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New Mass Limit for White Dwarfs: Super-Chandrasekhar Type Ia Supernova as a New Standard Candle

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Type Ia supernovae, sparked off by exploding white dwarfs of mass close to the Chandrasekhar limit, play the key role in understanding the expansion rate of the Universe. However, recent observations of several peculiar type Ia supernovae argue for its progenitor mass to be significantly super-Chandrasekhar. We show that strongly magnetized white dwarfs not only can violate the Chandrasekhar mass limit significantly, but exhibit a different mass limit. We establish from a foundational level that the generic mass limit of white dwarfs is 2.58 solar mass. This explains the origin of overluminous peculiar type Ia supernovae. Our finding further argues for a possible second standard candle, which has many far reaching implications, including a possible reconsideration of the expansion history of the Universe.

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Introduction.—Recently, some peculiar type Ia supernovae, e.g., SN 2006gz, SN 2007if, SN 2009dc, SN 2003fg, have been observed [1] with exceptionally higher luminosities but lower kinetic energies. The kinetic energy is proportional to the difference between the obtained nuclear energy, which arises from the synthesis of elements in the explosion through fusion, and the binding energy of the white dwarf. Most of the light curves of the above-mentioned peculiar supernovae appear to be overluminous and slow rising, which does not allow them to be calibrated as standard candles. This questions the use of all type Ia supernovae in measuring distances of far away regions and hence unraveling the expansion history of the Universe. However, assuming the progenitor to be a highly super-Chandrasekhar mass white dwarf reproduces the low kinetic energies and thus velocities seen in the above supernovae [2]. This is because a larger mass implies a larger binding energy of the star and hence a smaller velocity for the same and/or higher luminosity (due to nuclear fusions) than that observed in a standard type Ia supernova. The progenitor masses required to explain the above supernovae lie in the range 2.1–2.8$M_\odot$, with $M_\odot$ being the mass of sun, subject to the model chosen to estimate the nickel mass [1–6]. Now naturally the following vital questions arise. Is there any fundamental basis behind the formation of a highly super-Chandrasekhar white dwarf? How do we address the significant violation of the Chandrasekhar mass limit? What is the ultimate mass limit of a white dwarf? Here we plan to address all the above issues by exploiting the effects of the magnetic field in compact objects. This will lead, as we will show, to a natural explanation of the so-called "peculiar" type Ia supernovae. This might eventually lead these supernovae to be considered as altogether new standard candles. This has many far reaching implications, including a possible reconsideration of the expansion history of the Universe.

Before proceeding further, let us recall the physics of a white dwarf and its link to the type Ia supernova. When a star exhausts its nuclear fuel, it converts to one of the three compact objects: white dwarf, neutron star or black hole, depending on the initial mass of the evolving star. It is generally believed that its fate is a white dwarf when the mass of the initial star in its main sequence is $M_{\rm MS} \lesssim 5M_\odot$. Such a main sequence star undergoing collapse leading to a small volume consists of a lot of electrons. Being in a small volume, many such electrons tend to occupy same energy states, and hence they become degenerate, as the energy of a particle depends on its momentum which is determined by the total volume of the system. However, being a fermion, an electron obeys Pauli's exclusion principle which says that no two fermions can occupy the same quantum state. Hence, once all the energy levels up to the Fermi level (which is the maximum allowed energy of a fermion) are filled by the electrons, there is no available space for the remaining electrons in a small volume of a collapsing star, which expels the electrons to move out leading to an outward pressure. In a white dwarf, the inward gravitational force is balanced by the force due to outward pressure created by degenerate electrons. Chandrasekhar in one of his celebrated papers [7] showed that the mass of a white dwarf cannot be more than 1.44$M_\odot$, which sets the famous Chandrasekhar mass limit of a white dwarf.

If a white dwarf having mass close to the Chandrasekhar limit gains more mass (e.g., by accretion, when the mass is supplied by a companion star of the white dwarf), then its mass exceeds the Chandrasekhar limit, which leads to a gravitational force stronger than the outward force that arises due to the degenerate electrons. Hence, this leads to the contraction of the white dwarf and a subsequent increase of its temperature, which is favorable for the initiation of fusion reactions again. If the white dwarf mostly consists of carbon and oxygen, namely, the carbon-oxygen white dwarf (which is commonly the case), then nuclear fusion of carbon (and oxygen) takes place. Subsequently, within a few seconds, a substantial fraction of the white dwarf matter undergoes a runaway reaction which releases a huge amount of energy $\sim 10^{51}$ erg to unbind it in an explosion, namely type Ia supernova explosion. This eventually leads to a complete gravitational collapse of the star without leaving any remnant.

