Collectively Enhanced Interactions in Solid-State Spin Qubits

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We propose and analyze a technique to collectively enhance interactions between solid-state quantum registers composed from random networks of spin qubits. In such systems, disordered dipolar interactions generically result in localization. Here, we demonstrate the emergence of a single collective delocalized eigenmode as one turns on a transverse magnetic field. The interaction strength between this symmetric collective mode and a remote spin qubit is enhanced by the square root of the number of spins participating in the delocalized mode. Mediated by such collective enhancement, long-range quantum logic between remote spin registers can occur at distances consistent with optical addressing. A specific implementation utilizing nitrogen-vacancy defects in diamond is discussed and the effects of decoherence are considered.

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Harnessing collective phenomena by utilizing ensembles of identical particles is a powerful tool, which has been exploited in effects ranging from superradiance to scattering suppression [1]. The coherent dynamics resulting from interactions with individual constituents of an ensemble are often too weak to be observed directly; however, as evidenced by experiments in systems such as Rydberg atoms [2–4], cavity QED [5,6], atomic ensembles [7,8], and solid-state qubits [9], collective enhancement provides a natural route to overcoming this challenge. Here, we demonstrate that, for electronic spin quantum registers, such collective effects enable an extended coherent coupling over large distances—an essential prerequisite for quantum information processing.

Owing to favorable coherence properties, electronic spins associated with pointlike defects in solid-state systems have garnered significant recent interest as candidates for room-temperature quantum registers. Quantum control of such spins can be achieved using a combination of optical, magnetic, and electric fields. While our considerations apply to a variety of electronic spin qubits [10–12], here we focus on the nitrogen-vacancy (NV) center in diamond. The NV center harbors an electronic spin ($S = 1$), which can be optically initialized, coherently manipulated, and readout on subwavelength scales [13–15]. These results have sparked several recent proposals which utilize networks of NV registers as the platform for a scalable quantum information processor [16–19]. However, for any spin qubit candidate, two crucial challenges remain to be addressed: (1) the weakness of the magnetic dipolar interactions on distances compatible with individual optical addressing and (2) the disorder in spin positioning due to inherent imperfections during defect creation.

In this Letter, we present a novel approach to remote quantum logic which harnesses collectively enhanced interactions to overcome both of the above challenges. The key idea underlying our proposal is to associate a single, robust qubit with a collective, generally disordered spin ensemble (Fig. 1). If the spins behave in an aggregate fashion, such a qubit can produce a large state-dependent magnetic field, leading to enhanced long-range coupling between ensembles; this is reminiscent of tailored light-matter interactions achieved via atomic ensembles [20]. However, we note that quenched disorder naturally leads to localization in solid-state spin systems [21] due to random flip-flop interactions. Similar to Anderson localization [22], this implies that each eigenmode of the ensemble is composed of only a few spins. Here, we demonstrate the use of a uniform transverse magnetic field to overcome this issue. The applied field causes the symmetric W state [23,24] to become an approximate eigenstate of the Hamiltonian, thereby enabling us to harness it as a collective qubit. Moreover, we show that this particular state is largely insensitive to the underlying spin distribution and hence robust to effects of disorder.

To be specific, we now describe our proposal in the context of NV diamond color centers. The largest energy scale in this system (Δ) is set by a combination of the zero-field splitting (2.87 GHz) and the projection of

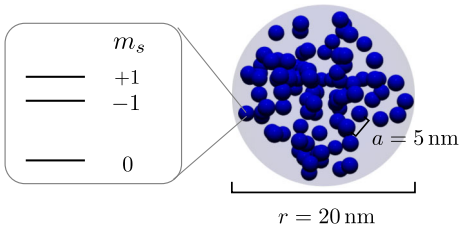

FIG. 1 (color online). High-density NV spin ensemble distributed randomly within a sphere of diameter r, with an average distance a. The NV centers have three internal spin states that are split by a zero-field splitting and a Zeeman field.

