operator $E = \sum_{n=0}^N K_n^* \otimes K_n$ (or equivalently of $T_{\delta t}$), the largest eigenvalue being $\mu_1 = 1$ by the conservation of probability of the map (1). The correlation length ξ is given by $\xi^{-1} = -\log|\mu_2|$. The correlations exhibit exponentially damped oscillations when the eigenvalue μ_2 is complex, in which case μ_2^* is also an eigenvalue of $T_{\delta t}$. It is more convenient to directly study the eigenvalues λ of the QMO $\mathcal{W}$. In the limit $\delta t \to 0$ temporal correlations between quantum jumps behave as $\langle A(t) B(t + t') \rangle_c \propto \exp(-t'/\tau)\cos(\omega t')$, with $\tau \equiv -1/\mathrm{Re}(\lambda_2)$ and $\omega \equiv \mathrm{Im}(\lambda_2)$, where λ_2 is the eigenvalue of $\mathcal{W}$ with the second largest real part, Fig. 1(c) (note that the eigenvalue with largest real part is $\lambda_1 = 0$).

Dynamical phase transitions will manifest in the closing of the spectral gap of the operator $\mathcal{W}$, i.e., $\lambda_2 \to 0$ [see Fig. 1(c)]. In the following we will analyze four example systems using the QMO spectrum and quantum jump trajectories calculated via the quantum jump Monte Carlo method discussed in Ref. [16].

(i) Three-level system.—This system [16], depicted in Fig. 2(a), is described by Hamiltonian $H_3 = \Omega_1 |0\rangle\langle 1| + \Omega_2 |0\rangle\langle 2| + \mathrm{H.c.}$ and a single jump operator $L = \sqrt{\kappa} |0\rangle\langle 1|$.

