Dissipation-Assisted Quantum Information Processing with Trapped Ions

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We introduce a scheme to perform dissipation-assisted quantum information processing in ion traps considering realistic decoherence rates, for example, due to motional heating. By means of continuous sympathetic cooling, we overcome the trap heating by showing that the damped vibrational excitations can still be exploited to mediate coherent interactions as well as collective dissipative effects. We describe how to control their relative strength experimentally, allowing for protocols of coherent or dissipative generation of entanglement. This scheme can be scaled to larger ion registers for coherent or dissipative many-body quantum simulations.

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Among the platforms for quantum computation (QC) and simulations (QS) [1], trapped ions [2] stand out as excellent small-scale prototypes, which have a well-defined road map towards large-scale devices based on microfabrication [3,4]. The success of this technology depends on the impact of various sources of decoherence, such as the anomalous heating induced by the electric noise emanating from the trap electrodes [5]. The strong ion-ion couplings, required for scalable QC or QS, demand that the ions lie closer to the electrodes of these miniaturized traps, where the heating is critical and must be carefully considered. A strategy to minimize it is to cryogenically cool the setup [6], or to clean the electrodes by laser ablation [7] or ion bombardment [8]. Although these approaches are promising, a substantial residual noise still exists. We propose to minimize it by applying sympathetic laser cooling *continuously* during the whole QC-QS protocol.

Sympathetic cooling requires active laser cooling of a subset of ions, and passive cooling of the remaining ions by Coulomb interaction. This technique may overcome the motional heating [9,10], and has already been implemented between sequential gates for QC [11]. However, the larger heating rates in surface traps would further require cooling during the gates. There are different schemes along these lines. (i) In the absence of fluctuating electric gradients, interactions can be mediated by vibrational modes robust to the heating, while continuously cooling the remaining modes [9]. (ii) By using far-detuned state-dependent forces [12], the mediated interactions do not rely on the motional coherence and can thus withstand a heating or cooling that is considerably weaker than the interactions. (iii) For ground-state cooled crystals, resolved-sideband cooling may provide a dissipative force that improves the success or fidelity of protocols that are shorter than the inverse of the heating rate [13]. Unfortunately, these requirements are not met in current surface traps. (i) Since ions lie close to the electrodes, electric gradients prevent the isolation of robust modes. (ii), (iii) Heating rates in room-temperature setups (1 phonon/ms [14]) coincide with the above protocol's speed [12,13].

In this work, we propose a dissipation-assisted protocol based on an always-on sympathetic cooling that overcomes the anomalous heating for surface traps. We identify regimes where the sympathetically cooled vibrational modes can be used as mediators of both coherent interactions and collective dissipation. Since we only require Doppler cooling, this proposal can be applied to larger registers for QC or QS.

Model.—We consider an array of two types of ions $\{\sigma, \tau\}$ confined in a radio frequency (rf) trap [Fig. 1(a)]. Two hyperfine ground states $\{|\uparrow\rangle, |\downarrow\rangle\}$ of the σ ions provide the playground for QC or QS, whereas the τ ions act as an auxiliary gadget to sympathetically cool the crystal. In particular, the τ ions are Doppler cooled by using a standing wave that is red detuned from a dipole-allowed transition with decay rate Γ_τ [15]. On the other hand, the σ ions are subjected to a spin-phonon coupling obtained from two Raman beams in a traveling-wave configuration [16]. When the laser cooling is strong, the atomic degrees of freedom of the τ ions can be traced out [17], and one

