Spin-Nematic and Spin-Density-Wave Orders in Spatially Anisotropic Frustrated Magnets in a Magnetic Field

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We develop a microscopic theory of finite-temperature spin-nematic orderings in three-dimensional spatially anisotropic magnets consisting of weakly coupled frustrated spin-$\frac{1}{2}$ chains with nearest-neighbor and next-nearest-neighbor couplings in a magnetic field. Combining a field theoretical technique with density-matrix renormalization group results, we complete finite-temperature phase diagrams in a wide magnetic-field range that possess spin-bond-nematic and incommensurate spin-density-wave ordered phases. The effects of a four-spin interaction are also studied. The relevance of our results to quasi-one-dimensional edge-shared cuprate magnets such as $LiCuVO_4$ is discussed.

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Introduction.—The quest for novel states of matter has been attracting much attention in condensed-matter physics. Among those states, recently, spin-nematic (quadrupolar) phases have been vividly discussed in the field of frustrated magnetism [1–10]. The spin-nematic phase is defined by the presence of a symmetrized rank-2 spin tensor order, such as $\langle S_{\boldsymbol{r}}^+ S_{\boldsymbol{r}'}^+ + \mathrm{H.c.}\rangle \neq 0$, and the absence of any spin (dipolar) moment. Geometrical frustration, which generally suppresses spin orders, is an important ingredient for the emergence of spin nematics [1]. In spin-$\frac{1}{2}$ magnets, the spin-nematic operators cannot be defined on a single site because of the commutation relation of spin-$\frac{1}{2}$ operators. They reside on bonds between different sites [1,3], which is a significant difference from the quadrupolar phases in higher-spin systems [7]. Due to this property, it is generally quite hard to develop theories of spin nematics in spin-$\frac{1}{2}$ magnets, particularly in two- or three-dimensional (3D) systems. Developing such a theory is a current important issue in magnetism.

Among the existing models predicting spin-nematic phases, the spin-$\frac{1}{2}$ frustrated chain with a ferromagnetic nearest-neighbor coupling $J_1 < 0$ and an antiferromagnetic (AF) next-nearest-neighbor one $J_2 > 0$ would be the most relevant in nature because this system is believed to be an effective model for a series of quasi-1D edge-shared cuprate magnets such as $LiCuVO_4$ [11–16], $Rb_2Cu_2Mo_3O_{12}$ [17], $PbCuSO_4(OH)_2$ [18,19], $LiCuSbO_4$ [20], and $LiCu_2O_2$ [21]. These quasi-1D magnets hence offer a promising playground for spin-nematic phases.

Low-energy properties of the spin-$\frac{1}{2}$ J_1-J_2 chain have been well understood, thanks to recent theoretical efforts [2–6]. The corresponding Hamiltonian is given by

$$\mathcal{H} = \sum_{n=1,2} \sum_j J_n \boldsymbol{S}_j \cdot \boldsymbol{S}_{j+n} - H \sum_j S_j^z, \qquad (1)$$

where $\boldsymbol{S}_j$ is the spin-$\frac{1}{2}$ operator on site j and H is an external field. Below the saturation field in the broad parameter range $-2.7 \lesssim J_1/J_2 < 0$, the nematic operator $S_j^{\pm} S_{j+1}^{\pm}$ and the longitudinal spin S_j^z exhibit quasi-long-range orders, while the transverse spin correlator $\langle S_j^{\pm} S_0^{\mp}\rangle$ decays exponentially due to the formation of two-magnon bound states [3]. This phase is called a spin-nematic Tomonaga-Luttinger (TL) liquid, and it expands down to a low-field regime. The nematic correlation is stronger than the incommensurate longitudinal spin correlation in the high-field regime, while the latter grows stronger in the low-field regime.

