$H = \frac{1}{2}$

Arjun Bagchi,¹ Stéphane Detournay,² Reza Fareghbal,³ and Joan Simón¹

¹School of Mathematics and Maxwell Institute, University of Edinburgh, Edinburgh EH9 3JZ, United Kingdom

²Center for the Fundamental Laws of Nature, Harvard University, Cambridge, Massachusetts 02138, USA³School of Particles and Accelerators, Institute for Passacreh in Eundamental Sciences (IPM), P.O. Box 19395, 5531, 3

School of Particles and Accelerators, Institute for Research in Fundamental Sciences (IPM), P.O. Box 19395-5531, Tehran, Iran

(Received 22 September 2012; published 2 April 2013)

We provide a first derivation of the Bekenstein-Hawking entropy of 3D flat cosmological horizons in terms of the counting of states in a dual field theory. These horizons appear in the flat limit of nonextremal rotating Banados-Teitleboim-Zanelli black holes and are remnants of the inner horizons. They also satisfy the first law of thermodynamics. We study flat holography as a limit of AdS_3/CFT_2 to semiclassically compute the density of states in the dual theory, which is given by a contraction of a 2D conformal field theory, exactly reproducing the bulk entropy in the limit of large charges. We comment on how the dual theory reproduces the bulk first law and how cosmological bulk excitations are matched with boundary quantum numbers.

DOI: [10.1103/PhysRevLett.110.141302](http://dx.doi.org/10.1103/PhysRevLett.110.141302) PACS numbers: 04.70.Dy, 04.20.Ha, 04.60.Rt, 11.25.Tq

Introduction.—Quantum gravity is the holy grail of modern theoretical physics. Perhaps the most enigmatic and most successful route to a theory of quantum gravity is the holographic principle $[1,2]$ $[1,2]$, which states that a theory of quantum gravity in some spacetime is exactly equivalent to a theory without gravity living on the boundary of that spacetime. Our current understanding of the holographic principle is mainly restricted to anti-de Sitter (AdS) spacetimes through the AdS/conformal field theory (CFT) corre-spondence [\[3](#page-4-2)]. One of the enduring successes of AdS/CFT has been the explanation of the entropy of AdS black holes by a counting of microstates in the dual field theory [\[4\]](#page-4-3).

Real astrophysical black holes, however, are asymptotically flat. The understanding of their features using holography, and indeed a better understanding of the holographic principle itself, calls for a formulation holography in flat spacetimes. In this Letter, we take significant strides in this direction and provide such an understanding of the entropy of horizons in 3D Minkowski spacetime in terms of the counting of states in a conjectured dual field theory. Our analysis can be looked at as the flat space analogue of the derivation of the AdS_3 Banados-Teitleboim-Zanelli (BTZ) black holes [\[5,](#page-4-4)[6](#page-4-5)] entropy in terms of the symmetries of the dual CFT₂ [[7](#page-4-6)]. The horizon that we study in this Letter is a cosmological horizon which arises in a certain quotient of Minkowski spacetime. This can also be understood as a remnant of the inner horizon of a nonextremal BTZ black hole in the flat space limit. The peculiarity of our setup actually also allows us to ask interesting questions about cosmology and provides for the first time an understanding of cosmological horizons in terms of a dual description.

Flat spacetimes can be obtained as large radius limits of AdS. Formulating flat space holography as a limit of AdS/ CFT has been used to extract features of the flat space S matrix from AdS/CFT correlators; see, e.g., Refs. [\[8](#page-4-7)[–11\]](#page-4-8). Our 3D analysis follows this philosophy and is primarily based on the symmetries of the theory. This was recently pursued in Refs. $[12-15]$ $[12-15]$ $[12-15]$ where these flat limits are realized as a contraction on the symmetry structure of AdS/ CFT. On the bulk side [[12](#page-4-9)[,15](#page-4-10)[,16\]](#page-4-11), the AdS symmetries are contracted to what is called the Bondi-Metzner-Sachs (BMS) algebra [[17](#page-4-12)–[20](#page-4-13)]. On the field theory side, the CFT symmetries are contracted to the so-called Galilean conformal algebras (GCA), previously considered in the context of the nonrelativistic limits of CFTs [\[21\]](#page-4-14). This connection was christened the BMS/GCA correspondence [\[13\]](#page-4-15), which holds even beyond our 3D context.

Here, we focus on $BMS₃/GCA₂$ and explore the extent to which the dual 2D Galilean conformal field theory (GCFT), invariant under GCA, can reproduce nontrivial features about asymptotic $\mathbb{R}^{1,2}$ spacetimes. We show the family of shifted-boost orbifolds of $\mathbb{R}^{1,2}$ [[22](#page-4-16)] correspond to the flat limit of the nonextremal BTZ black holes and carry nontrivial $BMS₃$ charges. Their causal structure contains a cosmological horizon, the remnant of the BTZ inner horizon, whose area, surface gravity, and rotation allow us to define a bulk entropy, temperature, and angular velocity, respectively. We show that these satisfy a first law of thermodynamics, analogous to the ones recently discussed in the literature $[23-30]$ $[23-30]$ $[23-30]$. We then derive an analogue of Cardy's formula for 2D GCFTs by computing their density of states semiclassically. Our main result is that our dual field theory calculation reproduces the bulk entropy. This constitutes the first ''microscopic'' derivation of the entropy of a cosmological horizon, in the spirit of Ref. [[7](#page-4-6)].

