Self-Consistent Multiple Complex-Kink Solutions in Bogoliubov–de Gennes and Chiral Gross-Neveu Systems

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We exhaust all exact self-consistent solutions of complex-valued fermionic condensates in the $(1 + 1)$-dimensional Bogoliubov–de Gennes and chiral Gross-Neveu systems under uniform boundary conditions. We obtain n complex (twisted) kinks, or gray solitons, with $2n$ parameters corresponding to their positions and phase shifts. Each soliton can be placed at an arbitrary position while the self-consistency requires its phase shift to be quantized by π/N for N flavors.

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Introduction.—The search for inhomogeneous self-consistent fermionic condensates including states such as the Fulde-Ferrell [1] and Larkin-Ovchinnikov [2] states having phase and amplitude modulations, respectively, in superconductors has attracted considerable attentions for more than half a century since theoretical predictions were made about their existence. While amplitude modulations are well studied in conducting polymers [3–7], the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state is theoretically shown to be a ground state of superconductors under a magnetic field [8]. Recently, the FFLO state has also been discussed in the context of an ultracold atomic Fermi gas [9,10]. In general, inhomogeneous self-consistent fermionic condensates with a gap function and fermionic excitations can be treated simultaneously using the Bogoliubov–de Gennes (BdG) and gap equations [11]. The gap functions are real and complex for conducting polymers [12] and superconductors, respectively. In the quantum field theory, these systems correspond to the Gross-Neveu (GN) model [13] and the Nambu–Jona-Lasinio (or chiral GN) model [14], which were proposed as models of dynamical chiral symmetry breaking in $1 + 1$ or $2 + 1$ dimensions. Therefore, BdG and (chiral) GN systems have been studied and developed together from the viewpoint of both condensed matter physics and high energy physics (see Ref. [15] for a review). For instance, fermion number fractionization is one of the topics that has been studied from this viewpoint [16,17]. Recently, it has been shown that the solutions in $1 + 1$ dimensions can be promoted to $3 + 1$ dimensions [18,19], thereby leading to extensive study of the modulated phases of these systems in terms of quantum chromodynamics (QCD) [20].

Inhomogeneous self-consistent solutions are often studied numerically because analytic solutions are generally difficult to obtain. However, several analytic solutions are available in the case of the real-valued condensates in $1 + 1$ dimensions, which describe the conducting polymers and the real GN model. Under uniform boundary conditions at spatial infinities, a real kink was constructed by Dashen *et al.* [21] by using the inverse scattering method, and later, it was reconstructed in polyacetylene [22] in the continuum limit of the lattice model [23]. Subsequently, a bound state of a kink and an antikink, which is called a polaron, was constructed in polyacetylene [24,25], for which achieving self-consistency in the system requires the distance between the kink and antikink to be fixed. Furthermore, three kinks (kink and polaron placed at arbitrary positions) [26,27] and more general solutions [28] were obtained. The attractive interaction between two polarons was also investigated [29]. For a periodic boundary condition, the existence of real kink crystals (the Larkin-Ovchinnikov state) has been known for a long time [3–7].

On the other hand, when compared with real condensates, only a few self-consistent solutions have thus far been obtained for complex condensates such as a complex (or twisted) kink or a gray soliton [30] and their crystals [31,32]. In these complex-valued crystals, both the amplitude and phase are modulated (the FFLO state), and this modulated phase has important applications in both superconductors and QCD, such as in the phase diagram of the chiral GN model [33]. An attempt to construct more general solutions was made [34,35] by using a technique of integrable systems known as the nonlinear Schrödinger or Ablowitz-Kaup-Newell-Segur hierarchy [36].

In this Letter, we exhaust all exact self-consistent solutions of complex condensates under uniform boundary conditions, and we find that they describe multiple twisted kinks. Unlike polarons in real condensates, where the distance between the kink and antikink is fixed, the situation is drastically simplified in our multiple twisted-kink solutions; we determine the filling rate of fermions for bound states of each kink, and we find that each kink can be placed at any position and has any phase shift quantized by π/N with the number of flavors N.