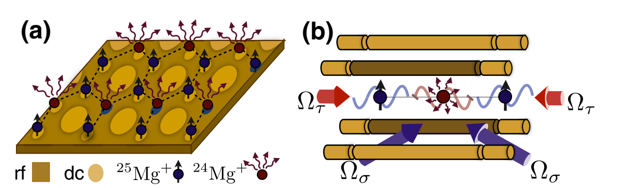

FIG. 1 (color online). (a) Coulomb crystal in a surface trap. The laser-cooled ions (red) assist the coherent or dissipative dynamics of the spins of the physical ions (blue). Both isotopes may be stored in the same individual minima without affecting the resultant lattice geometry. (b) Proof-of-principle experiment with three ions in a conventional rf trap. The red arrows correspond to a standing wave providing the Doppler cooling of the central ion, whereas the blue arrows lead to a running wave tuned to the axial red sideband of the outer ions.

obtains a master equation for the reduced dynamics of the σ spins and the collective vibrations

$$\dot{\mu} = -i[H_\sigma + H_{\text{ph}} + V_\sigma^{\text{ph}}, \mu] + \tilde{\mathcal{D}}(\mu),$$
$$\mu = \text{Tr}_{\tau,\text{atomic}}\{\rho\}. \tag{1}$$

Here, we have introduced the bare spin and phonon Hamiltonians $H_\sigma = \frac{1}{2}\sum_i \omega_0^\sigma \sigma_i^z$, $H_{\text{ph}} = \sum_n \omega_n b_n^\dagger b_n$, where ω_0^σ and ω_n are the electronic and longitudinal normal-mode frequencies [18]. Additionally, $\sigma_i^z = |\uparrow_i\rangle \times \langle\uparrow_i| - |\downarrow_i\rangle\langle\downarrow_i|$, and $b_n^\dagger$, b_n are the operators that create-annihilate phonons. The two crucial ingredients in (1) for our dissipation-assisted protocol are as follows.

(i) A spin-phonon coupling, provided by the Raman beams tuned to the so-called red sideband [17], leads to

$$V_\sigma^{\text{ph}} = \sum_{i,n} \mathcal{F}_{in}^\sigma \sigma_i^+ b_n e^{-i\omega_\sigma t} + \text{H.c.},$$
$$\mathcal{F}_{in}^\sigma = \frac{i\Omega_\sigma}{2}\eta_n^\sigma \mathcal{M}_{in} e^{i\phi_i},$$

where $\sigma_i^+ = |\uparrow_i\rangle\langle\downarrow_i|$, and the sum is extended to all σ ions and normal modes. Here, Ω_σ is the Rabi frequency of the Raman beams, $\omega_\sigma(\mathbf{k}_\sigma)$ is its frequency (wave vector), and $\phi_i = \mathbf{k}_\sigma \cdot \mathbf{r}_i^\sigma$ is defined in terms of the ion position $\mathbf{r}_i^\sigma$. The Lamb-Dicke parameter $\eta_n^\sigma = \mathbf{k}_\sigma \cdot \mathbf{e}_d/\sqrt{2m_\sigma\omega_n}$ describes the laser coupling to the nth normal mode, where the ith ion displacement along the direction $\mathbf{e}_d$ is given by $\mathcal{M}_{in}$, and m_σ is the ion mass.

(ii) An effective phonon damping, provided by the sympathetic Doppler cooling [17], which can be described by

$$\tilde{\mathcal{D}}(\mu) = \sum_n \{\Gamma_n^+(b_n^\dagger \mu b_n - b_n b_n^\dagger \mu)$$
$$+ \Gamma_n^-(b_n \mu b_n^\dagger - b_n^\dagger b_n \mu)\} + \text{H.c.}$$

Here, we have introduced Lorentzian-shaped cooling-heating couplings, which allow for an experimental control of the damping of the vibrational modes, and have the expression $\Gamma_n^\mp = \sum_l (\frac{1}{2}\Omega_\tau \eta_n^\tau \mathcal{M}_{ln})^2/[\frac{1}{2}\Gamma_\tau + i(-\Delta_\tau \pm \omega_n)]$. Note that we sum over all the laser-cooled τ ions. In these expressions, we have introduced the laser Rabi frequency Ω_τ, its detuning Δ_τ, and its wave vector $\mathbf{k}_\tau$ that determines $\eta_n^\tau = \mathbf{k}_\tau \cdot \mathbf{e}_p/\sqrt{2m_\tau\omega_n}$.