From these theoretical results, the quasi-1D cuprates are expected to possess incommensurate longitudinal spin-density-wave (SDW) and spin-nematic long-range orders, respectively, in low- and high-field regimes at sufficiently low temperatures. In fact, recent magnetization measurements of $LiCuVO_4$ at low temperatures have detected a new phase [12] near saturation, and it is expected to be a 3D spin-nematic phase. Some experiments on $LiCuVO_4$ in an intermediate-field regime find SDW oscillations [13–15] whose wave vectors agree with the result of the nematic TL-liquid theory [2,3,5]. Furthermore, the spin dynamics of $LiCuVO_4$ observed by NMR [16] seems to be consistent with the prediction from the same theory [5,6]. However, this nematic TL-liquid picture can be applicable only above the 3D ordering temperatures. We have to take into account interchain interactions to explain how 3D spin-nematic and SDW long-range ordered phases are induced with lowering temperature. A mean-field theory for the 3D nematic phase of quasi-1D spin-$\frac{1}{2}$ magnets [9] has been proposed recently, but it cannot be applied to the SDW phase and does not quantitatively describe finite-temperature effects. It is obscure how both nematic and SDW ordered phases are described in a unified way.

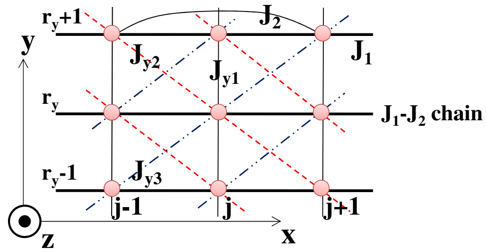

FIG. 1 (color online). Spatially anisotropic spin model consisting of weakly coupled spin-$\frac{1}{2}$ J_1-J_2 chains. We introduce interchain couplings J_{y_1,y_2,y_3} in the x-y plane. Similarly, J_{z_1,z_2,z_3} are present in the x-z plane.

A reliable theory for 3D orderings in weakly coupled spin-$\frac{1}{2}$ J_1-J_2 chains is strongly called for.

In this Letter, we develop a general theory for spin-nematic and incommensurate SDW orders in spatially anisotropic 3D magnets consisting of weakly coupled J_1-J_2 spin chains with arbitrary interchain couplings in a wide magnetic-field range. Combining field theoretical and numerical results for the J_1-J_2 spin chain, we obtain finite-temperature phase diagrams, which contain both spin-nematic and SDW phases at sufficiently low temperatures. We thereby reveal characteristic features in the ordering of weakly coupled J_1-J_2 chains, which cannot be predicted from the theory for the single J_1-J_2 chain. We also discuss the relevance of our results to real compounds such as $LiCuVO_4$.

Model.—Our model of a spatially anisotropic magnet is depicted in Fig. 1. The corresponding Hamiltonian is expressed as

$$\mathcal{H}_{3D} = \sum_{\boldsymbol{r}} \mathcal{H}_{\boldsymbol{r}} + \mathcal{H}_{\rm int}, \tag{2}$$

where $\boldsymbol{r} = (r_y, r_z)$ denotes the site index of the square lattice in the y-z plane, $\mathcal{H}_{\boldsymbol{r}}$ denotes the Hamiltonian (1) for the $\boldsymbol{r}$th J_1-J_2 chain along the x axis in a magnetic field H, and $\mathcal{H}_{\rm int}$ is the interchain interaction. In $\mathcal{H}_{\rm int}$, we introduce weak interchain Heisenberg-type exchange interactions with coupling constants J_{y_i} and J_{z_i} ($i = 1, 2, 3$) defined in the x-y and x-z planes, respectively [22].