Fundamental equations.—The fundamental equations which we consider in this Letter appear in both condensed matter and high energy physics. In the condensed matter language, they are the one-dimensional BdG system with the Andreev approximation consisting of the BdG equation for right movers (BdG_R)

$$\begin{pmatrix} -i\partial_x & \Delta(x) \\ \Delta(x)^* & i\partial_x \end{pmatrix}\begin{pmatrix} u_R \\ v_R \end{pmatrix} = \epsilon\begin{pmatrix} u_R \\ v_R \end{pmatrix}, \tag{1}$$

the BdG equation for left movers (BdG_L)

$$\begin{pmatrix} i\partial_x & \Delta(x) \\ \Delta(x)^* & -i\partial_x \end{pmatrix}\begin{pmatrix} u_L \\ v_L \end{pmatrix} = \epsilon\begin{pmatrix} u_L \\ v_L \end{pmatrix}, \tag{2}$$

and the gap equation as a self-consistent condition

$$-\frac{\Delta(x)}{g} = \sum_{\text{occupied states}} (u_R v_R^* + u_L v_L^*). \tag{3}$$

For a derivation from the second quantized Hamiltonian, see, e.g., Ref. [8].

In high energy physics, this problem is equivalent to the chiral GN model with N flavors

$$\mathcal{L} = \bar{\psi} i\partial\!\!\!/ \psi + \frac{g^2}{2N}[(\bar{\psi}\psi)^2 + (\bar{\psi} i\gamma_5 \psi)^2], \tag{4}$$

with $\psi(x) = (\psi_1(x), \cdots, \psi_N(x))^T$ [13,14,21,30]. Introducing the auxiliary fields $\sigma(x)$ and $\pi(x)$, this can be rewritten as

$$\mathcal{L} = \bar{\psi} i\partial\!\!\!/ \psi - g\bar{\psi}(\sigma + i\pi\gamma_5)\psi - \frac{N}{2}(\sigma^2 + \pi^2). \tag{5}$$

Eliminating $\sigma(x)$ and $\pi(x)$ by their equations of motion, $\sigma = -(g/N)\bar{\psi}\psi$ and $\pi = -(g/N)\bar{\psi} i\gamma_5\psi$, takes us back to Eq. (4). Instead, we integrate out $\psi(x)$ to obtain $Z = \int \mathcal{D}\sigma\mathcal{D}\pi \exp(iS_{\mathrm{eff}})$ with

$$S_{\mathrm{eff}} = N\{-i\ln\mathrm{Det}[i\partial\!\!\!/ - g(\sigma + i\pi\gamma_5)] - \tfrac{1}{2}(\sigma^2 + \pi^2)\}. \tag{6}$$

Defining $\Delta(x) = \sigma(x) + i\pi(x)$, the gap equation is obtained in the large-N limit as the stationary condition for $\Delta^*(x)$

$$\Delta(x) = -4i\frac{\delta}{\delta\Delta^*(x)}\ln\mathrm{Det}[i\partial\!\!\!/ - g(\sigma + i\pi\gamma_5)]. \tag{7}$$

In the Hartree-Fock formalism, we consider $H_R\psi_R = \epsilon\psi_R$ and $H_L\psi_L = \epsilon\psi_L$ with single-particle Hamiltonians $H_R = -i\gamma_5\partial_x + \gamma_0(\sigma + i\pi\gamma_5)$ and $H_L = +i\gamma_5\partial_x + \gamma_0(\sigma + i\pi\gamma_5)$, reducing to the BdG equations Eqs. (1) and (2) with $\gamma_0 = \sigma_1$, $\gamma_1 = -i\sigma_2$ and $\gamma_5 = \sigma_3$, while the consistency condition $\Delta = -(g/N)(\langle\bar{\psi}\psi\rangle + i\langle\bar{\psi} i\gamma_5\psi\rangle)$ reduces to Eq. (3).

Result from the inverse scattering theory.—First, we briefly summarize the mathematical expressions of the n-soliton solution and its eigenstates of the self-defocusing Zakharov-Shabat eigenvalue problem [37]

$$\begin{pmatrix} -i\partial_x & \Delta(x) \\ \Delta(x)^* & i\partial_x \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = \epsilon\begin{pmatrix} u \\ v \end{pmatrix} \tag{8}$$

obtained by the inverse scattering method [38]. The detailed derivation is provided in the Supplemental Material [39].

