Restriction on the Energy and Luminosity of e^+e^- Storage Rings due to Beamstrahlung

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The role of beamstrahlung in high-energy e^+e^- storage-ring colliders (SRCs) is examined. Particle loss due to the emission of single energetic beamstrahlung photons is shown to impose a fundamental limit on SRC luminosities at energies $2E_0 \gtrsim 140$ GeV for head-on collisions and $2E_0 \gtrsim 40$ GeV for crab-waist collisions. With beamstrahlung taken into account, we explore the viability of SRCs in the $2E_0 = 240$–500 GeV range, which is of interest in the precision study of the Higgs boson. At $2E_0 =$ 240 GeV, SRCs are found to be competitive with linear colliders; however, at $2E_0 = 400$–500 GeV, the attainable SRC luminosity would be a factor 15–25 smaller than desired.

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The ATLAS and CMS experiments at the LHC recently reported [1,2] an excess of events at $M = 125\ \mathrm{GeV/c^2}$, which may be evidence for the long-sought Higgs boson. The precision study of the Higgs boson's properties would require the construction of an energy- and luminosity-frontier e^+e^- collider [3].

The $2E_0 = 209$ GeV large electron positron (LEP) collider at CERN is generally considered to have been the last energy-frontier e^+e^- storage-ring collider (SRC) to be built, due to synchrotron-radiation energy losses, which are proportional to E_0^4/R. Linear e^+e^- colliders (LCs) are free from this limitation and allow multi-TeV energies to be reached. Two LC projects are in advanced stages of development: the $2E_0 = 500$ GeV ILC [4] and the $2E_0 = 500$–3000 GeV CLIC [5].

Nevertheless, several proposals [6,7] for a $2E_0 =$ 240 GeV SRC for the study of the Higgs boson in $e^+e^- \to HZ$ have recently been put forward [8]. Lower cost and reliance on firmly established technologies are cited as these projects' advantages over a LC. Moreover, it has been proposed that a $2E_0 = 240$ GeV SRC can provide superior luminosity, and that the "crab-waist" collision scheme [9,10] allows them to exceed the ILC and CLIC luminosities even at $2E_0 = 400$–500 GeV. Parameters of the recently proposed SRCs are summarized in Table I.

The present Letter examines the role of beamstrahlung, i.e., synchrotron radiation in the field of the opposing beam, in high-energy e^+e^- SRCs. First discussed in Ref. [11], beamstrahlung has been well-studied only in the LC case [12]. As we shall see, at energy-frontier e^+e^-, SRCs beamstrahlung determines the beam lifetime through the emission of single photons in the tail of the beamstrahlung spectra, thus severely limiting the luminosity.

At SRCs, the particles that lose a certain fraction of their energy in a beam collision leave the beam; this fraction η is typically around 0.01 (0.012 at LEP) and is known as the ring's energy acceptance. Beamstrahlung was negligible at all previously built SRCs. Its importance increases considerably with energy. Table I lists the beamstrahlung characteristics of the newly proposed SRCs assuming a 1% energy acceptance: the critical photon energy for the maximum beam field $E_{c,\max}$, the average number of beamstrahlung photons per electron per beam crossing n_γ, and the beamstrahlung-driven beam lifetime. Please note that once beamstrahlung is taken into account, the beam lifetime drops to unacceptable values, from a fraction of a second to as low as a few revolution periods.

At the SRCs considered in Table I, the beam lifetime due to the unavoidable radiative Bhabha scattering is 10 min or longer. One would therefore want the beam lifetime due to beamstrahlung to be at least 30 min. The simplest (but not optimum) way to suppress beamstrahlung is to decrease the number of particles per bunch with a simultaneous increase in the number of colliding bunches. As explained below, $E_{c,\max}$ should be reduced to $\approx 0.001E_0$. Thus, beamstrahlung causes a great drop in luminosity, especially at crab-waist SRCs: compare the proposed $\mathcal{L}$ and corrected (as suggested above) $\mathcal{L}_{\mathrm{corr}}$ rows in Table I.

To achieve a reasonable beam lifetime, one must make small the number of beamstrahlung photons with energies greater than the threshold energy $E_{\mathrm{th}} = \eta E_0$ that causes the electron to leave the beam. These photons belong to the high-energy tail of the beamstrahlung spectrum and have energies much greater than the critical energy. It will be shown below that the beam lifetime is determined by such single high-energy beamstrahlung photons, not by the energy spread due to the emission of multiple low-energy photons.