Spin-$\frac{1}{2}$ J_1-J_2 chain.—Under the condition $|J_{y_i,z_i}| \ll |J_{1,2}|$, it is reasonable to choose decoupled J_1-J_2 spin chains ($\mathcal{H}_{\boldsymbol{r}}$) as the starting point for analyzing the 3D model ($\mathcal{H}_{3D}$). The low-energy effective Hamiltonian for the nematic TL-liquid phase is given by

$$\mathcal{H}^{\boldsymbol{r}}_{\rm eff} = \int dx \sum_{\nu=\pm} \frac{v_\nu}{2}[K_\nu(\partial_x\theta^{\boldsymbol{r}}_\nu)^2 + K_\nu^{-1}(\partial_x\phi^{\boldsymbol{r}}_\nu)^2] + G_- \sin(\pi M)\sin(\sqrt{4\pi}\phi^{\boldsymbol{r}}_- + \pi M), \tag{3}$$

where $x = a_0 j$ (the length a_0 of the J_1 bond is set equal to unity), $(\phi^{\boldsymbol{r}}_\pm(x), \theta^{\boldsymbol{r}}_\pm(x))$ is the canonical pair of scalar boson fields, and $v_\pm$ and $K_\pm$ are, respectively, the excitation velocity and the TL-liquid parameter of the $(\phi_\pm, \theta_\pm)$ sector. The sine term makes ϕ_- pinned, inducing an excitation gap in the (ϕ_-, θ_-) sector. Physically, the gap corresponds to the magnon binding energy E_b. On the other hand, the (ϕ_+, θ_+) sector describes a massless TL liquid. Vertex operators are renormalized as $\langle e^{i\alpha\sqrt{\pi}\phi_+(x)}e^{-i\alpha\sqrt{\pi}\phi_+(0)}\rangle_+ = |2/x|^{\alpha^2K_+/2}$ for $|x| \gg 1$, in which $\langle\cdots\rangle_\pm$ denotes the average over the $(\phi_\pm, \theta_\pm)$ sector. Spin operators $\boldsymbol{S}_{j,\boldsymbol{r}}$ are also bosonized as

$$S^z_{j,\boldsymbol{r}} \approx M + \partial_x[\phi^{\boldsymbol{r}}_+ + (-1)^j\phi^{\boldsymbol{r}}_-]/\sqrt{\pi} + (-1)^q A_1 \times\cos\{\sqrt{\pi}[\phi^{\boldsymbol{r}}_+ + (-1)^j\phi^{\boldsymbol{r}}_-] + 2\pi M q\} + \cdots, \tag{4a}$$

$$S^+_{j,\boldsymbol{r}} \approx e^{i\sqrt{\pi}[\theta^{\boldsymbol{r}}_+ + (-1)^j\theta^{\boldsymbol{r}}_-]}[(-1)^q B_0 + B_1 \times\cos\{\sqrt{\pi}[\phi^{\boldsymbol{r}}_+ + (-1)^j\phi^{\boldsymbol{r}}_-] + 2\pi M q\} + \cdots], \tag{4b}$$

where $M = \langle S^z_{j,\boldsymbol{r}}\rangle$, $q = \frac{j}{2}(\frac{j-1}{2})$ for even (odd) j, and A_n and B_n are nonuniversal constants. Utilizing Eqs. (3) and (4), we can evaluate spin and nematic correlation functions at zero temperature ($T = 0$) as follows [3,5,6]:

$$\langle S^+_j S^-_0\rangle \approx B_0^2\cos(\pi j/2)(2/|j|)^{1/(2K_+)}g_-(x) + \cdots, \tag{5a}$$

$$\langle S^z_j S^z_0\rangle \approx M^2 + (A_1^2/2)|\langle e^{i\sqrt{\pi}\phi_-}\rangle_-|^2\cos[\pi j(M - 1/2)](2/|j|)^{K_+/2} + \cdots, \tag{5b}$$

$$\langle S^+_j S^+_{j+1} S^-_0 S^-_1\rangle \approx (-1)^j C_0|j|^{-2/K_+} + \cdots, \tag{5c}$$

where $g_-(x) = \langle e^{\pm i\sqrt{\pi}\theta_-(x)}e^{\mp i\sqrt{\pi}\theta_-(0)}\rangle_-$, C_0 is a constant, and we have omitted the index $\boldsymbol{r}$. The function $g_-(x)$ decays exponentially as $x^{-1/2}e^{-x/\xi_-}$. The parameter K_+, which is less than 2 in the low magnetization regime, monotonically increases with M [3] and $K_+ \to 4$ at the saturation. Thus, the spin-nematic (SN) correlation is stronger than the incommensurate SDW correlation in the high-field regime with $K_+ > 2$ and weaker in the low-field regime with $K_+ < 2$.