Asymptotic symmetries in 3D Minkowski spacetime.— The physical states in the Hilbert space of a quantum theory of gravity form representations of the symmetry structure of the theory at its asymptotic boundary, the asymptotic symmetry algebra (ASA) . In $AdS₃$, the seminal work of Ref. [[31](#page-4-19)] showed that the ASA is formed by two commuting copies of the Virasoro algebra $(\mathcal{L}_n, \mathcal{L}_n)$

$$
[\mathcal{L}_m, \mathcal{L}_n] = (m - n)\mathcal{L}_{m+n} + \frac{c}{12}m(m^2 - 1)\delta_{m+n,0}, \quad (1)
$$

with analogous brackets for \mathcal{L}_n and central charge \bar{c} = $c = \frac{3\ell}{2G}$, where ℓ is the radius of AdS₃ and G the Newton
constant. These Virasoro algebras are also physically realconstant. These Virasoro algebras are also physically realized as the local symmetry algebras of 2D CFTs. Through AdS/CFT, the two are identified, providing a match between the gravity and the dual CFT symmetries.

In 3D Minkowski spacetimes, the ASA is given by the $BMS₃$ algebra [[16](#page-4-11)]

$$
[L_m, L_n] = (m - n)L_{m+n} + c_{LL}m(m^2 - 1)\delta_{m+n,0},
$$

$$
[L_m, M_n] = (m - n)M_{m+n} + c_{LM}m(m^2 - 1)\delta_{m+n,0},
$$
 (2)

with $[M_n, M_m] = 0$, $c_{LL} = 0$, and $c_{LM} = 1/4$. These symmetries are defined at null infinity, with L_m being the diffeomorphisms of its spatial circle and M_n the angular dependent supertranslations and translations. This algebra is isomorphic to the 2D GCA, the symmetry algebra of GCFTs, which are best viewed as a limit of standard 2D $CFTs$. This forms the basis of the BMS/GCA correspondence in three bulk dimensions. The precise map can be described in terms of an Inönü-Wigner contraction of the two copies of the Virasoro algebra [[16](#page-4-11)]

$$
L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n}, \quad M_n = \epsilon(\mathcal{L}_n + \bar{\mathcal{L}}_{-n}), \quad \epsilon = G/\ell \to 0,
$$
\n(3)

where we take $\ell \rightarrow \infty$, keeping G fixed. This limit implemented on the algebra [\(1\)](#page-1-0) yields ([2](#page-1-1)), with central charges $c_{LL} = \frac{1}{12}(c - \bar{c}) = 0$ and $c_{LM} = \frac{G}{12\ell}(c + \bar{c}) = 1/4$.
A natural spacetime interpretation in terms of the

A natural spacetime interpretation in terms of the contraction of the Killing vectors of global AdS to the Killing vectors of flat space and the natural extension to the infinite set of asymptotic Killing vectors was recently worked out in Ref. [[14](#page-4-20)], where it was also shown that the $\ell \rightarrow \infty$ limit induces a spacetime contraction $(t, x) \rightarrow (\epsilon t, x)$ on the CFT living on the boundary cylinder. This limit is best interliving on the boundary cylinder. This limit is best interpreted as an ultrarelativistic limit [[32](#page-4-21)].

Spacetime analysis.—In the absence of sources, the most general solution to 3D pure gravity with a vanishing cosmological constant is locally flat [[34](#page-4-22)] and it can be written as [[12](#page-4-9)]

$$
ds^{2} = \Theta(\psi)du^{2} - 2drdu
$$

+ 2\left[\Xi(\psi) + \frac{u}{2}\Theta'\right]d\psi du + r^{2}d\psi^{2}. (4)

The asymptotic structure at null infinity is preserved by a set of diffeomorphisms, depending on two arbitrary functions of ψ whose modes [\[16](#page-4-11)[,35\]](#page-4-23)

$$
\ell_n = i e^{in\psi} \bigg[inu\partial_u - inr\partial_r + \bigg(1 + n^2 \frac{u}{r}\bigg)\partial_\psi \bigg], \qquad (5a)
$$

$$
m_n = i e^{in\psi} \partial_u, \qquad n \in \mathbb{Z}, \tag{5b}
$$

satisfy the centerless BMS₃ algebra. Note that $\ell_{\pm 1,0}$ and $m_{\pm 1,0}$ coincide asymptotically with the exact Killing
westers of Minkowski space forming the global $i \gtrsim 0.2, 1$ vectors of Minkowski space forming the global $i\Im(2, 1)$ subalgebra of $BMS₃$. The coefficients of the Fourier mode decomposition of the corresponding asymptotically conserved charges L_n and M_n determine the arbitrary functions $\Theta(\psi)$, $\Xi(\psi)$:

$$
\Theta = -1 + 8G \sum_{n} M_n e^{-in\psi}, \quad \Xi = 4G \sum_{n} L_n e^{-in\psi}. \tag{6}
$$