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the external Zeeman field along the NV axis. However, we would like to stress that the alignment of the field with the NV axis is not crucial, as its quantization axis is essentially given by the zero-field splitting. We assume that the Zeeman field is sufficiently strong to ensure that the $m_s = -1$ spin state is sufficiently far detuned and hence does not contribute to the effective dynamics. Thus, the number of $m_s = 1$ spins, m, is an approximately good quantum number and a perturbative description is justified. The second-largest energy scale arises due to the perturbation created by the transverse field Ω. To gain a qualitative understanding, let us restrict ourselves to the analytically tractable case where m is either 0 or 1. The effective Hamiltonian is $H_r = -\Delta|0\rangle\langle 0| + \sqrt{N}\Omega(|0\rangle\langle W| + \text{H.c.})$, where the state $|0\rangle$ has all ensemble spins polarized into $m_s = 0$, and the collective $|W\rangle$ state is fully symmetric with all spins sharing a single excitation,

$$|W\rangle = \frac{1}{\sqrt{N}}\sum_i |0\ldots 1_i \ldots\rangle. \tag{1}$$

Second order perturbation theory in $\sqrt{N}\Omega/\Delta$ yields $H_r' = -(\Delta + J)|\tilde{0}\rangle\langle\tilde{0}| + J|\tilde{W}\rangle\langle\tilde{W}|$, with $J = N\Omega^2/\Delta$ and the tilde referring to the perturbed eigenstates. Including higher m manifolds merely leads to a renormalization of J, without changing this qualitative picture (so long as we are in the perturbative limit). This is equally true in the presence of dipolar interactions, provided that the energy scale J is larger than the characteristic strength of the dipolar interaction V_{dd}. Thus, even with these additional terms, the new eigenstates will still have substantial overlap with the collective $|W\rangle$ state. This is in stark contrast to the situation without a transverse field, where strongly quenched disorder owing to random spin positions localizes all such eigenstates, even in three dimensions. Furthermore, the dipolar interaction naturally ensures that collective states with different m values will have different energies, leading to a "blockade-type" scenario, where manifolds with $m > 1$ are energetically inaccessible [23,24]. This allows us to selectively drive the transition between $|0\rangle$ and $|W\rangle$ without populating any other collective states, provided that the external driving Ω_{ext} is weaker than V_{dd}. This hierarchy of energy scales can be summarized as $\Delta \gg J \gg V_{dd} \gg \Omega_{\text{ext}}$.

Let us consider a three-dimensional ensemble of $N = 100$ NV centers randomly distributed within a diameter $r = 20$ nm, as depicted in Fig. 1. Such high density NV ensembles have been recently realized using long-time annealing of repeat-electron-irradiated diamond samples [25–28]. We will characterize our effective two-level system ($m_s = 0$, 1) using Pauli spin operators σ_α. Being magnetic dipoles, NV centers interact with one another via long-range magnetic dipolar interactions (ignoring energy nonconserving terms which are suppressed by the NV center's zero-field splitting),

$$V_{ij} = (1 - 3\cos^2\vartheta_{ij})\frac{\mu^2}{|\mathbf{r}_i - \mathbf{r}_j|^3} \times \left\{\frac{1}{4}[1 + \sigma_z^{(i)}][1 + \sigma_z^{(j)}] - \sigma_+^{(i)}\sigma_-^{(j)} - \sigma_-^{(i)}\sigma_+^{(j)}\right\}, \tag{2}$$

where $\mathbf{r}_i$ denotes the position, μ characterizes the magnetic dipole moment, and ϑ_{ij} is the angle between the NV axis and the vector connecting sites $\mathbf{r}_i$ and $\mathbf{r}_j$. The total Hamiltonian including both on-site and interaction terms is then given by $H = \Delta/2\sum_i \sigma_z^{(i)} + \Omega\sum_i \sigma_x^{(i)} + \sum_{i<j} V_{ij}$.

Now, let us turn to the enhanced coupling between an isolated NV defect (hereafter termed "qubit") and the collective ensemble, separated by the distance R. We envision the ensemble to be initialized into the $|0\rangle$ state, while the NV qubit is initialized to the $m_s = 1$ state. By ensuring that the qubit splitting is tuned resonant with only the $|W\rangle$ state, one finds that the effective dynamics are restricted to the single-excitation manifold of the combined qubit-ensemble system; to lowest order, these dynamics are governed by

$$H_{\text{eff}} = \sqrt{N_c}\frac{\mu^2}{R^3}(|1_q, 0\rangle\langle 0_q, W| + \text{H.c.}), \tag{3}$$

where N_c characterizes the approximate number of spins participating in the $|W\rangle$ state and the notation $|1_q, 0\rangle$ refers to the combined state with the NV qubit being in $m_s = 1$ and with the ensemble spins being in $|0\rangle$. Consistent with subwavelength techniques such as stimulated emission depletion microscopy ($R = 100$ nm), we will assume that the NV qubit can be manipulated and readout independently of the ensemble [29].