The critical energy for synchrotron radiation [13] is

$$E_c = \hbar\omega_{\mathrm{c}} = \hbar\frac{3\gamma^3 c}{2\rho}, \tag{1}$$

where ρ is the bending radius and $\gamma = E_0/mc^2$. The spectrum of photons per unit length with energy well above the critical energy [13] is

TABLE I. Parameters of LEP and several recently proposed storage-ring colliders [6,7]. "STR" refers to "SuperTRISTAN" [7]. Use of the crab-waist collision scheme [9,10] is denoted by "cr-w". The luminosities and the numbers of bunches for all projects are normalized to the total synchrotron-radiation power of 100 MW. Beamstrahlung-related quantities derived in this Letter are listed below the dividing horizontal line.

	LEP	LEP3	DLEP	STR1	STR2	STR3 cr-w	STR4 cr-w	STR5 cr-w	STR6 cr-w
$2E_0$, GeV	209	240	240	240	240	240	400	400	500
Circumference, km	27	27	53	40	60	40	40	60	80
Beam current, mA	4	7.2	14.4	14.5	23	14.7	1.5	2.7	1.55
Bunches per beam	4	3	60	20	49	15	1	1.4	2.2
N, 10^{11}	5.8	13.5	2.6	6	6	8.3	12.5	25	11.7
σ_z, mm	16	3	1.5	3	3	1.9	1.3	1.4	1.9
$\varepsilon_x/\varepsilon_y$, nm	48/0.25	20/0.15	5/0.05	23.3/0.09	24.6/0.09	3/0.011	2/0.011	3.2/0.017	3.4/0.013
β_x/β_y, mm	1500/50	150/1.2	200/2	80/2.5	80/2.5	26/0.25	20/0.2	30/0.32	34/0.26
σ_x/σ_y, μm	270/3.5	54/0.42	32/0.32	43/0.47	44/0.47	8.8/0.05	6.3/0.047	9.8/0.074	10.7/0.06
SR power, MW	22	100	100	100	100	100	100	100	100
Energy loss per turn, GeV	3.4	7	3.47	3.42	2.15	3.42	33.9	18.5	32.45
$\mathcal{L}$, 10^{34} cm^{-2} s^{-1}	0.013	1.3	1.6	1.7	2.7	17.6	4	7	2.2
$E_{c,\max}/E_0$, 10^{-3}	0.09	6.3	4.2	3.5	3.4	38	194	232	91
n_γ/electron	0.09	1.1	0.37	0.61	0.6	4.2	8.7	11.3	4.8
Lifetime (SR@IP), s [Eq. (4)]	$\sim\infty$	0.02	0.3	0.2	0.4	0.005	0.001	0.0005	0.005
$\mathcal{L}_{\rm corr}$, 10^{34} cm^{-2} s^{-1}	0.013	0.2	0.4	0.5	0.8	0.46	0.02	0.03	0.024

$$\frac{dn}{dx} = \sqrt{\frac{3\pi}{2}}\frac{\alpha\gamma}{2\pi\rho}\frac{e^{-u}}{\sqrt{u}}du, \tag{2}$$

where $u = E_\gamma/E_c$, $\alpha = e^2/\hbar c$. To evaluate the integral of this spectrum from the threshold energy ηE_0 to E_0, note that for the minimum value of $u \gg 1$, the exponent decreases rapidly, and so one can integrate only the exponent and use the minimum value of u outside the exponent. After the integration and substitution of ρ from Eq. (1), we obtain the number of photons emitted on the collision length l with energy $E_\gamma \geq \eta E_0$,

$$n_\gamma(E_\gamma \geq \eta E_0) \approx \frac{\alpha^2\eta l}{\sqrt{6\pi}r_e\gamma u^{3/2}}e^{-u}, \qquad u = \frac{\eta E_0}{E_c}, \tag{3}$$

where $r_e = e^2/mc^2$ is the classical radius of the electron.