The correlation length ξ_- is related to v_- via $v_- = \xi_- E_b$ under the assumption that the low-energy theory for the (ϕ_-, θ_-) sector has Lorentz invariance. The velocity v_+ has the relation $v_+ = 2K_+/(\pi\chi)$, where $\chi = \partial M/\partial H$ is the uniform susceptibility. Since K_+, ξ_-, E_b, and χ are all determined with reasonable accuracy by using the density-matrix renormalization group method [3,23], $v_\pm$ can be quantitatively evaluated as depicted in Fig. 2. The figure shows that v_- is always larger than v_+, in accordance with the perturbative formulas $v_\pm \approx v[1 \pm KJ_1/(\pi v) + \cdots]$ for $|J_1| \ll J_2$, in which v and K are, respectively, the spinon velocity and the TL-liquid parameter for the single AF-J_2 chain. We also note that v_+ approaches zero at $M \to \frac{1}{2}$.

Analysis of the 3D model.—Let us now analyze the 3D model (2) starting with the effective theory of the J_1-J_2 chain. We first bosonize all of the interchain couplings in $\mathcal{H}_{\rm int}$ through Eq. (4). To obtain the low-energy effective theory for Eq. (2), we trace out the massive (ϕ_-^r, θ_-^r) sectors in the Euclidean action $\mathcal{S}_{\rm tot} = \mathcal{S}_0 + \mathcal{S}_{\rm int}$ via the cumulant expansion $\mathcal{S}_{\rm eff}^{\rm 3D} = \mathcal{S}_0 + \langle \mathcal{S}_{\rm int}\rangle_- - \frac{1}{2}(\langle \mathcal{S}_{\rm int}^2\rangle_- - \langle \mathcal{S}_{\rm int}\rangle_-^2) + \cdots$, where $\mathcal{S}_0$ and $\mathcal{S}_{\rm int}$ are, respectively, the action for the TL-liquid part of the (ϕ_+^r, θ_+^r) sectors and that for the interchain couplings. This corresponds to the series expansion in $J_{y_i,z_i}/v_-$. The resultant effective Hamiltonian is expressed as $\mathcal{H}_{\rm eff}^{\rm 3D} = \mathcal{H}_0 + \mathcal{H}_{\rm SDW} + \mathcal{H}_{\rm SN} + \cdots$. Here, $\mathcal{H}_0 = \sum_{\boldsymbol r}\int dx \frac{v_+}{2}[K_+(\partial_x\theta_+^r)^2 + K_+^{-1}(\partial_x\phi_+^r)^2]$ is the TL-liquid part and $\mathcal{H}_{\rm SDW}$ and $\mathcal{H}_{\rm SN}$ are, respectively, obtained from the first- and second-order cumulants as follows:

$$\mathcal{H}_{\rm SDW} = G_{\rm SDW}\int\frac{dx}{2}\sum_{\boldsymbol r}\sum_{\substack{\alpha=y,z\\(\boldsymbol r'=\boldsymbol r+\boldsymbol e_\alpha)}}\{J_{\alpha1}\cos[\sqrt{\pi}(\phi_+^r - \phi_+^{r'})] - J_{\alpha2}\sin[\sqrt{\pi}(\phi_+^r - \phi_+^{r'}) - \pi M] + J_{\alpha3}\sin[\sqrt{\pi}(\phi_+^r - \phi_+^{r'}) + \pi M]\}, \tag{6a}$$

$$\mathcal{H}_{\rm SN} = G_{\rm SN}\int\frac{dx}{2}\sum_{\boldsymbol r}\sum_{\substack{\alpha=y,z\\(\boldsymbol r'=\boldsymbol r+\boldsymbol e_\alpha)}}[J_{\alpha1}^2 - (J_{\alpha2} - J_{\alpha3})^2]\cos[\sqrt{4\pi}(\theta_+^r - \theta_+^{r'})], \tag{6b}$$