The master equation (1) describes an array of spins coupled to a set of damped vibrational modes. The idea now is to use the quanta of these modes, i.e., the phonons, as mediators of a coherent spin-spin interaction. However, in addition to the coherent dynamics, the phonons also provide an indirect coupling to the electromagnetic reservoir leading to some collective dissipation on the spins. Our goal is to find suitable regimes where these collective effects still allow for QC or QS. To guide this search, note that the two-qubit gates implemented in different laboratories [19] use nearly resonant spin-dependent forces and rely on the motional coherence to suppress the residual spin-phonon entanglement. Since the motional coherence is absent in our case, we must work in the far off-resonant regime [12,20], where $|\mathcal{F}_{in}^\sigma| \ll |\delta_n| \ll \omega_n$, such that $\delta_n = \omega_\sigma - (\omega_0^\sigma - \omega_n)$. In this regime, motional excitations by the spin-phonon coupling are negligible. We identify below the additional conditions to tailor the coherent or dissipative phonon-mediated processes in the presence of laser cooling.

Collective Liouvillian.—For the values considered below, the laser-cooling rates reach $W_n \approx 10^{-2}\omega_n$. In this case, the cooling is very strong, and the vibrations reach the steady state very fast. Hence, we can apply the theory of Schrieffer-Wolf transformations for open systems [21] to trace out the phonons from Eq. (1), and obtain an effective Liouvillian

$$\dot{\mu}_\sigma = \mathcal{L}_{\text{eff}}(\mu_\sigma) = -i[H_{\text{eff}}, \mu_\sigma] + \mathcal{D}_{\text{eff}}(\mu_\sigma), \tag{2}$$

where $\mu_\sigma = \text{Tr}_{\text{ph}}\{\mu\}$. Here, the coherent Hamiltonian is

$$H_{\text{eff}} = \sum_{i>j}(J_{ij}^{\text{eff}}\sigma_i^+\sigma_j^- + \text{H.c.}) + \sum_{in}\frac{1}{2}B_{in}^{\text{eff}}\sigma_i^z,$$

which contains the phonon-mediated interactions of strength J_{ij}^{eff}, which describe processes where a phonon is virtually created and then absorbed elsewhere in the chain. These interactions can be used to implement gates for QC, or to explore spin models for QS. Additionally, we also find an ac-Stark shift, which can be interpreted as an effective magnetic field B_{in}^{eff}, arising from the processes where the phonon is created and absorbed by the same ion. Note that the same virtual phonon exchange also introduces an indirect dissipation in (2),

$$\mathcal{D}_{\text{eff}}(\mu_\sigma) = \sum_{i,j}\Gamma_{ij}'^{\text{eff}}(\sigma_i^+\mu_\sigma\sigma_j^- - \sigma_j^-\sigma_i^+\mu_\sigma + \text{H.c.})$$
$$+ \sum_{i,j}(\Gamma_{ij}^{\text{eff}} + \Gamma_{ij}'^{\text{eff}})$$
$$\times (\sigma_i^-\mu_\sigma\sigma_j^+ - \sigma_j^+\sigma_i^-\mu_\sigma + \text{H.c.}),$$

where Γ_{ij}^{eff}, $\Gamma_{ij}'^{\text{eff}}$ are the strengths of the collective processes of spontaneous and stimulated dissipation, respectively.