As all the commonly observed type Ia supernovae are produced by (almost) the same mechanism, namely the mass of the progenitor white dwarf exceeding the Chandrasekhar limit and subsequent processes, the underlying variations of luminosity as functions of time, namely light curves, appear alike for all the explosions. All of these supernovae exhibit consistent peak luminosity, the relation between the peak luminosity and width of the light curve [8,9], due to the uniform mass of white dwarfs (Chandrasekhar limit) which finally explode, e.g., because of the accretion process. Very importantly, the apparent stability of this value helps the underlying supernovae to be used as standard candles in order to measure the distances to their host galaxies. Since these supernovae are exceptionally bright, they can be observed across huge cosmic distances. The variation of their brightness with distance (or redshift) is an extremely important tool for measuring various cosmological parameters, which in turn shed light on the expansion history of the Universe. Their enormous importance is self-evident and was brought into prime focus by the awarding of the Nobel Prize in Physics in 2011, for the discovery (made possible by the observation of distant type Ia supernovae) that the Universe is undergoing an accelerated expansion [10,11].

Now we move on to our goal of establishing a new mass limit for super-Chandrasekhar white dwarfs, of whose formation there is no understanding from a foundational level—a caveat behind the hypothesis of the super-Chandrasekhar progenitor for the peculiar type Ia supernovae, raised by previous authors [2]. In fact they emphasized the pursuit of theoretical studies in order to assess the hypothesis. Although, based on a numerical code for stellar binary evolution, the rotating white dwarfs are suggested to hold mass up to $2.7M_\odot$ [12], a foundational level calculation is missing and there is no estimate of the mass limit of such a star either.

In this Letter, we show that (highly) magnetized white dwarfs not only can have mass $\sim 2.6M_\odot$, but also exhibit its ultimate limit of mass. Hence, we propose a fundamentally new mass limit for white dwarfs, which eventually helps in explaining light curves of peculiar type Ia supernovae. This may further lead to establishing these supernovae as modified standard candles for distance measurement.

The motivation behind our approach lies in the discovery of several isolated magnetized white dwarfs through the Sloan Digital Sky Survey (SDSS) with surface fields 10^5–10^9 G [13,14]. Hence their expected central fields could be 2–3 orders of magnitude higher. Moreover, about 25% of accreting white dwarfs, namely cataclysmic variables (CVs), are found to have high magnetic fields 10^7–10^8 G [15].

Equation of state.—As the aim is to exploit the effect of magnetic field in white dwarfs, we first recall degenerate electrons under the influence of a magnetic field, which are known to be Landau quantized [16]. The larger the magnetic field, the smaller is the number of Landau levels occupied [17] (see Supplemental Material [18]). Recent works [19–21] establish that Landau quantization due to a strong magnetic field modifies the equation of state (EoS) of the electron degenerate gas, which results in a significant modification of the the mass-radius relation of the underlying white dwarf. Interestingly, these white dwarfs are found to have super-Chandrasekhar masses. The main aim of this Letter is to obtain the maximum possible mass of such a white dwarf (which is magnetized), and therefore a (new) mass limit. Hence we look for the regime of high density of electron degenerate gas and the corresponding EoS, which further corresponds to the high Fermi energy (E_F) of the system. This is because high density corresponds to high momentum, which implies high energy (see Supplemental Material [18]). Note that the maximum Fermi energy ($E_{F\,\mathrm{max}}$) corresponds to the maximum central density of the star. Consequently, conservation of magnetic flux (technically speaking flux freezing theorem which is generally applicable for a compact star) argues for the maximum possible field of the system, which implies that only the ground Landau level will be occupied by the electrons. For the expressions of density, pressure and the EoS for such a highly magnetized system, see the Supplemental Material [18]. Hence, in the limit of $E_F \gg m_e c^2$, when m_e is the mass of the electrons and c the speed of light, for a given magnetic field exhibiting the system of one Landau level, the EoS is

$$P = K_m \rho^2, \tag{1}$$

when P and ρ are, respectively, the pressure and density of the gas and the constant K_m is given by

$$K_m = \frac{m_e c^2 \pi^2 \lambda_e^3}{(\mu_e m_H)^2 B_D}, \tag{2}$$

where $\lambda_e = \hbar / m_e c$, the Compton wavelength of electron, $\hbar$ the Planck's constant h divided by 2π, μ_e the mean molecular weight per electron, m_H the mass of hydrogen atom, and B_D the magnetic field in the units of 4.414×10^{13} G.