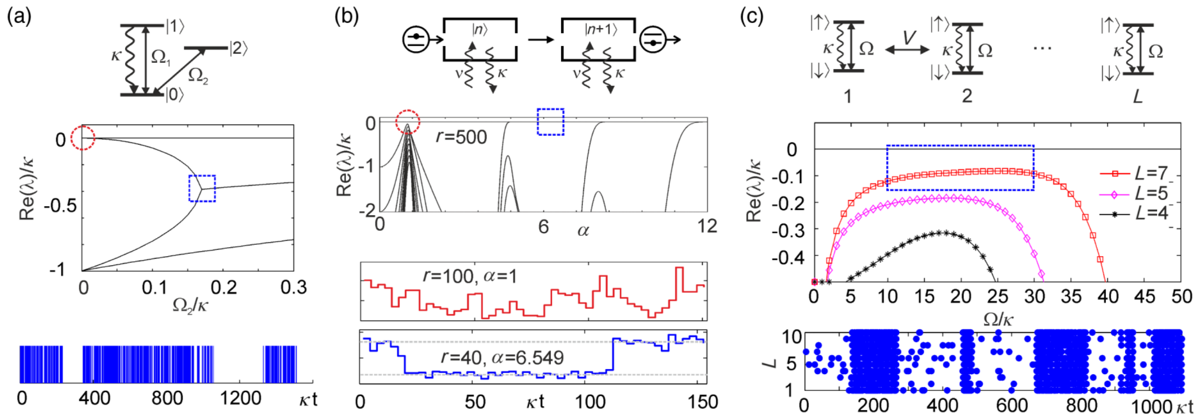

FIG. 2 (color online). Examples for open quantum systems where a dynamical transition is accompanied by a static transition. Each panel shows a sketch of the system, the real part of the spectrum of the QMO, and sample trajectories (from top to bottom). (a) The driven three-level system has a dynamical transition at $\Omega_2 = 0$ (circled region). For $\Omega_2 \gtrsim 0$ quantum jumps are highly intermittent (we show the case $\Omega_2 = 0.01\kappa$ and $\Omega_1 = 4\kappa$). In the boxed region λ_2 acquires an imaginary part and time correlations become oscillatory. (b) The micromaser displays both first- and second-order dynamical transitions. The two sample trajectories are taken at the second-order transition point (red circle and red trajectory) and within the coexistence region of two dynamical phases (blue box and blue trajectory). In order to facilitate the representation of the trajectory we have collected quantum jumps in time bins of length $t_{\mathrm{bin}} = 3/\kappa$. (c) The open Ising model with L spins has a genuine many-body dynamical first-order transition and the spectral gap closes when $L \to \infty$. In the parameter regime chosen ($V = 100\kappa$) the spectral gap becomes small for $10\kappa \leq \Omega \leq 30\kappa$. In this near coexistence region of two dynamical phases one observes strongly intermittent behavior of quantum jumps which are strongly correlated in the spatial direction (we show data for $L = 10$, $\Omega = 25\kappa$).

It exhibits a dynamical phase transition at $\Omega_2 = 0$ (where $\lambda_2 = 0$), which manifests in strongly intermittent behavior of photon emission when $\Omega_2 \ll \Omega_1$ [bottom of Fig. 2(a)] [21]. The reason is that at $\Omega_2 = 0$ the system decouples into a driven two-level system undergoing frequent quantum jumps and an inactive single dark level. This corresponds to a twofold degeneracy of the leading eigenvalue [Fig. 2(a)], which is lifted when $\Omega_2 > 0$. For $\Omega_2 \gtrsim 0$ the system can switch between these two phases on a time scale Ω_2^{-1}, resulting in strongly intermittent behavior reminiscent of a (smoothed) first-order transition [15]. From the perspective of the MPS this corresponds to the quantum phase transitions reported in Ref. [12]. Beyond the transition at $\Omega_2 = 0$, the three-level system also features a dynamical transition at finite Ω_2 where time correlations become oscillatory [see Fig. 2(a)]. Related transitions have been reported in a NMR experiment; see Refs. [22,23].

(ii) Micromaser.—This more complex single-body system has an infinite Hilbert space, and features a critical point and a sequence of first-order transitions [17,18]. It is modeled by a resonant single-mode cavity coupled to a finite temperature bath and pumped by excited two-level atoms which are sent into the cavity with a constant rate r; see Fig. 2(b). The Hamiltonian is zero and there are four jump operators, two nonlinear ones stemming from the atom-cavity interaction, $L_1 = \sqrt{r} a^\dagger \frac{\sin(\phi\sqrt{aa^\dagger})}{\sqrt{aa^\dagger}}$ and $L_2 = \sqrt{r}\cos(\phi\sqrt{aa^\dagger})$, and two from the cavity-bath interaction, $L_3 = \sqrt{\kappa} a$ and $L_4 = \sqrt{\nu} a^\dagger$. Here a, $a^\dagger$ are the raising or lowering operators of the cavity mode, κ and ν are the thermal relaxation and excitation rates, and ϕ encodes the atom-cavity interaction [18]. The spectrum of $\mathcal{W}$ is real and is depicted in Fig. 2(b) as a function of the "pump" parameter $\alpha = \phi/\sqrt{r}$. A sequence of first-order transitions is visible beyond $\alpha = 4$, and quantum jump trajectories (here we monitor quantum jumps associated to L_1) show the typical intermittent behavior. In contrast, in the vicinity of $\alpha = 1$ the spectral gap closes in a way that makes the spectrum dense. This is the onset of a second-order phase transition which strictly only occurs in the limit $r \to \infty$ [24]. Typical quantum jump trajectories near the critical point fluctuate very strongly [bottom of Fig. 2(b)]. Also here dynamical transitions are accompanied by a change in the statics: at first-order transitions the mean photon occupation of the cavity switches between two distinct values, while at the critical point the variance of the photon number undergoes a jump [17].