To find the correct regime for QC or QS purposes, we must compare the time scales derived from the expressions

$$J_{ij}^{\text{eff}} = -\sum_n \frac{\mathcal{F}_{in}^\sigma(\mathcal{F}_{jn}^\sigma)^*}{\tilde{\delta}_n^2 + W_n^2}\tilde{\delta}_n,$$
$$B_{in}^{\text{eff}} = -\frac{\mathcal{F}_{in}^\sigma(\mathcal{F}_{in}^\sigma)^*}{\tilde{\delta}_n^2 + W_n^2}\tilde{\delta}_n(2\bar{n}_n + 1),$$
$$\Gamma_{ij}^{\text{eff}} = +\sum_n \frac{\mathcal{F}_{in}^\sigma(\mathcal{F}_{jn}^\sigma)^*}{\tilde{\delta}_n^2 + W_n^2}W_n,$$
$$\Gamma_{ij}'^{\text{eff}} = \sum_n \frac{\mathcal{F}_{in}^\sigma(\mathcal{F}_{jn}^\sigma)^*}{\tilde{\delta}_n^2 + W_n^2}W_n\bar{n}_n.$$

Here, the laser cooling leads to the effective cooling rates $W_n = \text{Re}\{\Gamma_n^- - \Gamma_n^+\}$ that damp the ion vibrations, and to a

Lamb-type shift of the vibrational frequencies leading to $\tilde{\delta}_n = \delta_n + \mathrm{Im}\{\Gamma_n^+ - (\Gamma_n^-)^*\}$. Additionally, $\bar{n}_n = \mathrm{Re}(\Gamma_n^+)/W_n$ are the mean phonon numbers in the steady state of the laser cooling. From these expressions, it is clear that by tuning the ratio $\mathcal{R}_n = W_n(\bar{n}_n + 1)/|\tilde{\delta}_n|$, we control if the spin interactions prevail over the dissipation $\mathcal{R}_n \ll 1$ or vice versa $\mathcal{R}_n \gg 1$.

Coherent and dissipative generation of entanglement.— We consider the simplest scenario to test our scheme: a three-ion chain in a linear Paul trap [Fig. 1(b)]. To use realistic parameters, we consider $^{25}\mathrm{Mg}^+ - {}^{24}\mathrm{Mg}^+ - {}^{25}\mathrm{Mg}^+$, and set the axial trap frequency to $\omega_z/2\pi = 4.1\,\mathrm{MHz}$, which is possible by optimizing the trap voltages. The dipole-allowed transition $|g\rangle = |3S_{1/2}\rangle \leftrightarrow |e\rangle = |3P_{1/2}\rangle$ of $^{24}\mathrm{Mg}^+$, which is characterized by $\lambda_\tau \approx 280.3$ nm and $\Gamma_\tau/2\pi \approx 41.4$ MHz, shall be used for continuous sympathetic cooling. By applying an external magnetic field, we can encode the spins in a couple of Zeeman sublevels $|F, M\rangle$ of the ground-state manifold of $^{25}\mathrm{Mg}^+$, e.g., $|\uparrow\rangle = |2, 2\rangle$ and $|\downarrow\rangle = |3, 3\rangle$. This leads to a resonance frequency of $\omega_0^\sigma/2\pi \approx 1.79$ GHz, and a negligible decay rate of $\Gamma_\sigma/2\pi \approx 10^{-14}$ Hz. Finally, a pair of off-resonant lasers drives the axial red sideband through an excited state in the $3P_{3/2}$ manifold of $^{25}\mathrm{Mg}^+$, such that $\eta_1^\sigma \approx 0.16$.

