Absolute Bunch Length Measurement Using Coherent Diffraction Radiation

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The longitudinal electron beam properties are of crucial importance for many types of frontier accelerators, from storage rings to free electron lasers and energy recovery linacs. For the online control of the machine and its stable operation, nondestructive shot by shot bunch length measurements are needed. Among the various instrumentations proposed and installed in accelerators worldwide, the ones based on the measurement of the coherent radiation power represent the simplest and the more robust tools for operational control. The major limitation of these systems is that they usually can provide only relative bunch length estimation. In this Letter we present a novel experimental methodology to self-calibrate a simple equipment based on diffraction radiation from a gap providing a measurement of the second order moment of the longitudinal distribution. We present the theoretical basis of the proposed approach and validate it through a detailed campaign of measurements.

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Frontier fast dynamics science is strongly founded on accelerator-based pulsed photon sources operating in the spectral range from THz to hard x rays. The longitudinal electron bunch properties play a crucial role to guarantee high performance and good reliability of the most advanced accelerator-based facilities. Offstage of these impressive science advances, electron accelerators and instrumentation have been designed to provide and maintain picosecond and subpicosecond electron bunches, and monitor their longitudinal parameters. To this purpose in the last decades several instruments and techniques have been implemented to measure the electron bunch length: streak camera [1], transverse radio frequency deflecting cavity [2], electro-optic sampling [3], devices based on coherent radiation [4,5], and others. The first three can provide absolute bunch length measurements, but require complex and expensive equipment and are either destructive or not well suited for machine control. Coherent radiation spectroscopic measurements, attempting to reconstruct the beam profile from spectral information, are not yet fully mature, and most often not single shot. The simplest and most robust diagnostics for operational control are based on the measurement of coherent radiation power. These coherent diagnostics are routinely used in existing electron accelerators to measure the relative variation of the bunch length [6,7]. The resulting output signals are typically implemented in bunch length control feedback loops [8]. The great advantage of implementing these tools is the possibility to measure nondestructively the bunch length from shot to shot, without perturbing the electron beam. The drawback is that for an absolute estimation of the bunch length, external instrumentation like a transverse rf deflecting cavity is usually needed.

In this Letter we present a novel experimental methodology to self-calibrate a device based on diffraction radiation from a ceramic gap. We demonstrate that this provides the absolute measurement of the second order moment of the electron bunch longitudinal distribution. In the rest of the Letter, we will refer to this quantity as bunch length. The method is best suited for measuring bunches with lengths from picoseconds to subpicoseconds, it is independent from other external instruments and the required equipment is extremely simple and less expensive than other aforementioned instruments.

The general solution of the Maxwell problem of an ultrarelativistic electron bunch passing a gap in a waveguide and emitting radiation is provided by Palumbo [9] and relies on the seminal work of Bolotovskii [10]. We introduce a high frequency approximation of the result, that can be easily treated analytically to calculate the coherent radiation power as a function of the bunch length. The spectral content of the radiation of a subpicoseconds electron bunch is strongly dominated, in the millimeter wave and THz spectrum range, by the longitudinal form factor $F(\omega)$ defined as the Fourier transform of the longitudinal bunch distribution. We assume that the electron beam travels on axis and that it is transversally small compared to the transverse waveguide radius. In these conditions the transverse effects are negligible and we are allowed to describe the spectrum-angular distribution of the energy radiated by the electron bunch as in Ref. [11]:

$$\frac{d^2W}{d\omega d\Omega} = \frac{d^2W}{d\omega d\Omega}\bigg|_{1e^-}(N + N(N-1)|F(\omega)|^2), \quad (1)$$