(iii) Dissipative Ising model.—In a many-body system such as this one [3,4] [see Fig. 2(c)], the degeneracy leading to a phase transition is a collective effect and not imposed externally or through nonlinear jump operators. The Hamiltonian is $H_I = \Omega\sum_{k=1}^{L}\sigma_x^k + V\sum_{k=1}^{L}\sigma_z^k\sigma_z^{k+1}$, where σ_α^k are the Pauli spin matrices. The jump operators are $L_k = \sqrt{\kappa}\sigma_k^-$, which produce incoherent spin flips, $|\uparrow\rangle \to |\downarrow\rangle$, at a rate κ. In Fig. 2(c) we show the real part of the eigenvalues of the QMO $\mathcal{W}$ for various system sizes L. With increasing L the spectral gap closes over an entire (coexistence) region in parameter space and we expect it to approach zero when $L \to \infty$. This resembles a static phase transition in the MPS. For finite L and finite gap, the system switches between two dynamical phases on long but finite time scales and quantum jump trajectories are strongly intermittent. Once again, the dynamical transition can be traced back to a bistable static behavior: When the emission of photons is plentiful, the average magnetization is close to zero, while it is large and negative during the dark periods [4]. This would suggest that dynamical transitions can always be anticipated to occur by simply considering static or steady-state properties. The next example proves that this is not the case.

(iv) Dissipative quantum glass.—This system is related to the dissipative quantum glass models of Ref. [10]. It is a spin chain with Hamiltonian $H_g = \Omega \sum_{k=1}^{L} \sigma_x^k f_{k+1}^2(p)$ and jump operators $L_k = \sqrt{\kappa}\sigma_k^- f_{k+1}(p)$ (see Fig. 3). The operators $f_k(p)$ are *kinetic constraints*. If $f_k(p) = 1$, the system is just a set of noninteracting two-level systems with steady-state density matrix $\rho_{ss} = \otimes_{k=1}^{L}[\{(1/2) + (\omega\kappa)/(\kappa^2+\omega^2)\}P_k + \{(1/2) - (\omega\kappa)/(\kappa^2+\omega^2)\}Q_k]$, where $\omega \equiv \sqrt{16\Omega^2+\kappa^2}$, $Q_k = \frac{\mathbb{1}}{2} + \frac{\kappa}{2\omega}\sigma_z^k - \frac{2\Omega}{\omega}\sigma_y^k$, and $P_k = \mathbb{1} - Q_k$. Here the statics is clearly featureless. The problem becomes interacting and glassy if we choose the constraints to be $f_k(p) = pQ_k + (1-p)\mathbb{1}$ with $0 \le p \le 1$: For $p = 0$ we have the noninteracting problem and for $p = 1$ a fully constrained quantum glass [10]. When $p = 1$ the state of the spin on site k can only change if the state $|\phi\rangle_{k+1}$ of its neighbor satisfies $Q_{k+1}|\phi\rangle_{k+1} \neq 0$, leading to correlated dynamics in the system. The parameter p controls how glassy this dynamics is. Figure 3 shows that the spectral gap of the QMO would close at $p = 1$. Here we expect a dynamical first-order phase transition to occur in analogy with classical constrained models [14,25]. A crucial feature of this model is that the steady state for *any* value of p is the trivial ρ_{ss}; i.e., there is no change in the statics despite the change in the dynamics, as is evident from the example jump trajectories shown Fig. 3. As $p \to 1$ the system is most of the time inactive and quantum jumps become more and more localized in space and time, a phenomenon called dynamical heterogeneity, which is a hallmark of glassy relaxation [26]. In contrast to the three

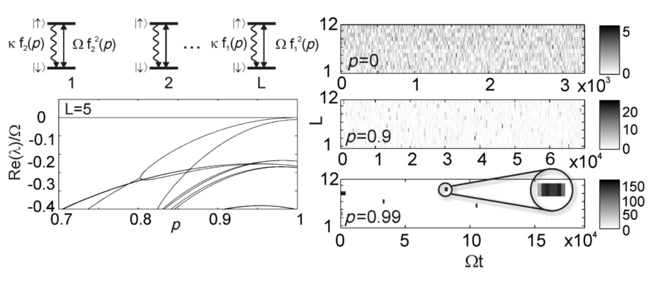

FIG. 3. Dissipative quantum glass model, with glassiness controlled by the parameter p: at $p = 0$ the system is noninteracting, for $p = 1$ dynamics is fully constrained. The spectral gap closes at $p = 1$ (we show the case of $\kappa = 8\Omega$). Trajectories shown consist of 5000 quantum jumps each. Dynamics changes drastically with p, but the stationary state remains invariant. Close to $p = 1$ emission periods are localized in time and space (see magnified region). For ease of visibility jumps are collected in 500 evenly spaced time bins.

systems described above, here the dramatic dynamical change with p is impossible to guess from static properties which are p independent and trivial throughout.