The regions of the beam where the field strength is the greatest contribute the most to the emission of the highest-energy photons. We need to find the critical energy for this field and the bunch size that yields an acceptable rate of beamstrahlung particle loss. The collision length $l \approx \sigma_z/2$ for head-on and $\approx \beta_y/2$ for crab-waist collisions. In the transverse direction, we can assume that the electron crosses the region with the strongest field with a 10% probability. The average number of beam collisions $n_{\rm col}$ experienced by an electron before it leaves the beam can be estimated from $0.1 n_{\rm col} n_\gamma = 1$, where n_γ is given by Eq. (3). Thus, $n_{\rm col}$ and the beam lifetime due to beamstrahlung τ are

$$n_{\rm col} \approx 10\frac{\sqrt{6\pi}r_e\gamma u^{3/2}}{\alpha^2\eta l}e^u, \qquad \tau = n_{\rm col}\frac{2\pi R}{c}. \tag{4}$$

Assuming $E_0 = 150$ GeV, $l = 0.1$ cm, $\eta = 0.01$, and a ring circumference of 50 km, from Eqs. (3) and (4) we get

$$u = \eta E_0/E_c \approx 8.5, \qquad E_c \approx 0.12\eta E_0 \sim 0.1\eta E_0. \tag{5}$$

The accuracy of this expression is quite good for any SRC because it depends on the values in front of the exponent in Eq. (4) only logarithmically.

Let us express the critical energy E_c via the beam parameters. In beam collisions, the electrical and magnetic forces are equal in magnitude and act on the particles in the oncoming beam in the same direction. Thus, we can use the effective doubled magnetic field. The maximum effective field for flat Gaussian beams $B \approx 2eN/\sigma_x\sigma_z$. The bending radius $\rho = pc/eB \approx \gamma mc^2/eB = \gamma\sigma_x\sigma_z/2r_e$. Substituting to Eq. (1), we find

$$\frac{E_c}{E_0} = \frac{3\gamma r_e^2 N}{\alpha\sigma_x\sigma_z}. \tag{6}$$

Combined with Eq. (5), this imposes a restriction on the beam parameters,

$$\frac{N}{\sigma_x\sigma_z} < 0.1\eta\frac{\alpha}{3\gamma r_e^2}. \tag{7}$$

This formula is the basis for the following discussion.

For Gaussian beams, the average number of beamstrahlung photons per electron for head-on collisions [12] $\langle n_\gamma\rangle \approx 2.12N\alpha r_e/\sigma_x$, their average energy $\langle E_\gamma\rangle \approx 0.31\langle E_c\rangle$, and the average critical energy $\langle E_c\rangle \approx 0.42E_{c,\max}$; hence, $\langle E_\gamma\rangle \approx 0.13E_{c,\max}$. Above, we considered the maximum field; i.e., E_c was equal to $E_{c,\max}$. Then, for the condition in Eq. (7) we obtain

$$\langle n_\gamma \rangle = \frac{0.07\eta\alpha^2\sigma_z}{r_e\gamma} \approx \frac{0.067(\sigma_z/\mathrm{mm})}{(E_0/100\ \mathrm{GeV})}\left(\frac{\eta}{0.01}\right), \quad (8)$$

$$\langle E_\gamma \rangle \approx 0.13 \times 0.1\eta E_0 \approx 1.3 \times 10^{-2}\eta E_0. \quad (9)$$

For crab-waist collisions, $\langle E_\gamma \rangle$ is the same while the interaction length is shorter, β_y instead of σ_z; therefore, the number of photons is proportionally smaller.

So when τ is large enough, τ is determined by the rare photons with energies $\gtrsim 8.5E_{c,\max}$, a factor $8.5/0.13 = 65$ greater than $\langle E_\gamma \rangle$.

The beam energy spread due to beamstrahlung can be estimated as follows. In the general case [14],

$$\frac{\sigma_E^2}{E_0^2} = \frac{\tau_s}{4E_0^2}\dot{n}_\gamma\langle E_\gamma^2 \rangle. \quad (10)$$