Let us assume that the gap function obeys the following asymptotically uniform boundary condition:

$$|\Delta(x)| \to m(>0), \qquad x \to \pm\infty. \tag{9}$$

Subsequently, we parametrize the energy and wave number of quasiparticles using the uniformizing variable s defined by

$$\epsilon(s) = \frac{m}{2}(s + s^{-1}), \qquad k(s) = \frac{m}{2}(s - s^{-1}). \tag{10}$$

We can easily verify that the dispersion relation $\epsilon^2 = k^2 + m^2$ holds for an arbitrary complex number s. Eigenstates corresponding to s on the real axis are scattering states, while those on the unit circle are bound states. Since s and s^* on the unit circle correspond to the same bound state, it is sufficient to consider the unit circle in the upper half plane when we count the number of bound states (see Fig. 1).

Let us consider the gap function $\Delta(x)$ which has n bound states and acts as a reflectionless potential for scattering states, i.e., the n-soliton solution. By writing the s values of the bound states as $s_j = e^{i\theta_j}$ where $j = 1, \ldots, n$ with $0 < \theta_j < \pi$, the eigenenergy and the complex wave number can be rewritten as

$$\kappa_j := -ik(s_j) = m\sin\theta_j, \qquad \epsilon_j := \epsilon(s_j) = m\cos\theta_j, \tag{11}$$

respectively. According to the inverse scattering theory, $\theta_1, \ldots, \theta_n$ are all different from each other and there is no degeneracy. We further introduce the following notation:

$$e_j(x) = \sqrt{\kappa_j}e^{\kappa_j(x - x_j)} \quad (j = 1, \ldots, n). \tag{12}$$

Here, the real constant x_j represents the position of the jth soliton up to an additive constant when solitons are well separated from each other, as shown below. Furthermore, we define the functions $f_1(x), \ldots, f_n(x)$ as solutions of the following linear equation:

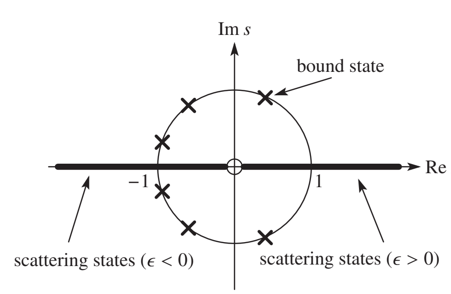

FIG. 1. Scattering and bound states in s plane. Scattering states with positive (negative) energy exist on the real and positive (negative) axis. Bound states exist on the unit circle, and s and s^* represent the same bound state.

$$f_j + e_j - \frac{2i}{m}\sum_{l=1}^{n}\frac{e_j e_l f_l}{s_j^{-1} - s_l} = 0 \qquad (j = 1, \ldots, n). \quad (13)$$

Here, the argument x is abbreviated.

By using the above notations, the n-soliton solution can be expressed as

$$\Delta(x) = m + 2i\sum_{j=1}^{n} s_j^{-1} e_j(x) f_j(x). \quad (14)$$

The complex-valued n-soliton solution has $2n$ parameters $s_1, \ldots, s_n$, $x_1, \ldots, x_n$, and this number of parameters is exactly twice that of the real-valued soliton solution. This $\Delta(x)$ has the following asymptotic form:

$$\Delta(x) \to \begin{cases} m & (x \to -\infty) \\ me^{-2i(\theta_1+\theta_2+\cdots+\theta_n)} & (x \to +\infty). \end{cases} \quad (15)$$

If the solitons are sufficiently separated from each other, the phase shift brought about by the jth soliton is $s_j^{-2} = e^{-2i\theta_j}$, and the position of the jth soliton X_j is given by

$$X_j = x_j + \frac{1}{\kappa_j}\sum_{l\,\mathrm{s.t.}\,x_l<x_j} \log\left|\frac{\sin\frac{\theta_l+\theta_j}{2}}{\sin\frac{\theta_l-\theta_j}{2}}\right|. \quad (16)$$

Figure 2 shows an example of the three-soliton solution.

The reduction to the real-valued soliton solution is obtained as follows. When the number of solitons is even ($n = 2n'$), the relations

$$s_{2j-1} = -s_{2j}^*, \qquad x_{2j-1} = x_{2j} \quad (j = 1, \ldots, n') \quad (17)$$

yield real-valued solutions. When the number of solitons is odd ($n = 2n' + 1$), we need to consider the term $s_{2n'+1} = e^{i\pi/2}$ in addition to Eq. (17) while $x_{2n'+1}$ remains arbitrary. By this reduction, we obtain $f_{2j-1}(x) = f_{2j}(x)^*$ and $f_{2n'+1}(x) = f_{2n'+1}(x)^*$, and the imaginary part of Eq. (14) vanishes.

