Avalanches and Dimensional Reduction Breakdown in the Critical Behavior of Disordered Systems

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We investigate the connection between a formal property of the critical behavior of several disordered systems, known as "dimensional reduction," and the presence in these systems at zero temperature of collective events known as "avalanches." Avalanches generically produce nonanalyticities in the functional dependence of the cumulants of the renormalized disorder. We show that this leads to a breakdown of the dimensional reduction predictions if and only if the fractal dimension characterizing the scaling properties of the avalanches is exactly equal to the difference between the dimension of space and the scaling dimension of the primary field. This is proven by combining scaling theory and the functional renormalization group. We therefore clarify the puzzle of why dimensional reduction remains valid in random field systems above a nontrivial dimension (but fails below), always applies to the statistics of branched polymer, and is always wrong in elastic models of interfaces in a random environment.

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In the theory of disordered systems, "dimensional reduction" (DR) is the property shared by several models that the long-distance physics in the presence of quenched disorder in some spatial dimension d is the same as that of the pure model with no disorder in a reduced spatial dimension $d-2$. In the known examples where it has been found through perturbation theory, i.e., the random field Ising model (RFIM) [1], elastic manifolds in a random environment [2] (abbreviated below as RM), the random field (RF) and random anisotropy (RA) $O(N)$ models [1,3], and the statistics of dilute branched polymers (BP) [4], it entails two conditions: (1) that the long-distance physics is controlled by a zero-temperature fixed point, so that it can be equally described from the solution (s) of a stochastic field equation at zero temperature ($T=0$), and (2) that an underlying supersymmetry emerges in the field-theoretical treatment of the stochastic equation [5–7]. DR, however, is known to be wrong in the RF and RA models in low enough dimension (a rigorous proof [8] exists for the RFIM in $d=2$ and 3) and the RM model [2]. On the other hand, it is proven to be right for the BP case in all dimensions below the upper critical one [9].

We have recently shown that the breakdown of DR and the spontaneous breaking of the underlying supersymmetry take place below a nontrivial critical dimension [10] in the RF $O(N)$ model: This dimension is close to 5 for the Ising ($N=1$) version and decreases continuously as N increases until it reaches 4 when N approaches 18 (the upper critical dimension is equal to 6 for random field systems) [12–14]. Describing this phenomenon requires a renormalization group (RG) approach that is *functional*, as the origin of the DR breakdown is the appearance of a nonanalytic dependence of the renormalized cumulants of the RF (a linear "cusp") in the dimensionless fields, and *nonperturbative*, as it takes place away from regimes where some form of perturbation analysis is possible (except for the $O(N)$ model when d is close to the lower critical dimension [12,14]). A similar conclusion was previously reached for the RM case, but there the DR predictions fail for all dimensions at and below the upper critical dimension and can be already assessed through a functional but perturbative RG [15–17].

The existence of a cusp in the cumulants of the renormalized disorder can be assigned to the presence of collective events known as "avalanches." In any typical sample of a disordered model, the ground state, which is the relevant configuration that describes the equilibrium properties of the system at $T=0$, abruptly changes for specific (sample-dependent) values of the external source; such a change is precisely an avalanche [18–24]. The same phenomenon is observed, still at $T=0$, when the system is driven by the external source without being allowed to equilibrate. The corresponding avalanches then take place out of equilibrium, between two metastable states of the system [23,25–28]. The fact that abrupt changes corresponding to discontinuous variations of the magnetization (in the language of magnetic systems) are found at $T=0$ should actually come as no surprise. In disordered systems, this can take place even in zero-dimensional models [14]. The central question, which we address in this Letter, is then the following: Under which conditions can avalanches influence the long-distance properties of a given disordered system and lead to a breakdown of DR?

Consider a disordered system of linear size L at $T=0$ in which avalanches are present. We use the language of magnetic systems and characterize configurations by the local magnetization. All considerations, however, apply equally well to configurations described by a continuous field, in or out of equilibrium, and to nonmagnetic systems. We also focus on situations in which the local order parameter (the "magnetization") is linearly coupled to the external source J, which for simplicity is taken as uniform in space. The avalanches can then be characterized by their size S (the overall change in the *total* magnetization) whose distribution is described by a density $\rho_L(S, J)$.