In our case, the damping time (due to radiation in bending magnets) $\tau_s \approx T_{\mathrm{rev}}E_0/\Delta E_{\mathrm{rev}}$, $\dot{n}_\gamma = \langle n_\gamma \rangle/T_{\mathrm{rev}}$, and $\langle E_\gamma^2 \rangle \approx 4.3\langle E_\gamma \rangle^2$ [15], which gives

$$\frac{\sigma_E^2}{E_0^2} \approx \frac{\langle n_\gamma \rangle\langle E_\gamma \rangle^2}{E_0\Delta E_{\mathrm{rev}}} = \frac{1.15 \times 10^{-9}(\sigma_z/\mathrm{mm})}{(E_0/100\ \mathrm{GeV})(\Delta E_{\mathrm{rev}}/E_0)}\left(\frac{\eta}{0.01}\right)^3, \quad (11)$$

where ΔE_{rev} is the energy loss per revolution and $\langle n_\gamma \rangle$ and $\langle E_\gamma \rangle$ are given by Eqs. (8) and (9). Taking the typical bunch length $\sigma_z = 5$ mm, $E_0 = 120$ GeV, and $\Delta E_{\mathrm{rev}}/E_0 = 0.05$, we get an estimate for the energy spread due to beamstrahlung [under the condition in Eq. (7)] $\sigma_E/E_0 \approx 3 \times 10^{-4}(\eta/0.01)^{3/2}$.

The beam energy spread due to synchrotron radiation (SR) in the bending magnets [14] is

$$\left(\frac{\sigma_E^2}{E_0^2}\right)_{\mathrm{SR}} = \frac{55\sqrt{3}}{128\pi\alpha J_s}\frac{mc^2}{E_0}\frac{\Delta E_{\mathrm{rev}}}{E_0} = \frac{0.016}{J_s E_0(\mathrm{GeV})}\frac{\Delta E_{\mathrm{rev}}}{E_0}, \quad (12)$$

where $1 < J_s < 2$ is the partition number. For the projects in Table I, σ_E/E_0 due to SR varies between 0.17% and 0.24%. For $E_0 = 120$ GeV, $\Delta E_{\mathrm{rev}}/E_0 = 0.05$, and for $J_s = 1.5$ one gets $(\sigma_E/E_0)_{\mathrm{SR}} \approx 2 \times 10^{-3}$. For the given example, the beamstrahlung energy spread becomes larger than that due to SR in rings at $\eta > 0.035$.

The energy spread due to beamstrahlung contributes to the beam lifetime (if the lifetime is large enough) when the energy acceptance $\eta \lesssim 6(\sigma_E/E_0)$; with Ref. (11) taken into account, this yields $\eta > 2.5(\Delta E_{\mathrm{rev}}/10\ \mathrm{GeV})/(\sigma_z/\mathrm{mm})$. For the typical $\Delta E_{\mathrm{rev}} = 5$ GeV, $\sigma_z = 5$ mm, we get $\eta > 0.25$, which is much larger than the realistic storage-ring energy acceptance $\eta = 0.01$–0.03. Therefore, the beam energy spread due to beamstrahlung never causes the beam lifetime; the lifetime is always determined by the emission of single photons.

In the crab-waist collision scheme [9,10], the beams collide at an angle $\theta \gg \sigma_x/\sigma_z$. The crab-waist scheme allows for higher luminosity when it is restricted only by the tune shift, characterized by the beam-beam strength parameter. One should work at a beam-beam strength parameter smaller than some threshold value, ≈ 0.15 for high-energy SRCs [6].

In head-on collisions, the vertical beam-beam strength parameter (further "beam-beam parameter") [14] is

$$\xi_y = \frac{Nr_e\beta_y}{2\pi\gamma\sigma_x\sigma_y} \approx \frac{Nr_e\sigma_z}{2\pi\gamma\sigma_x\sigma_y} \quad \text{for } \beta_y \approx \sigma_z. \quad (13)$$

In the crab-waist scheme [9],

$$\xi_y = \frac{Nr_e\beta_y^2}{\pi\gamma\sigma_x\sigma_y\sigma_z} \quad \text{for } \beta_y \approx \sigma_x/\theta. \quad (14)$$

The luminosity in head-on collisions is represented by

$$\mathcal{L} \approx \frac{N^2 f}{4\pi\sigma_x\sigma_y} \approx \frac{Nf\gamma\xi_y}{2r_e\sigma_z} \quad (15)$$

and in crab-waist collisions by

$$\mathcal{L} \approx \frac{N^2 f}{2\pi\sigma_y\sigma_z\theta} \approx \frac{N^2\beta_y f}{2\pi\sigma_x\sigma_y\sigma_z} \approx \frac{Nf\gamma\xi_y}{2r_e\beta_y}. \quad (16)$$

In the crab-waist scheme, one can make $\beta_y \ll \sigma_z$, which enhances the luminosity by a factor of σ_z/β_y compared to head-on collisions. For example, at the proposed Italian SuperB factory [10] this enhancement would be a factor of 20–30.