In this Letter, we study the most general zero mode solution labeled by $\Theta = 8GM$ and $\Xi = 4GJ$ with $M, J \ge 0$.
We show that these correspond to the shifted-boost orbi-We show that these correspond to the shifted-boost orbifold $(J \neq 0)$ [\[22\]](#page-4-16) and the boost orbifold $(J = 0)$ [\[36\]](#page-4-24) of $\mathbb{R}^{1,2}$. This claim can be derived by taking the flat limit, as in Ref. [\[22\]](#page-4-16), of the nonextremal BTZ black holes ($M\ell \neq J$),

$$
ds_{\text{BTZ}}^2 = \left(8GM - \frac{r^2}{\ell^2} \right) dt^2 + \frac{dr^2}{-8GM + \frac{r^2}{\ell^2} + \frac{16G^2 J^2}{r^2}} - 8GJdt d\phi + r^2 d\phi^2, \qquad \phi \sim \phi + 2\pi, \qquad (7)
$$

whose outer and inner horizons r_{\pm} are given by

$$
M = \frac{r_+^2 + r_-^2}{8G\ell^2}, \qquad J = \frac{r_+ r_-}{4G\ell}.
$$
 (8)

We refer to their flat $\ell \rightarrow \infty$ limit as the "flat BTZ" (FBTZ)

$$
ds_{\text{FBTZ}}^2 = \hat{r}_+^2 dt^2 - \frac{r^2 dr^2}{\hat{r}_+^2 (r^2 - r_0^2)} + r^2 d\phi^2 - 2\hat{r}_+ r_0 dt d\phi,
$$
\n(9)

where $r_+ \rightarrow \ell \sqrt{8GM} = \ell \hat{r}_+$ and $r_- \rightarrow \sqrt{\frac{2G}{M}}|J| = r_0$. These \mathbf{v} and \mathbf{v} and \mathbf{v} and \mathbf{v} and \mathbf{v} and \mathbf{v} are correspond to [\(4](#page-1-2)) under the coordinate transformation

$$
d\psi = d\phi + \frac{r_0 dr}{\hat{r}_+(r^2 - r_0^2)}, \qquad du = dt + \frac{r^2 dr}{\hat{r}_+^2 (r^2 - r_0^2)}.
$$

The null hypersurface $r = r_0$ is a Killing horizon with normal $\chi = \partial_u + (\hat{r}_+ / r_0) \partial_\phi$, surface gravity $\kappa = \hat{r}_+^2 / r_0$,
and angular velocity $\Omega = \hat{r}_+ / r_0$. Thus, we can associate a and angular velocity $\Omega = \hat{r}_+/r_0$. Thus, we can associate a Hawking temperature and entropy to it:

$$
T_{\text{FBTZ}} = \frac{\kappa}{2\pi} = \frac{\hat{r}_+^2}{2\pi r_0}, \qquad S_{\text{FBTZ}} = \frac{\pi |r_0|}{2G}.
$$

In fact, these quantities satisfy the first law of thermodynamics

$$
-T_{\text{FBTZ}}dS_{\text{FBTZ}} = dM - \Omega dJ,\tag{10}
$$

which is the remnant of the corresponding law satisfied by the BTZ inner horizon at finite ℓ

$$
-T_H^- dS^- = dM - \Omega^- dJ,\tag{11}
$$

where $T_H^ \Gamma_H = \frac{r_+^2 - r_-^2}{2\pi r_- \ell^2}, \ \Omega^{-1}$ $=\frac{r_+}{\ell r_-}$, and S^- -temperature, angular velocity, and entropy, respectively. $=\frac{\pi |r_-\|}{2G_3}$ are its

That [\(9](#page-1-3)) is a quotient manifests through the map to Cartesian coordinates (valid for $r > r_0$; a similar change of coordinates exists for $r < r_0$):

$$
X^{2} = \frac{r^{2} - r_{0}^{2}}{\hat{r}_{+}^{2}} \sinh^{2}(\hat{r}_{+} \phi), \qquad T^{2} = \frac{r^{2} - r_{0}^{2}}{\hat{r}_{+}^{2}} \cosh^{2}(\hat{r}_{+} \phi),
$$

$$
Y = r_{0} \phi - \hat{r}_{+} t,
$$
 (12)

so that $\xi = \partial_{\phi} = r_0 \partial_Y + \hat{r}_+(X \partial_Y + T \partial_X)$ acts like $X^{\pm} \sim$ $e^{\pm 2\pi \hat{r}_+} X^{\pm}$, $Y \sim Y + 2\pi r_0$. Equivalently, the action of ξ identifies points of $\mathbb{R}^{1,2}$ under a combined boost in the $(T, X) \mathbb{R}^{1,1}$ plane of rapidity \hat{r}_+ and a translation of length r_0 in the transverse Y direction. Thus, ([9](#page-1-3)) is the shiftedboost orbifold of $\mathbb{R}^{1,2}$ [[22](#page-4-16)].