The magnetization $m_L(J; \mathbf{h})$ is the spatial average of the local order-parameter field for a given sample characterized by the disorder realization $\mathbf{h}$. Its change between two values of the external source J_1 and J_2 (with, say, $J_2 > J_1$) is the sum of two contributions: A first one comes from the smooth changes in the configuration, and another one comes from the avalanches that take place between J_1 and J_2. As a consequence of the latter, the even moments of the difference $[m_L(J_1; \mathbf{h}) - m_L(J_2; \mathbf{h})]$, which are symmetric in the exchange of J_1 and J_2, display a linear cusp when $J_2 \to J_1$:

$$\overline{[m_L(J_2; \mathbf{h}) - m_L(J_1; \mathbf{h})]^{2p}} = |J_2 - J_1| L^{-2pd} \int_{S_{\min}}^{\infty} dS S^{2p} \rho_L(S, J) + O([J_2 - J_1]^2), \quad (1)$$

where $S_{\min}$ is a microscopic lower cutoff and the overline denotes the average over the quenched disorder.

It is easily realized that the nth moment is obtained by considering disorder averages over n copies of the *same* sample, with each copy coupled to a distinct external source J_a, $a = 1, \ldots, n$. For instance, the second moment is expressible in terms of a two-point Green's function at zero momentum $\tilde{G}_L(q = 0; J_1, J_2) = L^d[\overline{m_L(J_1; \mathbf{h}) m_L(J_2; \mathbf{h})} - \overline{m_L(J_1; \mathbf{h})}\, \overline{m_L(J_2; \mathbf{h})}]$, which is an extension to generic sources $J_1 \neq J_2$ of what is usually called the "disconnected" two-point function in the theory of disordered systems. One-particle irreducible (1PI) correlation functions (or proper vertices) [29] associated with the above Green's functions can be introduced along the same lines. From Eq. (1), with $p = 1$, one immediately derives that, for instance, the two-point Green's function $\tilde{G}_L(q = 0; J_1 = J - \delta J, J_2 = J + \delta J)$ has a nonanalytic dependence as $\delta J \to 0$, with the amplitude of the linear cusp related to the second moment of the avalanches. This can be transposed to the associated 1PI vertices and can be generalized to higher orders as well.

Avalanches, therefore, always induce a linear cusp in the functional dependence of the correlation functions associated with the cumulants of the renormalized disorder at $T = 0$. However, we are interested in situations where avalanches occur on all scales, as found for instance in the RFIM at criticality, in the rough phase or at the depinning transition of a RM, etc.

At large scale, when the correlation length and the extent of the largest typical avalanches have reached the system size L, one expects that the avalanche size distribution can be written in a scaling form [23,24,28,30]:

$$\rho_L(S, J) = \rho_{0,L}(J) S^{-\tau} \mathcal{D}\left(\frac{S}{S_L}, |J - J_c| S^{\psi}\right), \quad (2)$$

where $S_L \sim L^{d_f}$ is the size of the largest typical "critical" avalanches (see Ref. [28] for a careful discussion) which acts as a cutoff for $\mathcal{D}$ that decays exponentially for $S/S_L \gtrsim 1$. The critical conditions correspond to $J = J_c$ (for the RFIM at equilibrium, one has $J_c = 0$ due to the Z_2 symmetry, and, for the RM, there is no condition on J, as the whole phase is critical), and $\rho_{0,L}(J)$ is an overall factor such that $\rho_L(S)/\rho_{0,L}$ is normalized. This factor can be evaluated by considering the so-called "connected" susceptibility $\chi_{c,L}(J)$, which is the standard magnetic susceptibility divided by the temperature and which is obtained by deriving m_L with respect to J. $\chi_{c,L}(J)$ can be expressed as

$$\chi_{c,L}(J) = \chi_{c,L}^{\text{smooth}}(J) + \frac{1}{L^d} \int_{S_{\min}}^{\infty} dS S \rho_L(S, J). \quad (3)$$

For $J = J_c$, $\chi_{c,L}$ goes as $L^{2-\eta}$. Under the natural assumption that the contribution from the avalanches is at least of the order of the smooth one, by using Eqs. (2) and (3) and the fact that the first moment of the avalanches is dominated by large avalanches (which is true when $1 < \tau < 2$, a condition usually fulfilled), one then obtains that $\rho_{0,L}(J_c) L^{-d+(2-\tau)d_f} \sim L^{2-\eta}$. As a result, Eq. (1) leads to $\overline{[m_L(J_2; \mathbf{h}) - m_L(J_1; \mathbf{h})]^{2p}} \sim |J_2 - J_1| L^{2-\eta-(2p-1)(d-d_f)}$. The linear cusp in $\tilde{G}_L(q = 0; J - \delta J, J + \delta J)$ when $\delta J \to 0$ is then found as

$$\tilde{G}_L(q = 0; J_c - \delta J, J_c + \delta J) - \tilde{G}_L(q = 0; J_c, J_c) \sim |\delta J| L^{d_f + 2 - \eta}. \quad (4)$$