The isotopic mass ratio $m_\tau/m_\sigma \approx 0.96$ implies that the axial vibrational modes are almost unchanged with respect to the homogeneous chain, $\omega_n/2\pi \approx \{4.1, 7.1, 10.1\}\mathrm{MHz}$. To attain a wide range of values for the ratio $\mathcal{R}_n$, we tune the Raman lasers closer to the highest-frequency mode, the so-called Egyptian mode, such that the detunings are $\delta_n/2\pi \in \{6.2, 3.2, 0.3\}$ MHz. Note that, due to the large detuning from the remaining modes, the collective effects will be mediated by the Egyptian mode. To sympathetically cool it, we place the cooling isotope at the middle of the chain, such that it coincides with the node of a standing-wave laser [15]. This laser has a frequency that is red detuned from the transition, and we set the detuning to be $\Delta_\tau = -\Gamma_\tau/2$. This leads to a steady-state mean phonon number $\bar{n}_3 = 0.65$ independent of the standing-wave Rabi frequency. Therefore, we can modify it $0.2 \leq \Omega_\tau/\Gamma_\tau \leq 2$ in order to control the cooling rate W_3, and thus the ratio $\mathcal{R}_3$. This allows for reaching regimes of either dominant dissipation or interactions. We remark that the laser used for cooling $^{24}\mathrm{Mg}^+$ will be highly detuned from the cooling transition of $^{25}\mathrm{Mg}^+$ (i.e., $\Delta/2\pi \approx 2.7$ THz), such that the induced decay rate for the considered regime fulfills $\Gamma_\tau(\Omega_\tau/\Delta)^2/(2\pi) \leq 10^{-2}$ Hz. Therefore, this laser only contributes with off-resonant ac-Stark shifts that shall be considered later on. We now explore two possible applications.

(a) Coherent generation of entanglement. Our goal is to use the coherent phonon-mediated interaction in Eq. (2) to generate entanglement between the $^{25}\mathrm{Mg}^+$ ions. By setting $\Omega_\tau = 0.15\Gamma_\tau$, we obtain a cooling rate of $W_3/\omega_3 \approx 4.3 \times 10^{-3}$, such that $\mathcal{R}_3 \approx 7 \times 10^{-3}$, and the Hamiltonian part of the Liouvillian (2) thus dominates. Initializing the spin state in $|\psi_\sigma(0)\rangle = |\uparrow_1\downarrow_3\rangle$, and setting $\Omega_\sigma \eta_3^\sigma \approx 10W_3$, such that the distance between the ions is an integer multiple of the effective Raman wavelength, we obtain the Bell state $|\psi_B\rangle = \frac{1}{\sqrt{2}}(|\uparrow_1\downarrow_3\rangle - i|\downarrow_1\uparrow_3\rangle)$ for $t_f \approx 4$ ms [Fig. 2(a)]. In the numerical simulations, we have considered a realistic heating rate for macroscopic rf traps of $\Gamma_{ah} \approx 0.1$ phonon/ms, by substituting $\Gamma_n^+ \rightarrow \Gamma_n^+ + \Gamma_{an}$ in the dissipator of (1). From this figure, we observe that, even if the process is slower than the usual gates [19], it prevails over the phonon-mediated decoherence leading to errors as low as $\epsilon_B \sim 10^{-2}$. Note that such errors are not sufficient for fault-tolerance QC, which require $\epsilon_{ft} \sim 10^{-2}$–$10^{-4}$. On the one hand, we can achieve lower error rates by working with larger detunings. On the other hand, this leads to slower gates, which require an additional scheme to decouple from other sources of decoherence that shall be introduced below. Finally, for the anomalous heating rates in microfabricated surface traps $\Gamma_{ah} \approx 1$ phonon/ms, the same parameters lead to errors $\epsilon_B \approx 2 \times 10^{-2}$ for $t_f \approx 5$ ms, which illustrates the robustness of our scheme with respect to motional heating. Let us also advance that our protocol might be scaled directly to many ions for QS [17], which do not require such small error rates.

(b) Dissipative generation of entanglement. A different possibility would be to exploit the collective dissipation in