Since the ensemble of trajectories is fully encoded in the MPS $|\Psi\rangle$, the sample trajectories of Fig. 3 indicate that the dynamical transition is equivalent to a static transition within the MPS as $p \to 1$. Usually such static quantum transitions are accompanied by a singularity, such as a logarithmic divergence in the entanglement entropy of large spin blocks [27]. In our case this would be entanglement between quantum jumps in subsequent long time segments. The entropy of a large block, however, is just 2 times the von Neumann entropy of the stationary state $S_E = -2\,\mathrm{Tr}(\rho_{ss}\log\rho_{ss})$ of the system, where $\rho_{ss} \equiv \mathrm{Tr}_\psi|\Psi\rangle\langle\Psi|$ [28]. Hence, due to the invariance of the stationary state with p, this entanglement measure does not detect the transition.

The reason is that—as in glasses, and is likely also in complex many-body systems—the fields driving the transition do not couple directly to obvious static quantities, but do so to time-integrated observables. An example is "counting" fields introduced when computing full counting statistics [29] of dynamical observables [15]. Constrained models such as the one above are known to exhibit transitions in trajectories, which are evident in the moment generating function (MGF) of the number of quantum jumps, $Z_t(s) \equiv \sum_J P_t(J)e^{-sJ}$, where $P_t(J)$ is the probability of observing J jumps in time t. In the $t \to \infty$ limit $Z_t(s)$ becomes singular at some value $s = s_c$ of the counting field [14,15], indicating a phase transition in the ensemble of trajectories. At long times, the MGF is obtained from the largest eigenvalue of a deformation $\mathcal{W} \to \mathcal{W}_s$ of the QMO parametrized by s [14,15]. The field s therefore couples directly to the spectrum, so that a singularity of the MGF at s_c indicates the existence of close to degenerate but distinct dynamical states. By driving s one can single these states out; see Refs. [4,15,18] for details.

The MGF can be connected to the MPS through the norm, $Z_t(s) = \langle\Psi(s)|\Psi(s)\rangle$, of the deformation $|\Psi(s)\rangle \equiv e^{-s\hat{J}/2}|\Psi\rangle$, where $\hat{J}$ is an operator that counts the number of $n_{m>0}$ in a state $|n_1, \ldots, n_M\rangle$. In a thermodynamic analogy $Z_t(s)$ is like a partition sum over trajectories and s a chemical potential which favors or disfavors quantum jumps. The state $|\Psi(s)\rangle$ is thus a superposition of MPS like those of (2) but with each term weighed by a factor of $e^{-s/2}$ for each jump. The eigenstate $\rho(s)$ of $\mathcal{W}_s$ corresponding to the largest eigenvalue is related to $|\Psi(s)\rangle$ through $\mathrm{Tr}_\psi|\Psi(s)\rangle\langle\Psi(s)| = Z_t(s)\rho(s)$, where $\mathrm{Tr}\rho(s) = 1$ and $\rho(0) = \rho_{ss}$. The entanglement entropy of this state, $\tilde{S}_E = -2\,\mathrm{Tr}[\rho(s)\log\rho(s)]$, will depend on s, and will display nonanalytic behavior at s_c. At this level counting fields work in a similar manner to more standard static fields which drive phase transitions, but couple directly to the relevant dynamical order parameters that reveal transitions in quantum jump trajectories. This perspective should be useful in the study of dynamical phase transitions in systems where they are not obviously connected to a change in spatial correlations.

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