Mass limit of white dwarfs.—Now following the Lane-Emden formalism [22], we obtain the mass of the magnetized white dwarf

$$M = 4\pi^2 \rho_c \left(\frac{K_m}{2\pi G}\right)^{3/2}, \tag{3}$$

and the corresponding radius

$$R = \sqrt{\frac{\pi K_m}{2G}}, \tag{4}$$

when ρ_c is the central density of the white dwarf supplied as a boundary condition in addition to the condition that $d\rho/dr = 0$ at $r = 0$, when r is the radial distance from the center of the star such that at the surface $r = R$, and G is Newton's gravitation constant. See Supplemental Material [18] for detailed calculations.

Now the expression of ρ_c for a one Landau level system in the limit $E_F = E_{F\,\mathrm{max}} \gg m_e c^2$ is given by (see Supplemental Material [18])

$$\rho_c = \frac{\mu_e m_H}{\sqrt{2}\pi^2 \lambda_e^3} B_D^{3/2}. \tag{5}$$

Substituting ρ_c from Eq. (5) in Eq. (3), we obtain the mass of the white dwarf, independent of ρ_c and B_D, given by

$$M = \left(\frac{hc}{2G}\right)^{3/2} \frac{1}{(\mu_e m_H)^2} \approx \frac{10.312}{\mu_e^2} M_\odot, \tag{6}$$

and from Eqs. (2), (4), and (5), we obtain the radius

$$R = \left(\frac{\pi^{2/3} hc}{2^{7/3} (\mu_e m_H)^{4/3} G}\right)^{1/2} \rho_c^{-1/3} \rightarrow 0 \quad \text{as } \rho_c \rightarrow \infty, \tag{7}$$

which set the new limits for mass and radius, respectively. Note that the Chandrasekhar limit also corresponds to $\rho \rightarrow \infty$ and $R \rightarrow 0$. For $\mu_e = 2$, which is the case of a carbon-oxygen white dwarf,

$$M \approx 2.58 M_\odot. \tag{8}$$

Interestingly, while a high magnetic field introduces anisotropic effects into the star tending it to be an oblate spheroid, this does not affect the limiting mass as the corresponding radius tends to zero. However, for lighter white dwarfs with finite radii, the super-Chandrasekhar mass would have been achieved at a lower field, if the star is appropriately set to be a spheroid rather than a sphere, as justified earlier [19,20].

Scaling behaviors of the mass and radius of the white dwarf with its central density.—Now we provide general scaling laws for the mass and radius of the white dwarf describing their variations with its central density (as is known for nonmagnetized white dwarfs proposed by Chandrasekhar) and magnetic field strength. By Lane-Emden formalism

$$M \propto K^{3/2} \rho_c^{(3-n)/2n}, \tag{9}$$

where K depends on B_D and ρ_c for a magnetized white dwarf (see Supplemental Material [18]). For the extremely high density regime of the white dwarf, the EoS reduces to a polytropic form $P = K\rho^\Gamma$ (as can be verified from the Supplemental Material [18]), when $\Gamma = 1 + 1/n$, is the polytropic index. In this regime, $\Gamma = 2$, consequently $n = 1$ and $K = K_m \propto B_D^{-1} \propto \rho_c^{-2/3}$, as shown by Eqs. (1), (2), and (5). This finally renders M in Eq. (9) to be independent of ρ_c, as already shown by Eq. (6). Similarly, the scaling of radius is obtained as

$$R \propto K^{1/2} \rho_c^{(1-n)/2n}, \tag{10}$$

which reveals $R \propto \rho_c^{-1/3}$ for $n = 1$, as already shown by Eq. (7).

Justification of a high magnetic field in white dwarfs.—So far we have employed Landau quantization in search of a new mass limit giving rise to super-Chandrasekhar white dwarfs. However, the effect of Landau quantization becomes significant only at a high field $\sim B_D \times 10^{13}$ G $= B_{\mathrm{cr}}$. How can we justify such a high field in a white dwarf?

Let us consider the commonly observed phenomenon of a magnetized white dwarf accreting mass from its companion. Now the surface field of an accreting white dwarf, as observed, could be $\sim 10^9$ G $\ll B_{\mathrm{cr}}$ [13]. Its central field, however, can be several orders of magnitude higher $\sim 10^{12}$ G, which is also less than B_{cr}. Naturally, such a magnetized CV, commonly known as a polar, still lies on the mass radius relation obtained by Chandrasekhar. However, in contrast with Chandrasekhar's work (which did not include a magnetic field in the calculations), we will see that a nonzero initial field in the white dwarf, however ineffective for rendering Landau quantization effects, will prove to be crucial in supporting the additional mass accumulated due to accretion. As the magnetized white dwarf accretes mass, its total mass increases which in turn increases the gravitational power and hence the white dwarf contracts in size due to the increased gravitational pull. However, the total magnetic flux is conserved in such a process and, hence, as a result of the above decrease in the size of the star, the central (as well as surface) magnetic field also increases. Here we are interested in the evolution of the central field, since it is this field that is primarily responsible