This orbifold interpretation provides a global descrip-tion for the spacetime ([9](#page-1-3)) [\[37\]](#page-4-25). Whenever $X^+X^- > 0$,

$$
X^{\pm} = \frac{\tau}{\sqrt{2}} e^{\pm E(z+y)}, \qquad Y = z, \qquad E = \frac{\hat{r}_+}{r_0}, \qquad (13)
$$

the geometry describes a contracting universe $(\tau < 0)$ toward a cosmological horizon located at $\tau = 0$, i.e., $r = r_0$, and an expanding one ($\tau > 0$) from it

$$
ds^{2} = -d\tau^{2} + \frac{(E\tau)^{2}}{H^{2}}dy^{2} + H^{2}(dz + A)^{2}, \qquad (14)
$$

with $H^2 = 1 + (E\tau)^2$ and $A = (1 - H^{-2})dy$. In the region $-1/E^2 < 2X^+X^- < 0$ $1/E^2 < 2X^+X^- < 0$,

$$
X^{\pm} = \pm \frac{x}{\sqrt{2}} e^{\pm E(z+y)}, \qquad Y = z,
$$
 (15)

the geometry is static;

$$
ds^{2} = dx^{2} - \frac{(Ex)^{2}}{H^{2}}dy^{2} + H^{2}(dz + A)^{2}, \qquad (16)
$$

with $H^2 = 1 - (Ex)^2$ and $A = (1 - H^{-2})dy$, and
describes a Rindler space in the region $(Fx)^2 \ll 1$ with a describes a Rindler space in the region $(EX)^2 \ll 1$ with a Rindler horizon at $x = 0$ i.e. $r = r_0$ Finally whenever Rindler horizon at $x = 0$, i.e., $r = r_0$. Finally, whenever $2X^+X^- < -1/E^2$, the geometry has closed timelike
curves as nonextremal BTZ black holes do As in that curves, as nonextremal BTZ black holes do. As in that case, we will excise this region from spacetime, introducing a singularity at its boundary $2X^+X^- = -1/E^2$.

¼ -Dual field theory analysis.—Quantum gravity states in 3D flat space should transform under representations of the infinite 2D GCA. These are labeled by eigenvalues of L_0 , M_0 [[33](#page-4-26),[38](#page-4-27)]:

$$
L_0|h_L, h_M\rangle = h_L|h_L, h_M\rangle, \qquad M_0|h_L, h_M\rangle = h_M|h_L, h_M\rangle,
$$

where $h_L = \lim_{\epsilon \to 0} (h - \bar{h}), \qquad h_M = \lim_{\epsilon \to 0} \epsilon (h + \bar{h}).$ (17)

There exists the usual notion of primary states $|h_L, h_M\rangle_p$ in the 2D GCA annihilated by L_n , M_n for $n > 0$. Representations are built by acting with the raising operators L_{-n} , M_{-n} on them. They satisfy $h_L \ge 0$. Using the dictionary between $h \bar{h}$ and AdS_a mass and angular modictionary between h, \bar{h} and AdS₃ mass and angular momentum, we can relate $\{h_L, h_M\}$ to the BMS₃ charges

$$
h = \frac{1}{2}(\ell M + J) + \frac{c}{24},
$$

\n
$$
\bar{h} = \frac{1}{2}(\ell M - J) + \frac{\bar{c}}{24} \Rightarrow h_L = J,
$$

\n
$$
h_M = GM + \frac{c_{LM}}{2} = GM + \frac{1}{8}.
$$
\n(18)

This suggests the bound $h_M \ge 0$, saturated by $\mathbb{R}^{1,2}$ [[15\]](#page-4-10), given that $GM = -1/8$ for global AdS₃. This is confirmed
by a 2D GCFT unitarity bound derived from demanding by a 2D GCFT unitarity bound derived from demanding that the norm of a state of weight (h, h) at a level n be non-negative in the original 2D CFT. This gives $2nh +$ $\frac{c}{12} n(n^2 - 1) \ge 0$ and similarly for \bar{h} . Using the definitions (17) one can derive the analogous statement for 2D GCET [\(17\)](#page-2-0), one can derive the analogous statement for 2D GCFT

$$
nh_M + c_{LM}n(n^2 - 1) \ge 0 \Rightarrow h_M \ge 0, \qquad c_{LM} \ge 0. \tag{19}
$$

A more thorough analysis is required to better understand aspects of unitarity in 2D GCFTs.

Given the success of Cardy's formula in accounting for the entropy of BTZ black holes [\[7\]](#page-4-6), it is natural to wonder whether a counting of states in 2D GCFTs can reproduce the gravitational entropy S_{FBTZ} . To analyze this, define the partition function of 2D GCFT as

$$
Z_{\text{GCFT}}(\eta,\rho) = \sum d(h_L, h_M) e^{2\pi i (\eta h_L + \rho h_M)},\tag{20}
$$

where $d(h_L, h_M)$ is its density of states with charges $\{h_L, h_M\}$. To derive an analogue of Cardy's formula for the GCFT, it is crucial to use an analogue of modular invariance in the original 2D CFT for the 2D GCFT partition function. In particular, we need to derive the S transformation rules for 2D GCFTs. We shall first state this result and then motivate it as emerging as a limit of the original 2D CFT. The S transformation in 2D GCFTs reads

$$
S: (\eta, \rho) \to (-1/\eta, \rho/\eta^2). \tag{21}
$$

To understand this, let us start with the 2D CFT partition function and rewrite it using ([17](#page-2-0)) but at finite ϵ

$$
Z_{\text{CFT}} = \sum d_{\text{CFT}}(h, \bar{h}) e^{2\pi i (\tau h + \bar{\tau}\bar{h})}
$$