The bound state with $s = s_j(\leftrightarrow \epsilon = m\cos\theta_j)$ is given by

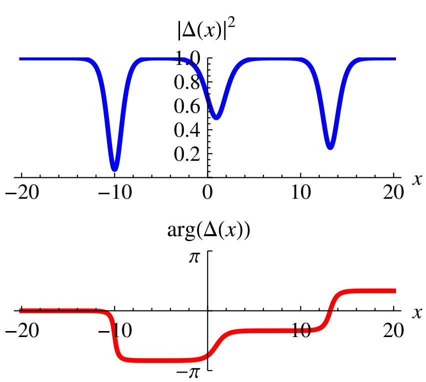

FIG. 2 (color online). Example of a three-kink solution. Here the parameters are $m = 1$, $s_1 = e^{(5/12)\pi i}$, $s_2 = e^{(2/3)\pi i}$, $s_3 = e^{(3/4)\pi i}$, $x_1 = -10$, $x_2 = 10$, and $x_3 = 0$. The positions of the solitons [Eq. (16)] are calculated as $X_1 = -10$, $X_2 = 13.18$, and $X_3 = 0.93$.

$$\begin{pmatrix} u_j(x) \\ v_j(x) \end{pmatrix} = \begin{pmatrix} f_j(x) \\ s_j f_j(x)^* \end{pmatrix} \qquad (j = 1, \ldots, n). \quad (18)$$

We can show that this state is already normalized, i.e., $\int dx(|u_j|^2 + |v_j|^2) = 1$ holds.

Finally, let s be real. Then, the scattering states are given by

$$\begin{pmatrix} u(x,s) \\ v(x,s) \end{pmatrix} = e^{ik(s)x}\left[\begin{pmatrix} 1 \\ s^{-1} \end{pmatrix} + \frac{2i}{m}\sum_{j=1}^{n}\frac{e_j(x)}{s_j - s}\begin{pmatrix} f_j(x) \\ s_j f_j(x)^* \end{pmatrix}\right], \quad (19)$$

which are obviously reflectionless as observed from the expression. The amplitudes of these solutions at $x = \pm\infty$ are

$$|u(\pm\infty, s)|^2 + |v(\pm\infty, s)|^2 = 1 + s^{-2}. \quad (20)$$

Occupation states and gap equation.—From this point onwards, we consider the occupation states of the BdG system with N internal degrees of freedom, or equivalently, the chiral GN model with N flavors. We first note that the following relation exists between the solutions of the right and left movers:

$$(\epsilon, u(x), v(x))\ \text{is a solution of}\ \mathrm{BdG}_R.$$
$$\leftrightarrow (-\epsilon^*, -v(x)^*, u(x)^*)\ \text{is a solution of}\ \mathrm{BdG}_L. \quad (21)$$

Thus, we can rewrite all quasiparticle wave functions of the left movers using those of the right movers. In the light of examining low-energy excited states of condensed matter systems, we consider the configurations in which all the negative-energy scattering states are filled by fermions and positive-energy states are completely vacant. As for bound states, we label the bound states of BdG_R as $(u_{j,R}, v_{j,R})$ where $j = 1, \ldots, n$, and we also label the corresponding bound states of BdG_L with the energy of the opposite sign related by Eq. (21) as $(u_{j,L}, v_{j,L})$. These states are assumed to be filled partially, and we write the occupation number as N_{jR} and N_{jL}, as schematically shown in Fig. 3. The gap equation subsequently becomes

$$\begin{aligned} -\frac{\Delta(x)}{g} &= \sum_{\substack{\mathrm{s.s.}\\ \epsilon<0}} N u_R v_R^* + \sum_{\substack{\mathrm{s.s.}\\ \epsilon<0}} N u_L v_L^* + \sum_{\mathrm{b.s.}} N_{jR} u_{j,R} v_{j,R}^* \\ &\quad + \sum_{\mathrm{b.s.}} N_{jL} u_{j,L} v_{j,L}^* \\ &= N\Big(\sum_{\substack{\mathrm{s.s.}\\ \epsilon<0}} u_R v_R^* - \sum_{\substack{\mathrm{s.s.}\\ \epsilon>0}} u_R v_R^*\Big) + \sum_{\mathrm{b.s.}} (N_{jR} - N_{jL}) u_{j,R} v_{j,R}^*. \end{aligned} \quad (22)$$