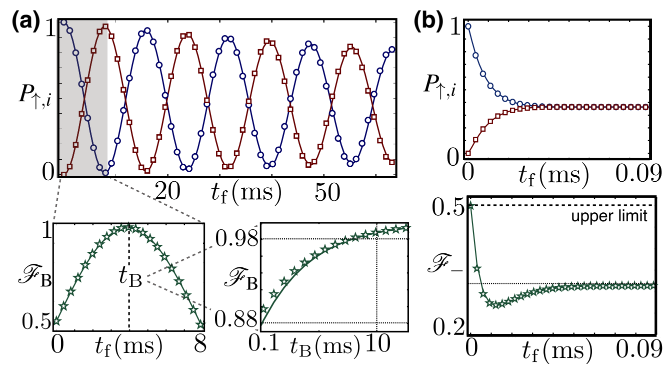

FIG. 2 (color online). (a) In the main panel, we show the coherent flip-flop dynamics for the three-ion setup, when the red sideband is tuned close to the highest-frequency vibrational mode. The solid lines represent the spin populations ($P_{\uparrow,1}$, blue; $P_{\uparrow,3}$, red) given by the original Liouvillian (1), while the symbols ($P_{\uparrow,1}$, circles; $P_{\uparrow,3}$, squares) correspond to the effective description (2). In the left-hand lower panel, the fidelity $\mathcal{F}_B = |\langle\psi_B|\mu_\sigma|\psi_B\rangle|$ with the Bell state $|\psi_B\rangle = \frac{1}{\sqrt{2}}(|\uparrow_1\downarrow_3\rangle - i|\downarrow_1\uparrow_3\rangle)$ for a single flip-flop exchange is displayed. An optimization of the fidelity for different gate times t_B is shown in the right-hand lower panel. In both cases, green solid lines represent the complete Liouvillian (1), whereas the stars follow from the effective description (2). (b) In the upper panel, the dissipative dynamics under Eq. (1) ($P_{\uparrow,1}$, blue solid line; $P_{\uparrow,3}$, red solid line) and Eq. (2) ($P_{\uparrow,1}$, circles; $P_{\uparrow,3}$, squares) is shown. In the lower panel, we display the fidelity $\mathcal{F}_- = |\langle\phi_-|\mu_\sigma|\phi_-\rangle|$ with the Bell state $|\phi_-\rangle = \frac{1}{\sqrt{2}}(|\uparrow_1\downarrow_3\rangle - |\downarrow_1\uparrow_3\rangle)$. Again, we use gray (green) solid lines for the complete Liouvillian (1) and stars for the effective description (2).

Eq. (2) to generate entanglement in the steady state. The idea is to set $\Omega_\tau = 2\Gamma_\tau$, such that the dissipative part of the Liouvillian (2) becomes dominating $\mathcal{R}_3 \approx 2.3$, and we can exploit a superradiant or subradiant phenomenon [22]. By controlling the ion-ion distance with respect to the Raman wavelength such that $\mathbf{k}_\sigma \cdot (\mathbf{r}_1^\sigma - \mathbf{r}_3^\sigma) = p\pi$, where $p \in \mathbb{Z}$, the decay rates fulfill $\Gamma_{11}^{\rm eff} = \Gamma_{33}^{\rm eff} = \Gamma_{\rm eff}$, and $\Gamma_{13}^{\rm eff} = \Gamma_{31}^{\rm eff} = (-1)^p\Gamma_{\rm eff}$ (equally for $\Gamma_{ij}^{\prime \rm eff}$). In this limit, the dissipator in (2) can be written as

$$\mathcal{D}_{\rm eff}(\mu_\sigma) = L_-\mu_\sigma L_-^\dagger + L_+\mu_\sigma L_+^\dagger - L_-^\dagger L_-\mu_\sigma - L_+^\dagger L_+\mu_\sigma + \text{H.c.},$$