Using Eqs. (13) and (14) and the restriction in Eq. (7), we find the minimum beam energy when beamstrahlung becomes important. For head-on collisions,

$$\gamma_{\min} = \left(\frac{0.1\eta\alpha\sigma_z^2}{6\pi r_e\xi_y\sigma_y}\right)^{1/2} \propto \frac{\sigma_z^{3/4}}{\xi_y^{1/2}\varepsilon_y^{1/4}}, \quad (17)$$

and for crab-waist collisions,

$$\gamma_{\min} = \left(\frac{0.1\eta\alpha\beta_y^2}{3\pi r_e\xi_y\sigma_y}\right)^{1/2} \propto \frac{\beta_y^{3/4}}{\xi_y^{1/2}\varepsilon_y^{1/4}}. \quad (18)$$

In the crab-waist scheme, beamstrahlung becomes important at much lower energies because $\beta_y \ll \sigma_z$. This can be understood from Eq. (14): a smaller β_y value corresponds to denser beams, leading to a higher beamstrahlung rate.

Here are some examples: (a) SuperB [10], crab waist, $E_0 = 7$ GeV, $\sigma_y = 20$ nm, $\beta_y = 0.2$ mm, $\xi_y = 0.16$. Then, $E_{\min} = 29$ GeV; i.e., beamstrahlung is not important. (b) The STR3 project (Table I), crab crossing, $E_0 = 120$ GeV, $\sigma_y = 50$ nm, $\beta_y = 0.25$ mm, $\xi_y \sim 0.2$. Then, $E_{\min} = 16.5$ GeV, a factor of 7 lower than E_0; thus, beamstrahlung is very important. (c) For projects STR1 and STR2, head-on, $E_0 = 120$ GeV, $\sigma_y = 500$ nm, $\sigma_z = 3$ mm, $\xi_y \sim 0.15$; $E_{\min} = 68$ GeV, and beamstrahlung is important.

We have shown that beamstrahlung restricts the maximum value of $N/\sigma_x\sigma_z$ and becomes important at energies

$E_0 \gtrsim 70$ GeV for e^+e^- storage rings with head-on collisions; when the crab-waist scheme is employed, this changes to the more strict $E_0 \gtrsim 20$ GeV. All newly proposed projects listed in Table I are affected as they have $E_0 \geq 120$ GeV.

Now, let us find the luminosity $\mathcal{L}$ when it is restricted by both beam-beam strength parameter and beamstrahlung. For head-on collisions,

$$\mathcal{L} \approx \frac{(Nf)N}{4\pi\sigma_x\sigma_y}, \qquad \xi_y \approx \frac{Nr_e\sigma_z}{2\pi\gamma\sigma_x\sigma_y}, \qquad \frac{N}{\sigma_x\sigma_z} \equiv k \approx 0.1\eta\frac{\alpha}{3\gamma r_e^2} \tag{19}$$

and $\sigma_y \approx \sqrt{\varepsilon_y\sigma_z}$. This can be rewritten as

$$\mathcal{L} \approx \frac{(Nf)k\sigma_z}{4\pi\sigma_y}, \quad \xi_y \approx \frac{kr_e\sigma_z^2}{2\pi\gamma\sigma_y}, \quad \sigma_y \approx \sqrt{\varepsilon_y\sigma_z}. \tag{20}$$

Thus, in the beamstrahlung-dominated regime the luminosity is proportional to the bunch length, and its maximum value is determined by the beam-beam strength parameter. Together, these equations give

$$\mathcal{L} \approx \frac{Nf}{4\pi}\left(\frac{0.1\eta\alpha}{3}\right)^{2/3}\left(\frac{2\pi\xi_y}{\gamma r_e^5\varepsilon_y}\right)^{1/3}, \tag{21}$$

$$\sigma_{z,\mathrm{opt}} = \varepsilon_y^{1/3}\left(\frac{6\pi\gamma^2 r_e\xi_y}{0.1\eta\alpha}\right)^{2/3}. \tag{22}$$