Here, the notation s.s. (b.s.) denotes the scattering (bound) states, and the relation of Eq. (21) is used to obtain the second equality in the above equation. Defining the filling rate by

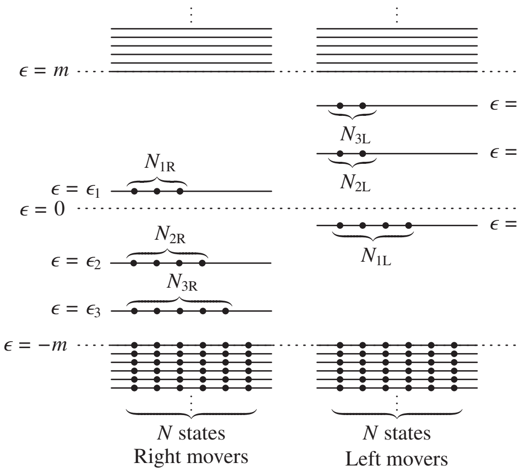

FIG. 3. Diagram of the occupation states considered in this Letter. In this example figure, the number of flavors is $N = 6$ and the number of solitons is $n = 3$. The filling rates defined by Eq. (23) are given by $\nu_1 = -1/6$, $\nu_2 = 1/3$, and $\nu_3 = 1/2$.

$$\nu_j := \frac{N_{jR} - N_{jL}}{N}, \qquad -1 \le \nu_j \le 1 \qquad (j = 1, \ldots, n), \tag{23}$$

the above equation can be rewritten as follows:

$$-\frac{\Delta(x)}{\tilde{g}} = \sum_{\substack{\text{s.s.}\\ \epsilon<0}} u_R v_R^* - \sum_{\substack{\text{s.s.}\\ \epsilon>0}} u_R v_R^* + \sum_{\text{b.s.}} \nu_j u_{j,R} v_{j,R}^* \tag{24}$$

with $\tilde{g} := Ng$. It is to be noted that the sum of positive-energy scattering states in Eq. (24) has a negative sign because of the Eq. (21) relation, and it is equivalent to the stationary condition of the action in the GN model, as given in Ref. [30]. Thus, we can again confirm the equivalence of the problems between the BdG and GN systems.

Henceforth, we always use the quantities of the BdG_R system, and we omit the subscript R. Considering the limit $L \to \infty$, where L denotes the system size, we replace the sum of the scattering states of the gap equation [Eq. (24)] by the corresponding integral. After renormalization of the coupling constant

$$\frac{1}{\tilde{g}} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{dk}{\sqrt{k^2 + m^2}}, \tag{25}$$

and subtracting the logarithmically divergent terms from both sides, we obtain the following expression:

$$0 = \sum_{\text{b.s.}} \nu_j u_j(x) v_j(x)^* + \sum_{\epsilon \gtrless 0} \int_{-\infty}^{\infty} \frac{dk}{2\pi} \left(\frac{\Delta(x)}{2\sqrt{k^2 + m^2}} - (\text{sgn}\epsilon) \frac{u_k(x) v_k(x)^*}{|u_{k,\infty}|^2 + |v_{k,\infty}|^2} \right), \tag{26}$$

where we have written the scattering states with the wave number k as $(u_k(x), v_k(x))$, and their amplitudes at infinity as $(u_{k,\infty}, v_{k,\infty})$. We note that $(u_k(x), v_k(x))$ for a positive energy and that for a negative energy are different from each other, though we use the same notation. It is convenient to rewrite the above integral in terms of the uniformizing variable s introduced in Eq. (10). Using the relation $\sqrt{k^2 + m^2} = \frac{m}{2}|s|(1 + s^{-2})$, we obtain

$$0 = \sum_{\text{b.s.}} \nu_j u_j(x) v_j(x)^* + \left[\int_{-\infty}^{0} - \int_{0}^{\infty} \right] \frac{ds}{2\pi} \times \left(\frac{m(1 + s^{-2}) u(x, s) v(x, s)^*}{2(|u(\infty, s)|^2 + |v(\infty, s)|^2)} - \frac{\Delta(x)}{2s} \right). \tag{27}$$

Here, we have written the scattering states labeled by s as $(u(x, s), v(x, s))$.

Self-consistent condition for the n-soliton solution.—We first present our main result in the following theorem, and then provide the proof.