with coupling constants $G_{\rm SDW} = A_1^2|\langle e^{i\sqrt{\pi}\phi_-}\rangle_-|^2$ [24] and $G_{\rm SN} = -\frac{B_0^4}{4v_-}\int dx v_- d\tau g_-(x,\tau)^2$ (τ is imaginary time). The summations run over all nearest-neighbor pairs of chains, where $\boldsymbol r' = \boldsymbol r + \boldsymbol e_\alpha$ ($\alpha = y, z$), $\boldsymbol e_\alpha$ denotes the unit vector along the α axis, and we have assumed that the field ϕ_+ smoothly varies in x. The first-order term $\mathcal{H}_{\rm SDW}$ contains an interchain interaction between the operators $e^{\pm i\sqrt{\pi}\phi_+^r}$, which essentially induces a 3D spin longitudinal order. Similarly, the term $\mathcal{H}_{\rm SN}$ contains an interchain interaction between the spin-nematic operators $S_{j,\boldsymbol r}^\pm S_{j+1,\boldsymbol r}^\pm \sim (-1)^j e^{\pm i\sqrt{4\pi}\theta_+^r}$, which enhances a 3D spin-nematic correlation. We should notice that the effective theory $\mathcal{H}_{\rm eff}^{\rm 3D}$ is reliable under the condition that temperature T is sufficiently smaller than the binding energy E_b and the velocities $v_\pm$.

Both the couplings $G_{\rm SDW,SN}$ can be numerically evaluated from the density-matrix renormalization group data of correlation functions [3,23]: $G_{\rm SDW}$ corresponds to the amplitude of the leading term of the longitudinal correlator $\langle S_j^z S_0^z\rangle$ given in Eq. (5) and $G_{\rm SN}$ can be evaluated as $G_{\rm SN} \approx \pi v_-^{-1}\sum_{j=1}^{L}(j/2)^{1/K_+}j\langle S_j^+S_0^-\rangle^2$. We have checked that the finite-size correction to the sum is small enough when the cutoff L is larger than ξ_-. We emphasize that there is no free parameter in $\mathcal{H}_{\rm eff}^{\rm 3D}$.

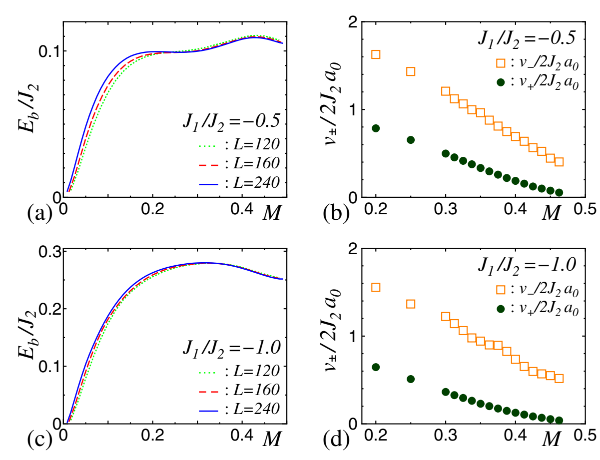

FIG. 2 (color online). (a),(c) Magnon binding energy E_b and (b),(d) excitation velocities $v_\pm$ as a function of M in the spin-nematic TL-liquid phase in the spin-$\frac{1}{2}$ J_1-J_2 chain at $T = 0$.