where we have introduced the collective jump operators $L_- = \sqrt{\Gamma_{\rm eff}(\bar{n}_3+1)}[\sigma_1^- + (-1)^p\sigma_3^-]$ and $L_+ = \sqrt{\Gamma_{\rm eff}\bar{n}_3}[\sigma_1^+ + (-1)^p\sigma_3^+]$. One can check that the symmetric or antisymmetric Bell states $|\phi_\pm\rangle = \frac{1}{\sqrt{2}}(|\uparrow_1\downarrow_3\rangle \pm |\downarrow_1\uparrow_3\rangle)$ are dark states of these jump operators for p odd or p even, respectively. These are the so-called subradiant decay channels [23], which allows us to get a mixed stationary state that is partially entangled. In particular, starting from $|\Psi_\sigma(0)\rangle = |\uparrow_1\downarrow_3\rangle$ for p even, and $\Omega_\sigma\eta_3^\sigma \approx W_3$, we obtain a decoherence-free entangled steady state $|\phi_-\rangle$ for $t \gg t_{ss} \approx 50\ \mu$s with fidelities around 30% [Fig. 2(b)] [24] . Note that this phononic subradiance is not affected by limitations in the ratio of the ion-ion distance with respect to the wavelength of the emitted light. The collective nature of the vibrations that mediate the subradiance allows us to surpass the limitations of the pioneering trapped-ion experiments [26]. We also note that the ultimate limit of 50% cannot be achieved due to the thermal contribution to Eq. (2). However, schemes originally formulated for cavities [27] can be adapted for our trapped-ion setting to reach unit fidelities.

Let us emphasize that, although we have considered a particular example, the scheme is also applicable to other ion species. For the regime of dominant dissipation, any ion will work equally well. Conversely, for dominant coherent interactions, the required strong sympathetic-cooling strengths and even larger detunings are likely to be optimized for crystals with two light isotopes. At this point, it is also worth commenting that the strong rates provided by standing-wave laser cooling are required to obtain the target states in time scales which are not prohibitively large. In light of the results shown in Fig. 2, the regime of coherent interactions necessarily requires standing-wave cooling. Conversely, the regime of leading dissipation is faster, and may also work with the more standard traveling-wave cooling. Let us note, however, that the experiments [28] show that standing-wave cooling with a precise positioning of the ions with respect to the standing wave is possible.

Sources of noise.—In addition to the motional heating, other sources of noise become relevant for the time scales of the above protocols 0.1–10 ms. In fact, fluctuating magnetic fields and laser intensities, together with thermal noise, lead to the dephasing term $H_n = \sum_i \frac{1}{2} \times [\sum_n B_{in}^{\rm eff} + F_i(t)]\sigma_i^z$. Here, $B_{in}^{\rm eff}$ in Eq. (2) introduces noise via fluctuations over the phonon steady state, and $F_i(t)$ is a random process that models the noise of external magnetic fields, or uncompensated ac-Stark shifts. This random noise, which typically has a short correlation time τ_c, leads to a dephasing rate $\Gamma_d/2\pi \sim 0.1$–1 kHz [17].

For any practical implementation of the proposed protocol, this dephasing should be carefully considered. The standard approach for prolonging the coherence of a system consists of a sequence of refocusing pulses, a technique known as pulsed dynamical decoupling [29]. Another approach, known as continuous dynamical decoupling, produces a similar effect by continuous drivings [30,31]. As recently demonstrated experimentally [32], this technique allows us to implement robust two-qubit gates exploiting a single sideband [31]. As a by-product, we show that this tool allows for slower gates, and thus smaller errors in principle. Moreover, it also provides a new gadget to tailor the collective Liouvillian (2).

We apply a continuous driving resonant with the spins, such that the bare spin Hamiltonian reads $H_\sigma = \frac{1}{2}\sum_i \omega_0^\sigma\sigma_i^z + [\Omega_d\sigma_i^+\cos(\omega_d t) + \text{H.c.}]$, with $\omega_d = \omega_0^\sigma$. In this regime, a modified Schrieffer-Wolf transformation leads to $\dot{\mu}_\sigma = \tilde{\mathcal{L}}_{\rm eff}(\mu_\sigma)$, where

$$\tilde{\mathcal{L}}_{\rm eff}(\mu_\sigma) = -i[\tilde{H}_{\rm eff} + \tilde{H}_n, \mu_\sigma] + \tilde{\mathcal{D}}_{\rm eff}(\mu_\sigma) + \tilde{\mathcal{D}}_n(\mu_\sigma). \tag{3}$$