where W is the energy emitted, Ω is the solid angle, N is the number of the electrons in the bunch, $\frac{d^2W}{d\omega d\Omega}|_{1e^-}$ is the spectrum-angular distribution of the energy radiated by a single electron. The term proportional to the number of electrons N is related to incoherent radiation, while the one proportional to N^2, modulated by the longitudinal form factor, is related to the coherent part of the radiation. In the millimeter waves and in the THz spectral range, the coherent term prevails over the incoherent one since $N|F(w)|^2 \gg 1$. Equation (1) applies to the emission of coherent radiation from any mechanism and is not specific to the source and type of radiation. We will introduce the minimum theoretical background for the case of radiation emitted from a gap in a waveguide to get to the high frequency approximation of $\frac{d^2W}{d\omega d\Omega}|_{1e^-}$. The problem of the radiation emission of an ultrarelativistic particle can be studied starting from the Maxwell equations, with the introduction of boundary conditions. This is the same approach used by pioneering works on transition and Cherenkov radiation by Bolotovskii *et al.* in Ref. [12] based on the evaluation of the radiated field with the help of the Wiener-Hopf factorization method described in Refs. [13,14]. A point charge moving in a discontinuous circular waveguide of radius a induces currents on the walls of the waveguide. At a boundary with free space, the currents become sources for an electromagnetic field radiating in the surrounding space. The exact solution of the problem for a particle incoming in a semi-infinite waveguide can be found in Ref. [10]. In the same way, the problem of a particle outgoing from the waveguide can be solved. The ceramic gap is modeled as two coaxially faced waveguides, at a distance from each other equal to the gap length, $2l$. With this model, at first order, the problem of emission from a gap can be understood using the superposition principle, i.e., considering that the total radiated field will be the sum of the outgoing particle field and the incoming particle one. The exact solution of the electromagnetic problem, including interferences between the two faced guides, can be found in Ref. [9]. For brevity, we report in Eq. (2) only the final expression of the spectrum-angular distribution for a single electron traveling on the axis of the waveguide, considering the structure in a spherical coordinate system (R, θ, ϕ), with the origin located at the center of the gap:

$$\left.\frac{d^2W(\theta)}{d\omega d\Omega}\right|_{1e^-} = \beta e^2 \frac{\sin^2\theta J_0^2(ka\sin\theta)}{4\pi^2 c(1-\beta\cos\theta)^2 I_0^2(\frac{ka}{\beta\gamma})} \times \left| \frac{L^-(\omega/v)\sqrt{1-\beta}e^{jk\ell(1-\beta\cos\theta)/\beta}}{L^-(k\cos\theta)\sqrt{1-\cos\theta}} + j\frac{L^+(\omega/v)\sqrt{1+\beta}e^{-jk\ell(1-\beta\cos\theta)/\beta}}{L^+(k\cos\theta)\sqrt{1+\cos\theta}}\right|^2, \tag{2}$$

where v is the speed of the electron, β is the ratio v/c, γ is the Lorentz factor, k is the wave number, a is the radius of the pipe, and e is the electron charge. $J_0(x)$ is the first type, zero order Bessel function, $I_0(x)$ is the modified zero order Bessel function. The functions $L^+(\alpha)$ and $L^-(\alpha)$ come from the factorization of the kernel $L(\alpha) = \pi a\Omega' J_0(\Omega' a)H_0^{(1)}(\Omega' a)$ as defined in Ref. [9] where $H_0^{(1)}(x)$ is the Hankel function $\Omega' = \sqrt{k^2-\alpha^2}$ and the variable α comes from the Fourier transform of the function in the space domain, i.e., of the functions of the variable z. Their formal expression involves complex integrals with singularities and no closed form is available in literature. Nevertheless, in the case of $ka \gg 1$, i.e., for high frequencies with respect to the beam pipe radius, it is possible to use the approximation: $L^+(\alpha) = L^-(\alpha) \approx 1$ so that Eq. (2) can be simplified to Eq. (3). In our experiment the detector central bandwidth is about 30 GHz, a is 20 mm, and ℓ is 3.18 mm. As shown in Ref. [9] the approximation for $ka > 10$ leads to a relative error of less than 7%.

$$\left.\frac{d^2W(\theta)}{d\omega d\Omega}\right|_{1e^-} = \beta e^2 \frac{\sin^2\theta J_0^2(ka\sin\theta)}{4\pi^2 c(1-\beta\cos\theta)^2 I_0^2(\frac{ka}{\beta\gamma})} \times \left|\sqrt{\frac{1-\beta}{1-\cos\theta}}e^{jk\ell(1-\beta\cos\theta)/\beta} + j\sqrt{\frac{1+\beta}{1+\cos\theta}}e^{-jk\ell(1-\beta\cos\theta)/\beta}\right|^2. \tag{3}$$

Both expressions in Eqs. (2) and (3) are valid for any beam energy. The first term of the sum in Eq. (3) is related to the "step in" (i.e., the electron entering the waveguide) and for ultrarelativistic electrons it tends to zero, and the gap emission is dominated by the second term, representing emission from "step out" (i.e., the electron exiting the waveguide). Figure 1 shows the spectrum-angular density of the energy radiated versus the frequency, for both incoming and outgoing particle cases, for a 330 MeV electron. The minima are related to the $J_0^2[ka\sin(\theta)]$ function in Eq. (3), as in the case of diffraction radiation from a hole, and their position in frequency is independent from the beam energy. For a bunch of N electrons with a given longitudinal current profile distribution, the coherent emission from the gap is calculated from Eq. (1), using the expression in Eq. (3) for the single electron emission. A bunch length variation induces a change of the longitudinal form factor. As the bunch becomes