=
$$
\sum d(h_L, h_M) e^{2\pi i [\eta h_L + (\rho/\epsilon)h_M]}
$$

where η , $\rho = \frac{1}{2}(\tau \pm \bar{\tau})$. At finite ϵ , the *S* transformation of the 2D CET reads the 2D CFT reads

$$
(\tau, \bar{\tau}) \to \left(-\frac{1}{\tau}, -\frac{1}{\bar{\tau}}\right) \Rightarrow (\eta, \rho) \to \left(\frac{\eta}{\rho^2 - \eta^2}, \frac{-\rho}{\rho^2 - \eta^2}\right).
$$
\n(22)

The Hamiltonian scaling in Z_{CFT} must be accompanied by a temperature rescaling $\rho \rightarrow \epsilon \rho$. 2D GCFT transforma-
tions (21) emerge from (22) after this rescaling Following tions [\(21\)](#page-2-1) emerge from ([22](#page-2-2)) after this rescaling. Following [\[39](#page-4-28)[,40\]](#page-4-29) closely, we define the quantity

$$
Z_{\text{GCFT}}^0(\eta,\rho) = e^{-\pi i \rho c_{LM}} Z_{\text{GCFT}}(\eta,\rho),\tag{23}
$$

in analogy to the modular invariant 2D CFT partition function on the plane $Z_{\text{CFT}}^0 = e^{-(\pi i/12)(c\tau - \bar{c}\bar{\tau})} Z_{\text{CFT}}$ satisfy-
ing Z_{CFT}^0 ($\tau \bar{\tau}$) = Z_{CFT}^0 ($-1/\tau$ -1/ $\bar{\tau}$) In the flat limit this ing $Z_{\text{CFT}}^0(\tau, \bar{\tau}) = Z_{\text{CFT}}^0(-1/\tau, -1/\bar{\tau})$. In the flat limit, this translates into $Z_{\text{GFT}}^0(\eta, \rho) = Z_{\text{GCT}}^0(-\frac{1}{\eta}, \frac{\rho}{\eta^2})$, the S transformation (21) (21) (21) , leading to

$$
Z_{\text{GCFT}}(\eta,\rho) = e^{i\pi c_{LM}\rho(1-1/\eta^2)} Z_{\text{GCFT}}\left(-\frac{1}{\eta},\frac{\rho}{\eta^2}\right).
$$
 (24)

By doing an inverse Laplace transformation, the density of states equals

$$
d(h_L, h_M) = \int d\eta d\rho e^{2\pi i f(\eta, \rho)} Z_{\text{GCFT}}(-1/\eta, \rho/\eta^2),
$$

where $f(\eta, \rho) = \left(\frac{c_{LM}}{2} - \frac{c_{LM}}{2\eta^2} - h_M\right)\rho - h_L \eta.$ (25)

In the limit of large charges, we evaluate (25) (25) (25) by a saddle-point approximation. There is a saddle at $\eta \approx$ $i\sqrt{c_{LM}}/\sqrt{2h_M}$ whenever $Z_{GCFT}(-1/\eta, \rho/\eta^2)$ is slowly
varying which occurs at positive i.e., peoptive GCET varying, which occurs at positive $i\rho$, i.e., negative GCFT temperature, a point we stress below. The 2D GCFT entropy is then

$$
S_{\text{GCFT}} = \log d(h_L, h_M) = 2\pi h_L \sqrt{\frac{c_{LM}}{2h_M}}.\tag{26}
$$

This is the analogue of the Cardy formula for 2D GCFT [[41\]](#page-4-30). Applying ([26](#page-3-1)) to the charges [\(18](#page-2-3)) describing the shifted-boost orbifold (in the limit of large charges), one finds

$$
S_{\text{GCFT}} = \frac{\pi|J|}{\sqrt{2GM}} = S_{\text{FBTZ}}.\tag{27}
$$

Thus, the 2D GCFT state counting exactly reproduces the entropy of the cosmological horizon.

Notice that 2D GCFT thermodynamic potentials equal

$$
\frac{\partial S}{\partial h_L} = \frac{\Omega^-}{T_H^-}, \qquad \frac{1}{T_{GCT}} = \frac{\partial S}{\partial h_M} = -\frac{1}{GT_H^-}.
$$
 (28)

Thus, a universal feature of 2D GCFTs is the negativity of their thermodynamic temperature and specific heat

$$
C_M = \frac{\partial h_M}{\partial T_{\text{GCFT}}} \bigg|_{h_L} = -\frac{\pi^2}{G} \frac{T_H^-}{(\Omega^-)^2}.
$$
 (29)

Our Cardy formula (26) is compatible with the bulk first law [\(11](#page-1-4)), capturing its peculiar sign through the negative temperature, since $h_M \ge 0$ in 2D GCFTs. We view this as a very interesting feature of these theories, given the negative specific heat that higher dimensional asymptotically flat black holes have, a feature that has always been difficult to reconcile with a dual field theory formulation.