To obtain the finite-temperature phase diagram, we apply the interchain mean-field (ICMF) approximation [25,26] to the effective Hamiltonian $\mathcal{H}_{\rm eff}^{\rm 3D}$. To this end, we introduce the "effective" SDW operator $\mathcal{O}_{\rm SDW} = e^{i\pi(1/2-M)j}e^{i\sqrt{\pi}\phi_+^r}$ and the spin-nematic operator $\mathcal{O}_{\rm SN} = (-1)^j e^{i\sqrt{4\pi}\theta_+^r}$. Within the ICMF approach, the finite-temperature dynamical susceptibilities of $\mathcal{O}_A$ (A = SDW or SN) above 3D ordering temperatures are calculated as

$$\chi_A(k_x, \boldsymbol k, \omega) = \frac{\chi_A^{\rm 1D}(k_x,\omega)}{1 + J_{\rm eff}^A(\boldsymbol k)\chi_A^{\rm 1D}(k_x,\omega)}, \tag{7}$$

where $\boldsymbol k = (k_y, k_z)$ is the wave vector in the y-z plane, ω is the frequency, and the effective coupling constants $J_{\rm eff}^A$ are given by

$$J_{\rm eff}^{\rm SDW}(\boldsymbol k) = G_{\rm SDW}\sum_{\alpha=y,z}[J_{\alpha_1}\cos k_\alpha - J_{\alpha_2}\sin(k_\alpha - \pi M) + J_{\alpha_3}\sin(k_\alpha + \pi M)], \tag{8a}$$

$$J_{\rm eff}^{\rm SN}(\boldsymbol k) = G_{\rm SN}\sum_{\alpha=y,z}[J_{\alpha_1}{}^2 - (J_{\alpha_2} - J_{\alpha_3})^2]\cos k_\alpha. \tag{8b}$$

The 1D susceptibilities $\chi_A^{\rm 1D}(k_x,\omega) = \frac{1}{2}\sum_j e^{-ik_xj}\times\int_0^\beta d\tau e^{i\omega_n\tau}\langle\mathcal{O}_A(j,\tau)\mathcal{O}_A^\dagger(0,0)\rangle|_{i\omega_n\to\omega+i\epsilon}$ are analytically computed by using the field theoretical technique ($\beta = 1/T$ and $\epsilon \to +0$) [27]. Those for SDW and spin-nematic operators respectively take the maximum at $k_x^{\rm max} = (\frac{1}{2} - M)\pi$ and π; $\chi_{\rm SDW}^{\rm 1D}(k_x^{\rm max}, 0) = \frac{2}{v_+}(\frac{4\pi}{\beta v_+})^{K_+/2-2}\times\sin(\frac{\pi K_+}{4})B(\frac{K_+}{8}, 1 - \frac{K_+}{4})^2$ and $\chi_{\rm SN}^{\rm 1D}(\pi, 0) = \frac{2}{v_+}(\frac{4\pi}{\beta v_+})^{2/K_+-2}\times\sin(\frac{\pi}{K_+})B(\frac{1}{2K_+}, 1 - \frac{1}{K_+})^2$, where $B(x, y)$ is the beta function.

The transition temperature of each order is obtained from the divergent point of its susceptibility at $\omega \to 0$, which is given by

$$1 + \mathrm{Min}_{\boldsymbol{k}}[J_{\mathrm{eff}}^{A}(\boldsymbol{k})]\chi_{A}^{\mathrm{1D}}(k_x^{\max}, 0) = 0. \quad (9)$$

The 3D ordered phase with the highest transition temperature is realized. From this ICMF scheme, we can determine the phase diagram for $\mathcal{H}_{\mathrm{3D}}$ with an arbitrary combination of J_{y_i,z_i}. This is a significant advantage compared with previous theories for spin-nematic phases. We note that, when J_{eff}^{A} approaches zero, the present framework becomes less reliable and we need to consider the subleading terms in $\mathcal{H}_{\mathrm{eff}}^{\mathrm{3D}}$.

From Eqs. (8) and (9), we find that the ordering wave numbers $k_{y,z}$ tend to be a commensurate value $k_{y,z} = 0$ or π (see also Ref. [24]). Thus, the SDW ordered phase has the wave vector $k_x = (\frac{1}{2} - M)\pi$ and $k_{y,z} = 0$ or π. This agrees with the experimental result in the intermediate-field phase of $LiCuVO_4$ [13,14]. For the spin-nematic ordered phase, we find the commensurate ordering vector $(k_x, k_{y(z)}) = (\pi, 0)$ for $|J_{y_1(z_1)}| > |J_{y_2(z_2)} - J_{y_3(z_3)}|$ and $(k_x, k_{y(z)}) = (\pi, \pi)$ for $|J_{y_1(z_1)}| < |J_{y_2(z_2)} - J_{y_3(z_3)}|$. This commensurate nature of $k_{x,y,z}$ in the nematic phase is consistent with Ref. [9].