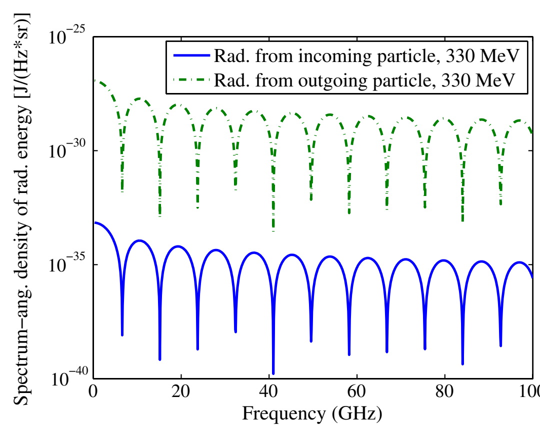

FIG. 1 (color online). Single electron spectrum-angular density at 330 MeV energy incoming and outgoing from a semi-infinite waveguide, at an angle of $\theta = \pi/2$.

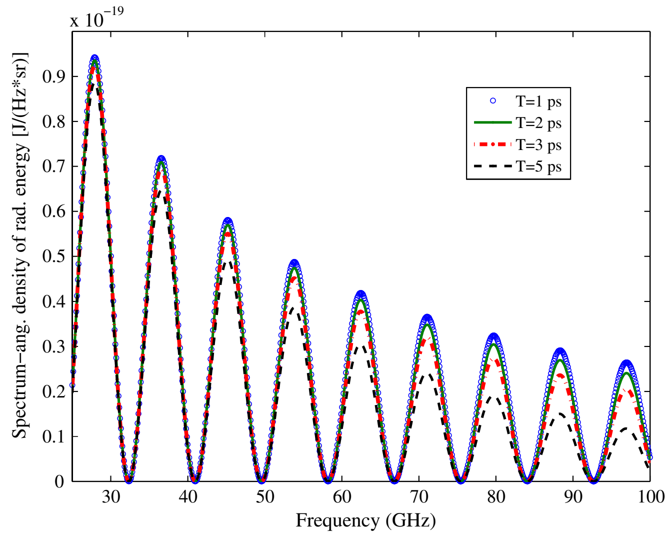

FIG. 2 (color online). Spectrum-angular density of energy radiated by a 1 nC rectangular electron bunch passing a gap in a waveguide, 330 MeV energy at an angle of $\theta = \pi/2$.

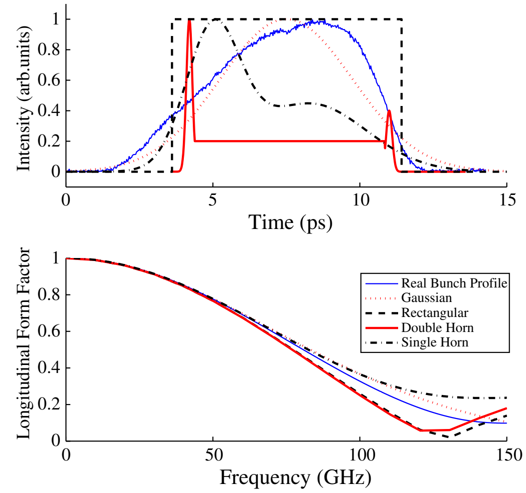

FIG. 3 (color online). Form factors comparison (at the bottom) between rectangular, Gaussian, single horn, double horn, and real bunch measured with the rf deflecting cavity. All profiles (on the top) have a rms length of 2.3 ps.