Similarly, for the crab-waist collision scheme,

$$\mathcal{L} \approx \frac{(Nf)N\beta_y}{2\pi\sigma_x\sigma_y\sigma_z}, \qquad \xi_y \approx \frac{Nr_e\beta_y^2}{\pi\gamma\sigma_x\sigma_y\sigma_z}, \qquad \frac{N}{\sigma_x\sigma_z} \equiv k \approx 0.1\eta\frac{\alpha}{3\gamma r_e^2}, \tag{23}$$

and $\sigma_y \approx \sqrt{\varepsilon_y\beta_y}$. Substituting, we obtain

$$\mathcal{L} \approx \frac{(Nf)k\beta_y}{2\pi\sigma_y}, \quad \frac{kr_e\beta_y^2}{\pi\gamma\sigma_y} \approx \xi_y, \quad \sigma_y \approx \sqrt{\varepsilon_y\beta_y}. \tag{24}$$

The corresponding solutions are

$$\mathcal{L} \approx \frac{Nf}{4\pi}\left(\frac{0.2\eta\alpha}{3}\right)^{2/3}\left(\frac{2\pi\xi_y}{\gamma r_e^5\varepsilon_y}\right)^{1/3}, \tag{25}$$

$$\beta_{y,\mathrm{opt}} = \varepsilon_y^{1/3}\left(\frac{3\pi\gamma^2 r_e\xi_y}{0.1\eta\alpha}\right)^{2/3}. \tag{26}$$

We have obtained a very important result: *in the beamstrahlung-dominated regime, the luminosities attainable in crab-waist and head-on collisions are practically the same.* The gain from using the crab-waist scheme is only a factor of $2^{2/3} \sim 1$, contrary to the low-energy case, where the gain may be greater than 1 order of magnitude. For this reason, from this point on we will consider only the case of head-on collisions.

From the above considerations, one can find the ratio of the luminosities with and without taking beamstrahlung into account: it is equal to $\sigma_z/\sigma_{z,\mathrm{opt}}$ for head-on collisions and $\beta_y/\beta_{y,\mathrm{opt}}$ for crab-waist collisions and scales as $1/E_0^{4/3}$ for $\gamma > \gamma_{\min}$. In practical units,

$$\frac{\sigma_{z,\mathrm{opt}}}{\mathrm{mm}} \approx \frac{2\xi_y^{2/3}}{\eta^{2/3}}\left(\frac{\varepsilon_y}{\mathrm{nm}}\right)^{1/3}\left(\frac{E_0}{100\ \mathrm{GeV}}\right)^{4/3}; \qquad \frac{\beta_{y,\mathrm{opt}}}{\sigma_{z,\mathrm{opt}}} \approx 0.63. \tag{27}$$

For example, for $\xi_y = 0.15$, $\eta = 0.01$, $E_0 = 100$ GeV, and the vertical emittances from Table I ($\varepsilon_y = 0.01$ to 0.15 nm), we get $\sigma_{z,\mathrm{opt}} = 2.5$ to 6.4 mm.

According to Eq. (21), the maximum luminosity at high-energy SRCs with beamstrahlung taken into account is

$$\mathcal{L} \approx h\frac{N^2 f}{4\pi\sigma_x\sigma_y} = h\frac{Nf}{4\pi}\left(\frac{0.1\eta\alpha}{3}\right)^{2/3}\left(\frac{2\pi\xi_y}{\gamma r_e^5\varepsilon_y}\right)^{1/3}, \tag{28}$$

where h is the hourglass loss factor, $f = n_\mathrm{b}c/2\pi R$ is the collision rate, R is the average ring radius, and n_b is the number of bunches in the beam.

The energy loss by one electron in a circular orbit of radius R_b [13] $\delta E = 4\pi e^2\gamma^4/3R_b$, so then the power radiated by the two beams in the ring

$$P = 2\delta E\frac{cNn_\mathrm{b}}{2\pi R} = \frac{4e^2\gamma^4 cNn_\mathrm{b}}{3RR_\mathrm{b}}. \tag{29}$$

Substituting Nn_b from Eq. (29) to Eq. (28), we obtain

TABLE II. Realistically achievable luminosities and other beam parameters for the projects listed in Table I at synchrotron-radiation power $P = 100$ MW. Only the parameters that differ from those in Table I are shown.