We show typical examples of obtained phase diagrams in Fig. 3. When interchain couplings are not frustrated, as the J_{y_1,z_1} dominant cases of Figs. 3(a) and 3(b), the SDW ordered phase is largely enhanced and the nematic ordered phase is reduced to a higher-field regime compared to the crossover line ($K_+ = 2$) in the J_1-J_2 chain. This is because the effective couplings $J_{\mathrm{eff}}^{\mathrm{SDW}}$ and $J_{\mathrm{eff}}^{\mathrm{SN}}$ are respectively generated from the first- and second-order cumulants, and therefore $J_{\mathrm{eff}}^{\mathrm{SDW}}$ is generally larger than $J_{\mathrm{eff}}^{\mathrm{SN}}$ in non-frustrated systems with weak interchain couplings. When both the couplings J_{y_2,y_3} are dominant, we find a similar tendency. We note that a model with dominant J_{y_2,y_3} has been proposed for $LiCuVO_4$ [11], where a new phase expected to be a 3D nematic phase has been observed only near the saturation [12]. From the calculations for the cases of $|J_1|/J_2 = 0.5$, 1.0, and 2.0, we find that the nematic phase region in the M-T phase diagram generally becomes smaller with increase in $|J_1|/J_2$ since the value $g_-(x)$ in G_{SN} decreases. When there is a certain frustration in interchain couplings, however, the nematic phase region can expand, as shown in Fig. 3(c). When the signs of J_{y_1} and $J_{y_2}(J_{y_3})$ are opposite, $J_{\mathrm{eff}}^{\mathrm{SDW}}$ becomes small and the 3D nematic phase expands down to a relatively lower-field regime. We emphasize that our theory succeeds in quantitatively analyzing the competition between SDW and nematic ordered phases in quasi-1D magnets.

Effects of a four-spin term.—Finally, we study the effects of an interchain four-spin interaction. The Hamiltonian we consider is

$$\mathcal{H}_4 = -J_4 \sum_{j,\langle \boldsymbol{r},\boldsymbol{r}'\rangle} S_{j,\boldsymbol{r}}^{+} S_{j+1,\boldsymbol{r}}^{+} S_{j,\boldsymbol{r}'}^{-} S_{j+1,\boldsymbol{r}'}^{-} + \mathrm{H.c.} \quad (10)$$