shorter, the longitudinal form factor extends towards higher frequencies. This determines an increase of the intensity emitted. Figure 2 shows the energy radiated by a rectangular electron bunch passing through the gap, for different bunch lengths. The spectrum-angular density is higher at all frequencies for a 1 ps long bunch relative to the case of 5 ps as a consequence of the longitudinal form factor modulating the single electron emission. The intensity drops as the frequency increases, mostly due to the single particle distribution behavior. Moreover, the relative variation of intensity at any frequency is larger for longer bunches. For example, when passing from 5 to 3 ps it is higher than passing from 2 to 1 ps. Eventually as the electron bunch gets even shorter the intensity variation becomes negligible over the same frequency range. This asymptotic behavior is the key property that we have exploited in the method we propose to perform an absolute bunch length measurement. Finally, the difference of intensity between shorter and longer bunches is more pronounced at higher frequency, as a consequence of a larger difference in the longitudinal form factors. The energy W radiated by the bunch in the gap region, is calculated integrating the spectrum-angular density over frequency and solid angle. Experimentally, this means over the detector band ($\Delta\omega$), and angular acceptance ($\Delta\Omega_{\rm rad}$):

$$W = N^2 \int_{\Delta\omega} \int_{\Delta\Omega_{\rm rad}} \frac{d^2W(\theta)}{d\omega d\Omega}\bigg|_{1e^-} |F(\omega)|^2 d\omega d\Omega$$
$$= N^2 \int_{\Delta\omega} \int_{\Delta\Omega_{\rm rad}} \beta e^2 \frac{\sin^2\theta J_0^2(ka\sin\theta)}{4\pi^2 c(1-\beta\cos\theta)^2 I_0^2(\frac{ka}{\beta\gamma})}$$
$$\times \left|\sqrt{\frac{1+\beta}{1+\cos\theta}} e^{-jk\ell(1-\beta\cos\theta)/\beta}\right|^2 |F(\omega)|^2 \cdot d\omega d\Omega. \quad (4)$$

In general, given a range of expected bunch lengths, several aspects have to be considered in choosing the best central wavelength of the detector for the measurement. The central frequency has to be chosen in the low frequency side of the main lobe of the longitudinal form factor, where differences of longitudinal profiles, for bunches with the same rms length, induce a negligible change in the $F(\omega)$. As an example, in our experimental case, the rms bunch length σ_z is 2.3 ps and the central frequency of the detector is 30 GHz. Figure 3 shows the longitudinal form factor of different bunch profiles, including rectangular, Gaussian, single horn, double horn, and a "real bunch profile," as measured by a rf deflecting cavity. All of them have the same rms bunch length of 2.3 ps. From the figure it is clear that at 30 GHz the measurement is not sensitive to the details of the longitudinal distribution. This is true for all the frequencies below 30 GHz. In order to apply the method to shorter electron bunches we could consider, e.g., $\sigma_z = 0.23$ ps, and evaluate which central frequency is best suited for the measurement. In this case all frequencies lower than or equal to 300 GHz will provide a measurement insensitive to the detail of the longitudinal profile. Figure 4 compares the normalized emitted energy W calculated using Eq. (4) for a central frequency of 30 GHz (solid line) and 300 GHz (dot-dashed line). The energy variation as a function of the bunch length is more pronounced at 300 GHz than at 30 GHz. For the ideal case of a noise free detector both frequencies could be adopted. The first practical limit of applicability depends on the signal to noise ratio of the measurement. Considering a noise level of 2% for both 30 and 300 GHz as the bunch gets shorter the signal variation will eventually become smaller than the noise level. In Fig. 4, for the 30 GHz, this situation corresponds to a bunch length of 0.5 ps, while for 300 GHz, it corresponds to about 0.03 ps. This means that for equally noisy detectors the choice of a higher central frequency is

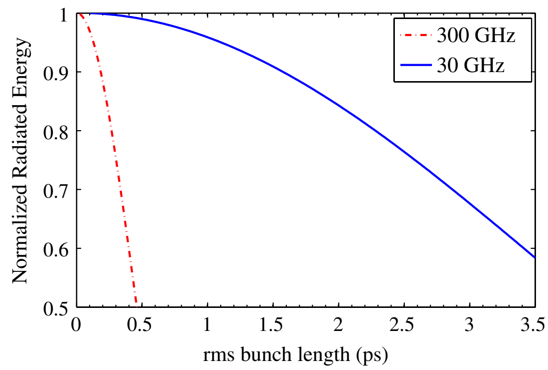

FIG. 4 (color online). Energy radiated by a rectangular electron bunch at 30 GHz, solid line, and at 300 GHz, dot-dashed line (both normalized to the asymptotic value).

better. When considering a comparison between frequencies in a real measurement setup, many factors have to be considered: the ratio of absolute intensities, the ratio of angular acceptance of the receiver (horn antennas), the specific bandwidth and the sensitivity of the detectors. Overall, the measurement performed at 300 GHz on an electron bunch with $\sigma_z = 0.23$ ps is expected to have a smaller signal to noise ratio than a measurement performed at 30 GHz on an electron bunch with $\sigma_z = 2.3$ ps. The significant advantage of the proposed method is that by exploiting the asymptotic behavior there is no need for an absolute detector calibration and for a detailed knowledge of the transfer function of the full detection system. Indeed, for pulsed operation in the spectral range from mm waves to THz, absolute calibrations are an issue. In the proposed method, linearity is the only requirement for the detection system.