The amplitude of the cusp therefore diverges as $L \to \infty$. [One should, however, keep in mind that the whole function $\tilde{G}_L(q = 0)$ itself diverges as $L^{4-\bar{\eta}}$ at criticality.] From Eq. (4), it is easily derived that the associated 1PI correlation function $\Delta_L(q = 0; m_1, m_2)$, which is the second cumulant of the renormalized disorder, also has a cusp in $|m_2 - m_1|$ as $m_2 \to m_1$. Indeed, after introducing $m_1 = m_c - \delta m$ and $m_2 = m_c + \delta m$, where $\delta m \to 0$ and m_c corresponds to the value of the average magnetizationat criticality, and using the relation between Green's functions and 1PI functions [29] as well as $\delta m = \overline{m_L(J_c + \delta J; \mathbf{h})} - \overline{m_L(J_c - \delta J; \mathbf{h})} \simeq \delta J \chi_{c,L}(J_c)$ when $\delta J \to 0$, we derive

$$\Delta_L(q = 0; m_c - \delta m, m_c + \delta m) - \Delta_L(q = 0; m_c, m_c) \simeq \chi_{c,L}(J_c)^{-2}[\tilde{G}_L(q = 0; J_c - \delta J, J_c + \delta J) - \tilde{G}_L(q = 0; J_c, J_c)] \sim |\delta m| L^{d_f - 2(2-\eta)}. \quad (5)$$

As already stressed, the functional RG (FRG) is a powerful and necessary framework to describe the critical behavior of the disordered systems of interest. Within such an approach, which is a version of Wilson's continuous RG [31–34], the fluctuations are progressively taken into account by introducing an infrared cutoff that enforces the decoupling of the low- and high-momentum modes at a running scale k. Flow equations then describe the evolution as one decreases k, and for $k = 0$ all fluctuations are included. Contrary to the standard RG that considers only a few coupling constants, the FRG accounts for an infinity of couplings through the flow of full functions. Here, one ends up with flow equations for the moments of the renormalized disorder. For instance, an equation is obtained for the second cumulant of the renormalized random field or random force $\Delta_k(q = 0; \phi_1, \phi_2)$ [12–17], which is the quantity already considered in the previous sections, with ϕ and k playing here the same role as the local magnetization m and the inverse system size $1/L$.

In order to reach the fixed point that controls the long-distance behavior under study, one must introduce scaling dimensions and convert the quantities appearing in the RG flow equations from "dimensionful" to "dimensionless." For the cases of interest, we have stressed that the fixed point is at *zero temperature*. Temperature is then a dangerously irrelevant variable, and an associated exponent $\theta > 0$ is introduced through an appropriate definition of a renormalized temperature T_k [12–17,35]: $T_k \sim k^{\theta}$. Near the zero-temperature fixed point, the dimension d_ϕ of the field ϕ is modified from its standard value of $(d - 2 + \eta)/2$, with η the anomalous dimension, by a term involving the temperature exponent: $d_\phi = (d - 2 + \eta - \theta)/2 = (d - 4 + \bar{\eta})/2$, where we have also introduced the additional anomalous dimension $\bar{\eta}$ through the relation $\theta = 2 + \eta - \bar{\eta}$. Similarly, the second cumulant Δ_k has the scaling dimension of a two-point 1PI vertex $2 - \eta$, modified by the temperature exponent, i.e., $2 - \eta - \theta = -2\eta + \bar{\eta}$; it can be put in dimensionless form as $\Delta_k(q = 0; \phi_1, \phi_2) \sim k^{-(2\eta-\bar{\eta})}\delta_k(0; \varphi_1, \varphi_2)$, where φ is the dimensionless field.