Cosmological interpretation.—The shifted-boost orbi-fold ([9\)](#page-1-3) is only invariant under the translation ∂_Y and the boost $X\partial_T + T\partial_X$. Its asymptotic structure at null infinity still satisfies the $BMS₃$ boundary conditions, in the same way as BTZ black holes do preserve $AdS₃$ asymptotics. Thus, an observer at null infinity can still assign nontrivial 2D GCA charges M_0 and L_0 to this geometry and use the 2D GCFT partition function to count the number of states carrying these charges.

A cosmological observer, who sees some contraction and expansion of the Universe, will measure some temperature due to particle creation by the cosmological horizon. The latter is measured with respect to his cosmological clock. To discuss how this description is related to the one in 2D GCFT, consider a bulk scalar field excitation Φ with Fourier decomposition $\Phi =$ $G_{pn}(\tau)e^{i(py+nz/r_0)}$. p corresponds to the boost quantum
number whereas $p \in \mathbb{Z}$ is momentum along the orbifold number, whereas $n \in \mathbb{Z}$ is momentum along the orbifold direction z. It was shown in Ref. $[42]$ $[42]$ $[42]$ that excitations satisfying Dirichlet boundary conditions at the singularity have an asymptotic behavior at $|\tau| \to \pm \infty$

$$
\Phi \to (1/\sqrt{\omega|\tau|})e^{i(\pm\omega|\tau|+py)}e^{inz/r_0},\tag{30}
$$

where for large charges (or massless fields) ω^2 = $(p - n/r_0)^2$. Using [\(12\)](#page-2-4) and [\(13\)](#page-2-5), one can relate this cosmological description to the BMS one as cosmological description to the BMS one as

$$
\Phi \rightarrow (\sqrt{\hat{r}_{+}/\omega r})e^{i\omega r/\hat{r}_{+}}e^{i[\hat{r}_{+}t(p-n/r_{0})+n\phi]}.
$$
 (31)

Thus, frequencies ω measured by the cosmological observer agree with those measured at infinity $[\hat{r}_+(p - n/r_0)]$, up to the rescaling \hat{r} , which also ensures the matching of the the rescaling \hat{r}_{+} , which also ensures the matching of the Hawking temperature due to particle creation radiation with the surface gravity temperature T_{FBTZ} [[42\]](#page-4-31).

Discussions.—In the present Letter, we have advocated that flat space holography in three bulk dimensions can be understood as an appropriate limit of AdS_3/CFT_2 . We have provided nontrivial evidence for this by showing that a counting of states in the dual GCFT leads to a Cardy-like formula which exactly reproduces the Bekenstein-Hawking entropy of the bulk cosmological horizon. The dual field theory also ''knows'' that this cosmological horizon is a remnant of the BTZ inner horizon and has a peculiar first law. This is reflected in the sign of the GCFT temperature and specific heat.

We had earlier motivated our Letter by stressing how flat space holography would facilitate the study of real astrophysical black holes. Let us explain how our present analysis would be useful for this. Real black holes live in 4D, and hence the obvious way to understand them is to generalize our construction to higher dimensions and use the BMS/GCA correspondence. But, there is actually another and possibly computationally simpler way out. Entropy counting for a special class of black holes, called extremal black holes, is successful today by looking at the symmetries near the horizon, realizing that there is always an AdS₂ factor [\[43\]](#page-4-32) and using AdS_2/CFT_1 techniques [[44\]](#page-4-33). The more general class of nonextremal black holes, including the Schwarzschild and the Kerr black holes, cannot be tackled by this method. Nonextremal black holes, however, contain a universal 2D Rindler spacetime in the ''nearhorizon" limit. In the present Letter, we have found a dual explanation of a horizon which can be looked upon as a 2D Rindler horizon in a certain patch. It is very likely that, by using techniques similar to the ones adopted for extremal black holes, we would be successful in describing nonextremal black holes by referring to the results of this Letter.

Before we close, let us comment on a couple of important points. The representations of M_0 are nondiagonal [\[33](#page-4-26)[,35,](#page-4-23)[40](#page-4-29)], a feature reminiscent of structures encountered in logarithmic CFTs. Thus, unitarity in 2D GCFTs deserves a more careful analysis. This off-diagonal nature is also relevant for entropy considerations. In fact, the density of states in ([20](#page-2-6)) is only a good approximation in the Cardy regime of large charges to the full partition function $Z_{GCFT} = Tre^{2\pi i \overline{\eta} L_0} e^{2\pi i \rho \overline{M}_0}$. Outside this regime, the off-diagonal corrections to the Cardy-like formula will the off-diagonal corrections to the Cardy-like formula will become important.

Our setup is also interesting because it allows us to address questions in cosmological backgrounds using a holographic description. There are open questions in this context. The shifted-boost orbifold has a classically stable Cauchy horizon [[45](#page-4-34)] when its singularity is interpreted as an orientifold in string theory [\[42\]](#page-4-31). It would be desirable to have a quantum version of this statement when including coupling to matter. It is also natural to investigate whether these solutions can be interpreted as thermal states in the 2D GCFT dual, for instance, by computing their quasinormal modes.