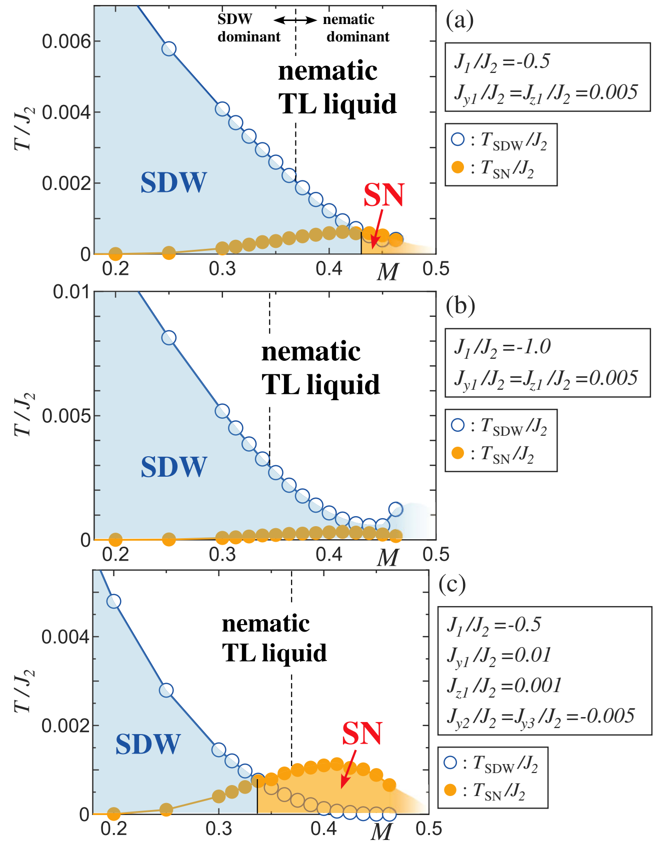

FIG. 3 (color online). Phase diagrams of the weakly coupled J_1-J_2 chains (2) in the M-T plane, which are derived from the ICMF approach. The temperatures $T_{\mathrm{SDW(SN)}}$ denote the 3D SDW (nematic) transition points. The vertical dashed lines denote the crossover lines between nematic dominant and SDW dominant TL liquids in the 1D J_1-J_2 chain.

This interaction is a part of the spin-phonon coupling $\mathcal{H}_{\mathrm{sp}} = -J_{\mathrm{sp}}\sum_{j,\langle \boldsymbol{r},\boldsymbol{r}'\rangle}(\boldsymbol{S}_{j,\boldsymbol{r}} \cdot \boldsymbol{S}_{j,\boldsymbol{r}'})(\boldsymbol{S}_{j+1,\boldsymbol{r}} \cdot \boldsymbol{S}_{j+1,\boldsymbol{r}'})$ and therefore it really exists in some compounds. One easily finds that Eq. (10) enhances the spin-nematic ordering. Applying the field theoretical strategy to the system $\mathcal{H}_{\mathrm{3D}} + \mathcal{H}_4$, we find that $J_{\mathrm{eff}}^{\mathrm{SN}}$ is replaced with $J_{\mathrm{eff}}^{\mathrm{SN}} - 4J_4C_0(\cos k_y + \cos k_z)$. We thus obtain the phase diagram for $\mathcal{H}_{\mathrm{3D}} + \mathcal{H}_4$, as shown in Fig. 4. Comparing Figs. 3(a) and 4, we see that an interchain four-spin interaction definitely enhances the 3D nematic phase even if its coupling constant J_4 is small. Since J_4 is usually positive, it favors ferrotype nematic ordering along the y and z axes; i.e., $k_{y,z} = 0$.

Conclusion.—We have constructed finite-temperature phase diagrams for 3D spatially anisotropic magnets, which consist of weakly coupled spin-$\frac{1}{2}$ J_1-J_2 chains in an applied magnetic field. Incommensurate SDW and spin-nematic ordered phases appear at sufficiently low temperatures, triggered by the nematic TL-liquid properties in the J_1-J_2 spin chains. We reveal several natures of orderings in the coupled J_1-J_2 chains: The 3D nematic ordered phase is generally smaller than the 1D nematic dominant region,

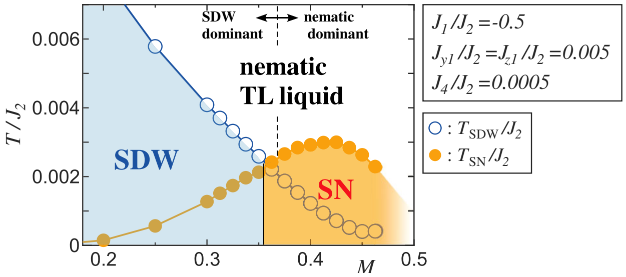

FIG. 4 (color online). Phase diagram of the weakly coupled J_1-J_2 spin chains (2) with a four-spin interaction $\mathcal{H}_4$.

while it can be larger if we somewhat tune the interchain couplings. The ordering wave numbers $k_{y,z}$ tend to be 0 or π, and a small four-spin interaction $\mathcal{H}_4$ efficiently helps the 3D nematic ordering. We finally note that our theory can also be applied to AF-J_1 systems.

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