DR corresponds to $\theta = 2$ ($\bar{\eta} = \eta$) and to all other exponents equal to their value in the system without disorder in dimension $d - 2$. The main outcome of the FRG studies is that DR breakdown is related to the presence of a cusp in the functional dependence of the dimensionless second cumulant of the renormalized random field or force $\delta_k(0; \varphi_1, \varphi_2)$ in the vicinity of the $T = 0$ fixed point [12–17]. More concretely, after introducing $\varphi = (\varphi_1 + \varphi_2)/2$ and $\delta\varphi = (\varphi_2 - \varphi_1)/2$, the "cuspy" behavior that changes the critical exponents from their DR prediction is of the form [13,14]

$$\delta_*(0; \varphi - \delta\varphi, \varphi + \delta\varphi) = \delta_{*,0}(\varphi) + \delta_{*,a}(\varphi)|\delta\varphi| + O(\delta\varphi^2), \tag{6}$$

when $\delta\varphi \to 0$, with $\delta_{*,a} < 0$; the star indicates the fixed-point value at $k = 0$. As a result of a nonzero $\delta_{*,a}$, the exponent θ takes a nontrivial d-dependent value <2 and η and $\bar{\eta}$ differ from the dimensional reduction values, with $\bar{\eta} \neq \eta$.

The connection between the quantities computed through the FRG and those previously discussed is made by associating the infrared cutoff k with $1/L$ [34]. Equation (5) can then be expressed in a dimensionless form by dividing the cumulant Δ_L and the magnetization δm by their scaling dimensions $L^{2\eta-\bar{\eta}}$ and $L^{-(d-4+\bar{\eta})/2}$, respectively. We immediately obtain that the amplitude of the linear cusp in dimensionless form scales as

$$L^{d_f-2(2-\eta)-(2\eta-\bar{\eta})-(d-4+\bar{\eta})/2} = L^{d_f-(d+4-\bar{\eta})/2}, \tag{7}$$

which can also be rewritten as $L^{d_f-(d-d_\phi)}$. By comparison with Eq. (6), one therefore finds that the cusp persists in the dimensionless quantities when $L \to \infty$, i.e., at the fixed point, *if and only if* $d_f = d - d_\phi$. If $d_f < d - d_\phi$, the cusp is only subdominant and does not affect the leading critical behavior and the associated exponents. (Note that the condition $d_f > d - d_\phi$ is not compatible with the result of the FRG studies, in which proper renormalized theories have always been found with no stronger nonanalyticities than the linear cusp.)

We conclude from the above derivation that DR breaks down *iff* $d_f = d - d_\phi$. On the other hand, DR remains valid if $d_f < d - d_\phi$, despite the presence of the avalanches and of a cusp in the dimensionful cumulants of the effective disorder at $T = 0$. In the latter case, the difference $(d - d_\phi - d_f)$ can be reinterpreted and computed in the FRG: When perturbing the "cuspless" fixed-point value of the dimensionless cumulant with a function that itself displays a linear cusp, the amplitude of the cuspy perturbation should go to zero as $k \to 0$ so that

$$\delta_k(0; \varphi - \delta\varphi, \varphi + \delta\varphi) \simeq \delta_*(0; \varphi - \delta\varphi, \varphi + \delta\varphi) + k^{\lambda} f_\lambda(\varphi, \delta\varphi), \tag{8}$$

with $\lambda = (d - d_\phi) - d_f > 0$ and $f_\lambda(\varphi, \delta\varphi) \simeq |\delta\varphi| f_\lambda(\varphi)$ when $\delta\varphi \to 0$. The fractal dimension d_f can then be obtained from an investigation of the irrelevant directions associated with nonanalytic eigenfunctions around the fixed point. This is what we have done by solving the nonperturbative FRG equations derived in Ref. [14] for a function δ_k of the form given in Eq. (8) (see the Supplemental Material [36]).

We are now in a position to discuss the consequences of the above conditions for several systems in which DR is predicted by standard perturbation theory. Consider first the mean-field limit. The exponents τ and d_f in Eq. (2) can easily be derived for fully connected models [30], and one finds $\tau = 3/2$ and $d_f = 4$ [37]. At the upper critical dimension $d_{\rm uc}$, the anomalous dimension $\bar{\eta} = 0$ so that $d_\phi = (d_{\rm uc} - 4)/2$. One should thus compare $d_f = 4$ and $d - d_\phi = d_{\rm uc}/2 + 2$. For RF and RA models (we include here models with N-component fields which we expect to behave in a similar manner as that of the single-component one), $d_{\rm uc} = 6$ so that DR should apply.