To support the qualitative picture presented above, we now perform exact diagonalization of the interacting spin Hamiltonian. In the majority of the numerics, we restrict ourselves to $m \leq 2$ excitations; however, we check the validity of our results by including the $m = 3$ manifold for slightly smaller system sizes. For each eigenstate $|\phi\rangle$, we calculate the collective enhancement factor, defined as

$$N_c = \left(\sum_i^N \langle 0_1 \ldots 1_i \ldots 0_N|\phi\rangle\right)^2, \tag{4}$$

which for a symmetric eigenmode characterizes the number of participating ensemble spins. As expected, in the absence of a transverse field, disorder localizes all eigenstates and, as depicted in Fig. 2 [dark gray (blue) circles], $N_c \ll N$ for all eigenstates. On the other hand, In the case of a moderate transverse field $\Omega = \mu B$, with $B \approx 40$ G, one finds the existence of a single eigenstate with $N_c \approx 70 \sim N$. While the specific details of this state depend on the microscopic details (e.g., spin distribution within the ensemble, higher-order couplings to the $m_s = -1$ state, and magnitude of the applied transverse field), its collective nature is rather robust. In particular, as one varies the strength of the transverse field B, there exists a large parameter regime where $N_c > 50$ (Fig. 2). This result

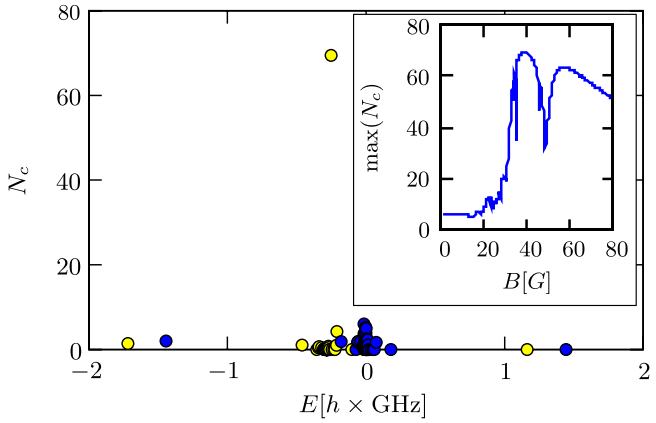

FIG. 2 (color online). Comparison of the collective enhancement N_c for $B = 0$ [dark gray (blue)] and $B = 40$ G [light gray (yellow)] within the single excitation manifold. In the latter case there is a collectively enhanced state with $N_c \approx 70$, corresponding to an increase by more than 1 order of magnitude ($\Delta = h \times 4$ GHz). The inset shows the maximum value of N_c depending on the transverse field strength B.

clearly supports our previous analytical arguments on the existence of a symmetric collective mode. The dips in N_c are associated with resonance effects, which arise when other eigenstates become near-degenerate with the collective state. Finally, the decrease of N_c for large values of Ω signals the breakdown of perturbation theory as $\sqrt{N}\Omega/\Delta$ approaches unity.

We now perform simulations of the combined qubit-ensemble system. As previously discussed, the system is initialized to $|1_q, 0\rangle$ and the qubit splitting is tuned resonant with the energy of the collective mode; the resulting dynamics is evinced in Fig. 3. Interestingly, the probability of