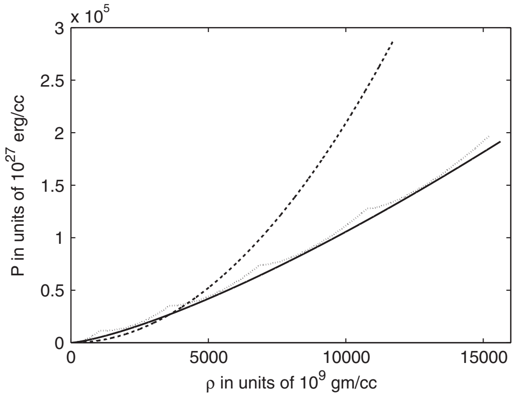

FIG. 1. Equation of states. The solid line represents Chandrasekhar's equation of state (corresponding to zero magnetic field and hence infinitely many Landau levels). The dotted and dashed lines represent the 5-level (corresponding to a very strong magnetic field) and 1-level (corresponding to the ultimate equation of state for an extreme magnetic field) systems of Landau quantization, respectively. $E_{F\,\mathrm{max}} = 200 m_e c^2$.

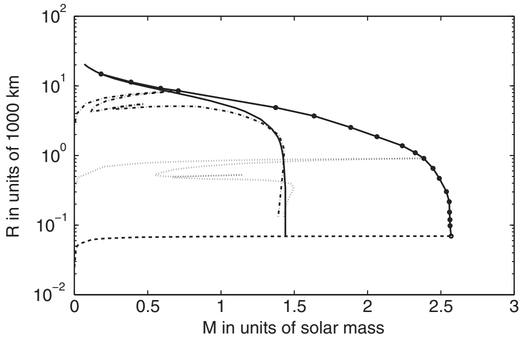

FIG. 2. Mass-Radius relations. The pure solid line represents Chandrasekhar's result and the one marked with filled circles represents the evolutionary track of the white dwarf with the increase of the magnetic field. The dot-dashed, dotted, and dashed lines represent the white dwarfs with 50 124-level, 200-level, and 1-level systems of Landau quantization, respectively (corresponding to increasing magnetic fields). $E_{F\,\mathrm{max}} = 200 m_e c^2$.

for rendering super-Chandrasekhar mass to the white dwarf, as justified in Ref. [19]. Since accretion is a continuous process, the deposition of matter on the surface of the white dwarf, followed by its contraction and subsequent increase of magnetic field, continues in a cycle. In such a process, eventually the central magnetic field could exceed B_{cr}. As a result, the EoS of the electron degenerate matter gets modified as shown in Fig. 1. Hence, the inward gravitational force is balanced by the outward force due to this modified pressure, and a quasiequilibrium state is attained. In this way, a very high magnetic field is generated, which in turn prevents the white dwarf from collapsing, thus making it more massive. Subsequently, with the continuation of accretion the white dwarf approaches the new mass limit $\sim 2.58 M_\odot$, as obtained above, which sparks off a violent thermonuclear reaction with further accretion, thus exploding it and giving rise to a super-Chandrasekhar type Ia supernova. The evolution of the mass-radius relation of a polar into that of a super-Chandrasekhar white dwarf of the maximum possible mass is shown in Fig. 2, along with a few typical mass-radius relations for different magnetic field strengths describing possible stars in intermediate quasiequilibrium states. The ultimate white dwarf, corresponding to the maximum mass $\sim 2.58 M_\odot$, lies on the mass-radius relation for a one Landau level system, but the intermediate white dwarfs having weaker magnetic fields correspond to multilevel systems. The one Landau level system corresponds to the central magnetic field 8.8×10^{17} G. The intermediate systems of the 200 level and 50 124 level correspond to central magnetic fields 4.4×10^{15} G and 1.7×10^{13} G, respectively (see Supplemental Material [18] for relevant formula), when $E_{F\,\mathrm{max}} = 200 m_e c^2$. This value of $E_{F\,\mathrm{max}}$ is found to produce the theoretical mass limit with good numerical accuracy. Note that one can in principle go up to higher $E_{F\,\mathrm{max}}$, which will lead to a further decrease in the radius of the white dwarf keeping the mass practically the same. In order to construct this evolutionary track, we consider the values of ρ_c corresponding to the density at the ground-to-first Landau level transition of the respective EoSs, since only this choice leads to the maximum possible mass. We emphasize here that the range of masses for the super-Chandrasekhar progenitors obtained from observations is not very strict. Hence, if the new mass limit obtained by us is taken into account, one could possibly do away with the mass distribution altogether. However, if the distribution is indeed real, then it could be attributed to the difference in accretion rates found in different CVs.

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