- [1] G. 't Hooft, in Salamfestschrift (World Scientific, Singapore, 1993).
- [2] L. Susskind, [J. Math. Phys. \(N.Y.\)](http://dx.doi.org/10.1063/1.531249) 36, 6377 (1995).
- [3] J. M. Maldacena, Adv. Theor. Math. Phys. 2, 231 (1998).
- [4] A. Strominger and C. Vafa, *[Phys. Lett. B](http://dx.doi.org/10.1016/0370-2693(96)00345-0)* 379, 99 (1996).
- [5] M. Bañados, C. Teitelboim, and J. Zanelli, *[Phys. Rev. Lett.](http://dx.doi.org/10.1103/PhysRevLett.69.1849)* 69[, 1849 \(1992\).](http://dx.doi.org/10.1103/PhysRevLett.69.1849)
- [6] M. Bañados, M. Henneaux, C. Teitelboim, and J. Zanelli, Phys. Rev. D 48[, 1506 \(1993\)](http://dx.doi.org/10.1103/PhysRevD.48.1506).
- [7] A. Strominger, [J. High Energy Phys. 02 \(1998\) 009.](http://dx.doi.org/10.1088/1126-6708/1998/02/009)
- [8] J. Polchinski, [arXiv:hep-th/9901076.](http://arXiv.org/abs/hep-th/9901076)
- [9] L. Susskind, [arXiv:hep-th/9901079.](http://arXiv.org/abs/hep-th/9901079)
- [10] S. B. Giddings, *Phys. Rev. D* **61**[, 106008 \(2000\).](http://dx.doi.org/10.1103/PhysRevD.61.106008)
- [11] M. Gary, S. B. Giddings, and J. Penedones, *[Phys. Rev. D](http://dx.doi.org/10.1103/PhysRevD.80.085005)* 80[, 085005 \(2009\)](http://dx.doi.org/10.1103/PhysRevD.80.085005).
- [12] G. Barnich and C. Troessaert, [J. High Energy Phys. 05](http://dx.doi.org/10.1007/JHEP05(2010)062) [\(2010\) 062.](http://dx.doi.org/10.1007/JHEP05(2010)062)
- [13] A. Bagchi, *Phys. Rev. Lett.* **105**[, 171601 \(2010\)](http://dx.doi.org/10.1103/PhysRevLett.105.171601).
- [14] A. Bagchi and R. Fareghbal, [arXiv:1203.5795.](http://arXiv.org/abs/1203.5795)
- [15] G. Barnich, A. Gomberoff, and H. A. Gonzalez, *[Phys. Rev.](http://dx.doi.org/10.1103/PhysRevD.86.024020)* D **86**[, 024020 \(2012\)](http://dx.doi.org/10.1103/PhysRevD.86.024020).
- [16] G. Barnich and G. Compere, [Classical Quantum Gravity](http://dx.doi.org/10.1088/0264-9381/24/5/F01) 24[, F15 \(2007\).](http://dx.doi.org/10.1088/0264-9381/24/5/F01)
- [17] H. Bondi, M. van der Burg, and A. Metzner, [Proc. R. Soc.](http://dx.doi.org/10.1098/rspa.1962.0161) ^A 269[, 21 \(1962\)](http://dx.doi.org/10.1098/rspa.1962.0161).
- [18] R. Sachs, *Phys. Rev.* **128**[, 2851 \(1962\)](http://dx.doi.org/10.1103/PhysRev.128.2851).
- [19] R. Sachs, [Proc. R. Soc. A](http://dx.doi.org/10.1098/rspa.1962.0206) **270**, 103 (1962).
- [20] A. Ashtekar, J. Bicak, and B. G. Schmidt, *[Phys. Rev. D](http://dx.doi.org/10.1103/PhysRevD.55.669)* 55, [669 \(1997\)](http://dx.doi.org/10.1103/PhysRevD.55.669).
- [21] A. Bagchi and R. Gopakumar, [J. High Energy Phys. 07](http://dx.doi.org/10.1088/1126-6708/2009/07/037) [\(2009\) 037.](http://dx.doi.org/10.1088/1126-6708/2009/07/037)
- [22] L. Cornalba and M.S. Costa, *[Phys. Rev. D](http://dx.doi.org/10.1103/PhysRevD.66.066001)* 66, 066001 [\(2002\)](http://dx.doi.org/10.1103/PhysRevD.66.066001).
- [23] M. Cvetic and F. Larsen, *Phys. Rev. D* 56[, 4994 \(1997\).](http://dx.doi.org/10.1103/PhysRevD.56.4994)
- [24] M.-I. Park, [Classical Quantum Gravity](http://dx.doi.org/10.1088/0264-9381/25/9/095013) 25, 095013 (2008).
- [25] M. Ansorg and J. Hennig, [Classical Quantum Gravity](http://dx.doi.org/10.1088/0264-9381/25/22/222001) 25, [222001 \(2008\).](http://dx.doi.org/10.1088/0264-9381/25/22/222001)
- [26] M. Ansorg and J. Hennig, *[Phys. Rev. Lett.](http://dx.doi.org/10.1103/PhysRevLett.102.221102)* **102**, 221102 [\(2009\)](http://dx.doi.org/10.1103/PhysRevLett.102.221102).