The self-calibration method was tested on the FERMI@Elettra free electron laser, using the coherent bunch length monitor (CBLM) diagnostics [15] installed after the first magnetic chicane at about 330 MeV. The test is based on a direct comparison with the transverse deflecting cavity [16]. The CBLM system is equipped with a ceramic gap and a 30 GHz Schottky diode from Millitech Inc. model DXP-28. The magnetic chicane was operated at the nominal value of 85 mrad (corresponding to a $R_{56} = 41$ mm). When the upstream accelerating structures (linac L01) rf phase is set at 90° (i.e., on crest condition) the beam is not compressed and has a nominal rms bunch length of about 2.5 ps. Moving the L01 phase at higher values, the bunch length is progressively decreased, reaching at 115° a theoretical compression factor of about 5, producing sub-ps bunches. The highest value of the compression brings the diode output signal to the asymptotic value. For the emitted power levels the diode is working in its "square root of power" region. So for a coherent radiation depending on N^2 the output signal of the diode depends linearly on the charge. This was verified experimentally scanning the charge from 100 to 350 pC while keeping constant the bunch length (checked with the rf deflecting

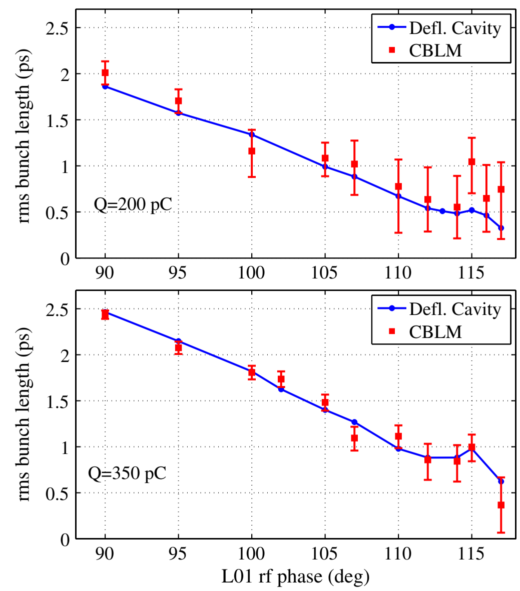

FIG. 5 (color online). Comparison of bunch length measurements performed with the rf deflecting cavity and with the CBLM. The error bars are the standard deviation calculated over 50 consecutive bunches at the same compression factor.

cavity installed downstream to the CBLM equipment). The self-calibration procedure consists of changing the compression factor until the rf diode signal reaches the asymptote and register this signal level as reference. At lower compression factor, the ratio between the rf diode signal and the asymptotic level is uniquely identified. Using the curve plotted in Fig. 4, which is derived from Eq. (4), one can convert the rf diode signal, normalized to the saturation level, in absolute rms bunch length.

In order to verify this method, we have varied the bunch charge and the compression factor and we have measured the bunch length with the rf deflecting cavity. For each L01 rf phase, corresponding to a given compression factor, the experimental data are obtained by averaging over 50 consecutive bunches acquired simultaneously with the CBLM and the rf deflecting cavity. Experimental results for electron bunches with a charge of 200 pC and of 350 pC are reported in Fig. 5. The CBLM absolute bunch length measurements are in very good agreement with the deflecting cavity measurements. The saturation of the CBLM signal is reached at 200 pC when the L01 rf phase is set at 114°. At 350 pC it is necessary to detune L01 down to 118°, since in the latter case the uncompressed bunch is longer. Nevertheless, in both cases the saturation condition is met when the rf deflecting cavity measures a rms bunch length close to 0.5 ps, confirming that this method does not depend on the bunch charge.

In conclusion, by exploiting the asymptotic dependence of the power emitted as a function of the bunch length and using a theoretical high frequency approximation of single particle emission, we propose a new method for self-calibrating a diagnostics based on coherent radiation emitted from a gap in a waveguide. The capability to provide absolute bunch length measurements was validated experimentally, proving the independence of the proposed method from the bunch charge. The results can be applied to different accelerators providing a shot by shot, nondestructive, absolute bunch length measurement with unprecedented simplicity.

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