	LEP	LEP3	DLEP	STR1	STR2	STR3 cr-w	STR4 cr-w	STR5 cr-w	STR6 cr-w
$2E_0$, GeV	209	240	240	240	240	240	400	400	500
Circumference, km	27	27	53	40	60	40	40	60	80
Bunches per beam	~2	~7	70	24	53	240	36	45	31
N, 10^{11}	33	5.9	2.35	3.9	4	0.4	0.34	0.6	0.65
σ_z, mm	8.1	8.1	5.7	6.9	6.9	3.4	6.7	7.8	9.6
σ_y, μm	1.4	1.1	0.53	0.78	0.78	0.19	0.27	0.36	0.35
$\mathcal{L}$, 10^{34} cm^{-2} s^{-1}	0.47	0.31	0.89	0.55	0.83	1.1	0.12	0.16	0.087

$$\mathcal{L} \approx h\frac{(0.1\eta\alpha)^{2/3}PR}{32\pi^2\gamma^{13/3}r_e^3}\left(\frac{R_b}{R}\right)\left(\frac{6\pi\xi_y r_e}{\varepsilon_y}\right)^{1/3}, \qquad (30)$$

or in practical units,

$$\frac{\mathcal{L}}{10^{34}\ \mathrm{cm}^{-2}\,\mathrm{s}^{-1}} \approx \frac{100h\eta^{2/3}\xi_y^{1/3}}{(E_0/100\ \mathrm{GeV})^{13/3}(\varepsilon_y/\mathrm{nm})^{1/3}} \times\left(\frac{P}{100\ \mathrm{MW}}\right)\left(\frac{2\pi R}{100\ \mathrm{km}}\right)\frac{R_b}{R}. \qquad (31)$$

Once the vertical emittance is given as an input parameter, we find the luminosity and the optimum bunch length by applying Eq. (27). Beamstrahlung and the beam-beam strength parameter determine only the combination N/σ_x; additional technical arguments are needed to find N and σ_x separately. When they are fixed, the optimal number of bunches n_b is found from the total SR power, expressed in Eq. (29).

In Table II, we present the luminosities and beam parameters for the rings listed in Table I after beamstrahlung is taken into account. The following assumptions are made: SR power $P = 100$ MW, $R_b/R = 0.7$, $h = 0.8$, $\xi_y = 0.15$, $\eta = 0.01$; the values of ε_y, ε_x, and β_x are taken from Table I.

Comparing Tables I and II, one can see that at $2E_o = 240$ GeV taking beamstrahlung into account lowers the luminosities at storage-ring colliders with crab-waist collisions by a factor of 15. Nevertheless, these luminosities are comparable to those at the ILC, $\mathcal{L}_{\mathrm{ILC}} \approx (0.55\text{–}0.7)\times 10^{34}\ \mathrm{cm}^{-2}\,\mathrm{s}^{-1}$ at $2E_0 = 240$ GeV [16]. However, at $2E_0 = 500$ GeV the ILC can achieve $\mathcal{L}_{\mathrm{ILC}} \approx (1.5\text{–}2)\times 10^{34}\ \mathrm{cm}^{-2}\,\mathrm{s}^{-1}$, which is a factor 15–25 greater than the luminosities achievable at storage rings.

In conclusion, we have shown that the beamstrahlung phenomenon must be properly taken into account in the design and optimization of high-energy e^+e^- SRCs. We have demonstrated that beamstrahlung suppresses the luminosities as $1/E_0^{4/3}$ at energies $E_0 \gtrsim 70$ GeV for head-on collisions and $E_0 \gtrsim 20$ GeV for crab-waist collisions. Beamstrahlung makes the luminosities attainable in head-on and crab-waist collisions approximately equal above these threshold energies. At $2E_0 = 240$–500 GeV, beamstrahlung lowers the luminosity of crab-waist rings by a factor of 15–40. Some increase in SRC luminosities can be achieved at rings with larger radius, larger energy acceptance, and smaller beam vertical emittance.

We also conclude that the luminosities attainable at e^+e^- storage rings (at one interaction point) and linear colliders are comparable at $2E_0 = 240$ GeV. However, at $2E_0 = 500$ GeV, storage-ring luminosities are smaller by a factor of 15–25. Linear colliders remain the most promising instrument for energies $2E_0 \gtrsim 250$ GeV.

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