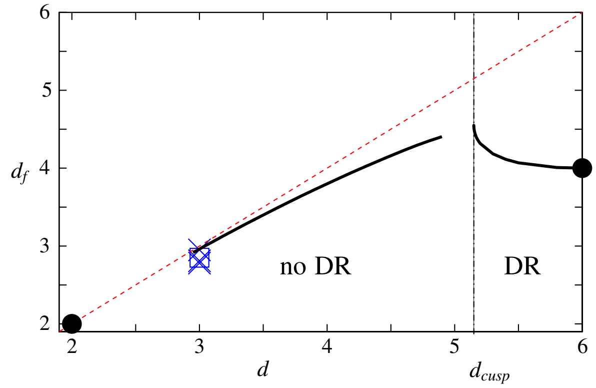

FIG. 1 (color online). Theoretical prediction for the avalanche fractal dimension d_f versus d for the equilibrium RFIM. Filled circles indicate the known values at $d_{\rm lc}$ and $d_{\rm uc}$; The crosses and square are the numerical estimates for the out-of-equilibrium [23,27,28] and equilibrium [23] behaviors in $d = 3$, respectively. The dashed line is the upper bound ($d_f \leq d$). The numerical resolution of the FRG flow equations becomes extremely difficult in low dimension, typically for $d \lesssim 2.9$, and when approaching $d_{\rm DR} \simeq d_{\rm cusp} \simeq 5.1$ from below, so that we have no results there.

The same is true for the BP statistics for which $d_{\rm uc} = 8$ [39]. On the other hand, for the RM case, $d_{\rm uc} = 4$: A failure of DR is then expected, possibly only in logarithmic corrections at $d = d_{\rm uc}$ but more severe as one lowers the dimension.

Below $d_{\rm uc}$, there must be a nonzero range of dimensions for which the DR predictions correctly describe the critical behavior of the RF, RA, and BP models but likely not that of the RM one. Actually, the latter has been studied in great detail through the perturbative FRG in $d = 4 - \epsilon$ [19,25,38]. It was found that d_f is equal to $d + \zeta$, where ζ is the exponent describing the roughness of the interface (for a single-component displacement field). As the dimension d_ϕ of the field is itself equal to $-\zeta$, it follows that the equality $d_f = d - d_\phi$ is always verified and that DR never applies, as indeed found by direct computation of the critical exponents within the FRG or in computer simulations. This conclusion is valid for the pinned phase, in equilibrium, and for the depinning threshold in the driven case.

For the RFIM at equilibrium, we have shown through a nonperturbative FRG that DR breaks down below a nontrivial dimension $d_{\rm cusp} \simeq 5.1$ [10,13,14]. According to the above conditions, the avalanche exponent d_f should then be equal to $d - d_\phi = (d + 4 - \bar{\eta})/2$ below $d_{\rm cusp}$ and to $(d + 4 - \bar{\eta})/2 - \lambda$, where λ is the eigenvalue associated with the irrelevant cuspy directions around the cuspless fixed point, above $d_{\rm cusp}$. In Fig. 1, we plot the theoretical prediction for d_f based on the above relations and on the computation of d_ϕ and λ from the solution of the nonperturbative FRG flow equations [14] (see the Supplemental Material [36]). At the lower critical dimension, $d_{\rm lc} = 2$, one expects the avalanches to be compact even at criticality, with therefore $d_f = d = 2$ (see also Ref. [27]). As $d_\phi = 0$, one then finds that $d_f = d - d_\phi$, as predicted.

Finally, for the BP statistics, so long as DR applies, $\bar{\eta} = 2\eta$ and is negative. In consequence, $d - d_\phi = d + 4 - \bar{\eta} > d + 4$. As the fractal dimension d_f should also be less than the dimension of the embedding space, one can see that $d_f \leq d < (d + 4)/2$ when $d \lesssim 4$. We therefore conclude that DR applies, at least, when $d \leq 4$ and in the vicinity of the upper critical dimension $d_{\rm uc} = 8$ (see above); the existence of an intermediate range of dimensions characterized by DR breakdown is highly unlikely. This is of course in agreement with the known exact results [9].

To summarize, we have related the breakdown of DR to the scaling characteristics of avalanches and clarified that the intriguing result of why DR fails below a nontrivial dimension for the RFIM is always broken for random elastic manifolds but applies to the branched-polymer problem. Already, in this latter example, the present results and formalism go beyond the realm of disordered systems. They may also be useful in other quite different contexts, such as turbulence, structural glasses, hysteresis in a variety of materials, socioeconomic phenomena, or non-Abelian gauge theories.

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the RFIM, the two are numerically almost indistinguishable, and, for the RF $O(N)$ model, one finds $N_{\rm DR} = 18$ and $N_{\rm cusp} = 18.393$ when approaching $d = 4$ [11].

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