In the limit of a strong driving [17], the Hamiltonian above corresponds to an interacting Ising model

$$\tilde{H}_{\rm eff} = \sum_{i>j}\tilde{J}_{ij}^{\rm eff}\sigma_i^x\sigma_j^x + \sum_{in}\frac{1}{2}\Omega_d\sigma_i^x,$$

and we obtain a collective phonon-mediated dephasing

$$\tilde{\mathcal{D}}_{\rm eff}(\mu_\sigma) = \sum_{i,j}\tilde{\Gamma}_{ij}^{\rm eff}(2\sigma_i^x\mu_\sigma\sigma_j^x - \sigma_j^x\sigma_i^x\mu_\sigma - \mu_\sigma\sigma_j^x\sigma_i^x),$$

where we have assumed that the surface-trap array is designed such that $\mathbf{k}_L \cdot \mathbf{r}_i^\sigma = 2\pi p$, with $p \in \mathbb{Z}$. We emphasize that the interaction strengths and dissipation rates,

$$\tilde{J}_{ij}^{\rm eff} = -\sum_n \frac{|\mathcal{F}_{in}^\sigma\mathcal{F}_{jn}^\sigma|}{\tilde{\delta}_n^2 + W_n^2}\frac{\tilde{\delta}_n}{2},$$

$$\tilde{\Gamma}_{ij}^{\rm eff} = \sum_n \frac{|\mathcal{F}_{in}^\sigma\mathcal{F}_{jn}^\sigma|}{\tilde{\delta}_n^2 + W_n^2}\frac{W_n}{2}\left(\bar{n}_n + \frac{1}{2}\right),$$

lead to a very similar control parameter $\tilde{\mathcal{R}}_n = W_n(\bar{n}_n + \frac{1}{2})/|\tilde{\delta}_n|$. Accordingly, under the same assumptions as we considered above, we can interpolate between regimes where the coherent Ising interactions dominate over the collective dephasing, or vice versa. In addition to the new range of possibilities offered by this collective Liouvillian, note that the dephasing noise terms contribute to the new Liouvillian (3) with

$$\tilde{H}_n = \sum_i \frac{1}{2}\tilde{\Omega}_d \sigma_i^x,$$
$$\tilde{\mathcal{D}}_n(\mu_\sigma) = \sum_i \sum_{\alpha=y,z} \frac{1}{2}\tilde{\Gamma}_d(\sigma_i^\alpha \mu_\sigma \sigma_i^\alpha - \mu_\sigma),$$

where we have assumed that the noise is local, and introduced $\tilde{\Omega}_d = (B_{jn}^{\rm eff}\tilde{\delta}_n\tau_c + \Gamma_d)/2\Omega_d\tau_c$, and $\tilde{\Gamma}_d = \Gamma_d/(\Omega_d\tau_c)^2$ in the limit of a strong driving $\Omega_d\tau_c \gg 1$ [17]. According to the above constraints, these noisy terms are suppressed by a sufficiently strong driving. To be more specific, for noise correlation times on the order of $\tau_c = 10^{-2}/\Gamma_d$ and detunings $\delta_n/2\pi \sim 0.1$–1 MHz, it suffices to apply drivings with $\Omega_d/2\pi \approx 10$ MHz to reduce the noise by more than 2 orders of magnitude. Therefore, we can decouple from the noise efficiently, while preserving the collective part of the Liouvillian for QC or QS. In contrast to Fig. 2(a), the new Liouvillian (3) allows for the coherent generation of all four Bell states.

Conclusions.—We have proposed a scheme based on sympathetic cooling to overcome the anomalous heating in surface traps, while allowing for QC or QS. The sympathetic cooling becomes a tool to tailor the collective effects of the Liouvillian. By controlling a single parameter, namely, the laser-cooling power, we have shown how the Liouvillian interpolates between regimes of dominating coherent interactions or collective dissipation, both of which allow for generation of entanglement. Moreover, this control may also be exploited for coherent or dissipative many-body QS.

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