Theorem:—Let $\Delta(x)$ be an n-soliton solution given by Eq. (14). The gap equation [Eq. (27)] holds if and only if the filling rate ν_j satisfies

$$\nu_j = \frac{2\theta_j - \pi}{\pi} \qquad (j = 1, \ldots, n). \tag{28}$$

Here, we remark on certain aspects of this theorem: (1) This theorem provides all self-consistent solutions under the uniform boundary condition [Eq. (9)], because $\Delta(x)$ needs to be a reflectionless potential in order for the gap equation to hold [21,28,30], and n-soliton solutions cover all reflectionless potentials with n bound states. (2) The filling rate ν_j for the jth bound state only depends on the phase shift of the jth soliton, and it is not affected by other soliton parameters. Thus, the self-consistent condition is decoupled for each bound state (or each soliton). (3) The parameter x_j, where $j = 1, \ldots, n$, which represents the position of the soliton up to an additive constant [Eq. (16)], is arbitrary and is not related to the self-consistency. This contrasts with the case of real-valued condensates. Because they must be real, the distance between two solitons must be fixed to a specific value, such as in the case of the polarons in polyacetylene [24–26] and the topologically trivial soliton in the GN model [27]. (4) For the N-flavor system, the possible values of the filling rate are given by $\nu = \frac{N-1}{N}, \frac{N-2}{N}, \ldots, -\frac{N-1}{N}$. Correspondingly, the possible phase shift of each soliton is also discretized. For example, only the trivial value $\nu = 0$ is allowed for $N = 1$, which corresponds to the real kink $2\theta = \pi$. When $N = 2$, the values $\nu = -\frac{1}{2}, 0, \frac{1}{2}$ are allowed, which correspond to $\theta = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$. The cases $N = 1$ and 2 correspond to s-wave superconductors and polyacetylene, respectively. The cases of N are also obtained as a dimensional reduction of nonrelativistic field theories in 3 + 1 dimensions, for which N is the number of patches of the Fermi surface [19]. On the other hand, any soliton solution can be self-consistent when $N = \infty$.

Proof.—Upon substituting Eqs. (14), (19), and (20), into the integrand of the gap equation [Eq. (27)], the terms $1 + s^{-2}$ and $|u(\pm\infty, s)|^2 + |v(\pm\infty, s)|^2$ in the first term in the bracket undergo cancellation, thereby yielding

$$\frac{m}{2}u(x,s)v(x,s)^* - \frac{\Delta(x)}{2s}$$
$$= 2\sum_j s_j^{-1} e_j f_j \frac{\sin\theta_j}{|s-s_j|^2} - \frac{4i}{m}\sum_{j,l}\frac{e_j f_j e_l f_l}{1-s_j s_l}\frac{\sin\theta_j}{|s-s_j|^2}$$
$$= -2\sum_j s_j^{-1} f_j^2 \frac{\sin\theta_j}{|s-s_j|^2}. \quad (29)$$

Here, Eq. (13) is used to obtain the last line. Using the formula $\int |s-s_j|^{-2} ds = (\sin\theta_j)^{-1}\tan^{-1}[(s-\cos\theta_j)/(\sin\theta_j)]$, we can perform the integration and obtain

$$\left[\int_{-\infty}^{0} - \int_{0}^{\infty}\right]\frac{ds}{2\pi}\left(\frac{m}{2}u(x,s)v(x,s)^* - \frac{\Delta(x)}{2s}\right)$$
$$= -\sum_j s_j^{-1} f_j^2 \frac{2\theta_j - \pi}{\pi}. \quad (30)$$

Recalling that the bound states are given by Eq. (18), we finally obtain

$$[\text{rhs of Eq. (27)}] = \sum_j s_j^{-1} f_j^2\left(v_j - \frac{2\theta_j - \pi}{\pi}\right). \quad (31)$$

Since the functions $f_1(x)^2, \ldots, f_n(x)^2$ are linearly independent of each other, the theorem holds. ■

Summary.—In summary, we have constructed all the exact self-consistent solutions of complex condensates under uniform boundary conditions. Our multiple n-twisted kink solution contains $2n$ parameters, and each kink has one bound state. Each kink can be placed at any position, while the self-consistency of the system requires the phase shift of each kink to be quantized by π/N with the number of flavors N. Our solution describes multiple gray solitons in ultracold atomic fermion gases, and our predictions require experimental verification. The dynamics and scattering of these solitons should be studied as a future research topic. Further research also needs to be conducted on the construction of self-consistent solutions under nonuniform boundary conditions.

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