- [27] M. Cvetic, G. W. Gibbons, and C. N. Pope, [Phys. Rev.](http://dx.doi.org/10.1103/PhysRevLett.106.121301) Lett. **106**[, 121301 \(2011\).](http://dx.doi.org/10.1103/PhysRevLett.106.121301)
- [28] A. Castro and M.J. Rodriguez, *[Phys. Rev. D](http://dx.doi.org/10.1103/PhysRevD.86.024008)* **86**, 024008 [\(2012\)](http://dx.doi.org/10.1103/PhysRevD.86.024008).
- [29] S. Detournay, *Phys. Rev. Lett.* **109**[, 031101 \(2012\).](http://dx.doi.org/10.1103/PhysRevLett.109.031101)
- [30] B. Chen, S.-x. Liu, and J.-j. Zhang, [arXiv:1206.2015.](http://arXiv.org/abs/1206.2015)
- [31] J.D. Brown and M. Henneaux, [Commun. Math. Phys.](http://dx.doi.org/10.1007/BF01211590) 104, [207 \(1986\)](http://dx.doi.org/10.1007/BF01211590).
- [32] One can look at an inequivalent contraction of the two Virasoros $L_n = \mathcal{L}_n + \bar{\mathcal{L}}_n^{\bullet}$, $M_n = \epsilon(\mathcal{L}_n - \bar{\mathcal{L}}_n)$ [[33](#page-4-26)], which can be understood as a nonrelativistic $(x, t) \rightarrow (\epsilon x, t)$
contraction in terms of spacetime. This vields the same contraction in terms of spacetime. This yields the same algebra [\(2](#page-1-1)). We will continue calling (2) $GCA₂$ in the dual theory to be consistent with previous literature.
- [33] A. Bagchi, R. Gopakumar, I. Mandal, and A. Miwa, [J. High Energy Phys. 08 \(2010\) 004.](http://dx.doi.org/10.1007/JHEP08(2010)004)
- [34] S. Deser, R. Jackiw, and G. 't Hooft, [Ann. Phys. \(N.Y.\)](http://dx.doi.org/10.1016/0003-4916(84)90085-X) 152[, 220 \(1984\).](http://dx.doi.org/10.1016/0003-4916(84)90085-X)
- [35] A. Bagchi, S. Detournay, and D. Grumiller, *[Phys. Rev.](http://dx.doi.org/10.1103/PhysRevLett.109.151301)* Lett. 109[, 151301 \(2012\).](http://dx.doi.org/10.1103/PhysRevLett.109.151301)
- [36] J. Khoury, B. A. Ovrut, N. Seiberg, P. J. Steinhardt, and N. Turok, Phys. Rev. D 65[, 086007 \(2002\).](http://dx.doi.org/10.1103/PhysRevD.65.086007)
- [37] L. Cornalba and M. S. Costa, [Fortschr. Phys.](http://dx.doi.org/10.1002/prop.200310123) 52, 145 [\(2004\)](http://dx.doi.org/10.1002/prop.200310123).
- [38] A. Bagchi and I. Mandal, *[Phys. Lett. B](http://dx.doi.org/10.1016/j.physletb.2009.04.030)* **675**, 393 (2009).
- [39] S. Carlip, [Classical Quantum Gravity](http://dx.doi.org/10.1088/0264-9381/15/11/020) 15, 3609 (1998).
- [40] K. Hotta, T. Kubota, and T. Nishinaka, [Nucl. Phys.](http://dx.doi.org/10.1016/j.nuclphysb.2010.05.015) **B838**, [358 \(2010\)](http://dx.doi.org/10.1016/j.nuclphysb.2010.05.015).
- [41] A similar Cardy-like $GCA₂$ formula was derived in the context of nonrelativistic AdS/CFT in Ref. [[40](#page-4-29)]. Our derivation relies on a different contraction of the original CFT and is hence different. To arrive at the results of Ref. [\[40\]](#page-4-29), one needs to use the scaling $\eta \rightarrow \epsilon \eta$ in the S
transformation and follow the steps outlined above transformation and follow the steps outlined above.
- [42] L. Cornalba, M. Costa, and C. Kounnas, [Nucl. Phys.](http://dx.doi.org/10.1016/S0550-3213(02)00446-7) **B637**, [378 \(2002\)](http://dx.doi.org/10.1016/S0550-3213(02)00446-7).
- [43] M. Guica, T. Hartman, W. Song, and A. Strominger, *[Phys.](http://dx.doi.org/10.1103/PhysRevD.80.124008)* Rev. D 80[, 124008 \(2009\)](http://dx.doi.org/10.1103/PhysRevD.80.124008).
- [44] A. Sen, [Int. J. Mod. Phys. A](http://dx.doi.org/10.1142/S0217751X09045893) 24, 4225 (2009).
- [45] L. Cornalba and M. S. Costa, [Classical Quantum Gravity](http://dx.doi.org/10.1088/0264-9381/20/18/309) 20[, 3969 \(2003\).](http://dx.doi.org/10.1088/0264-9381/20/18/309)