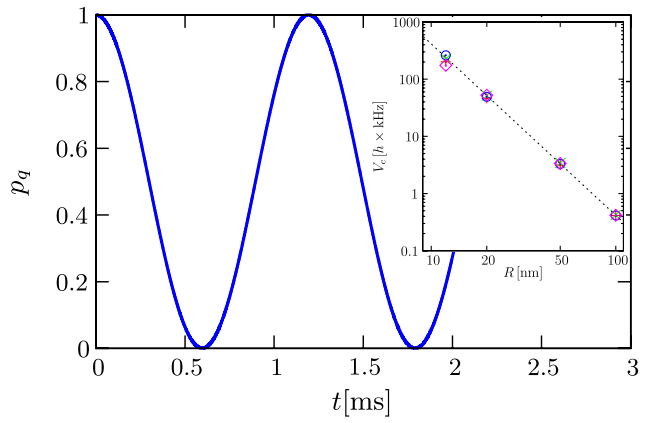

FIG. 3 (color online). Collectively enhanced Rabi oscillations between an isolated NV qubit and a NV ensemble. The probability to find the qubit in the $m_s = 1$ state, p_q, goes to zero within a time $t_\pi \approx 600$ μs. The inset shows the collectively enhanced coupling strength V_c between the qubit and the ensemble for four different realizations. The dashed line shows the asymptotic $1/R^3$ dependence.

finding the qubit in the $m_s = 1$ state, p_q, exhibits collectively enhanced Rabi oscillations. The frequency of these oscillations is enhanced by nearly an order of magnitude relative to that expected for bare dipolar interactions between two individual NV qubits at a similar distance. The numerics also allow us to obtain the time required for an interaction-induced π pulse, t_π, and from this, one can derive the effective distance R associated with H_{eff}. Surprisingly, in all cases, we observe that this distance corresponds not to R-r, but instead to the distance between the NV qubit and the *center* of the ensemble. We study the effects of putting the qubit closer to the ensemble by calculating the collectively enhanced coupling strength V_c (as extracted from the numerically obtained t_π). As shown in Fig. 3, we find that only for distances very close to the ensemble does the collective enhancement deviate from the asymptotic $1/R^3$ scaling; e.g., the qubit is coupled to individual spins rather than to the entire ensemble.

Experimental realization and decoherence.—Thus far, our discussion has assumed that both the NV qubit and the ensemble spins are perfectly decoupled from the environment. In any experimentally realistic scenario, however, there are two natural error sources which will be present: spin decoherence and spin relaxation. We are particularly interested in the scaling of the error rates with N, as this may adversely affect the scaling fidelity of our proposed long-range gates [19]. As the error processes act locally on individual spins, we first calculate the error rate for a single spin and multiply the result by N to obtain the rate for the collective state. For simplicity, we assume that the collective state is the previously described $|W\rangle$ state in which a single excitation is shared among all N spins.

First, let us consider the effects of spin decoherence. The worst-case scenario for such decoherence is given by the leaking out into nonsymmetric states. Consequently, the error probability after a single T_2 decoherence event on spin i is given by the probability to leave the $|W\rangle$ state,

$$p_{\bar{W}} = p_{T_2}[1 - |\langle W|\sigma_z^{(i)}|W\rangle|^2] = \frac{4}{N} p_{T_2}\left(1 - \frac{1}{N}\right), \quad (5)$$

where p_{T_2} is the single spin decoherence rate. For large N, this result is essentially independent of N (after weighing with the number of spins); therefore, the effect of T_2 processes on such a collective $|W\rangle$ state does not get enhanced by system size and, in fact, is only slightly worse than for a single spin; i.e., it can be expressed in terms of an effective coherence time T_2^{eff}.

Second, we consider the errors arising from phonon-induced spin relaxation processes (T_1). Here, we must distinguish between processes which flip an ensemble spin from $m_s = 1$ to $m_s = 0$, and the reverse. This asymmetry can easily be seen by noting that the $|W\rangle$ state has only one spin in $m_s = 1$, while all other spins are in $m_s = 0$. We denote the error probability associated with these two events as $p_{T_1}^{1\to 0}$ and $p_{T_1}^{0\to 1}$, respectively. For $p_{T_1}^{1\to 0}$,

the state $|0\rangle$ with all ensemble spins in $m_s = 0$ is not affected at all, while the probability to flip from the $|W\rangle$ state into $|0\rangle$ is given by

$$p_{W\to 0} = p_{T_1^{1\to 0}} |\langle 0|\sigma_-^{(i)}|W\rangle|^2 = \frac{p_{T_1^{1\to 0}}}{N}, \tag{6}$$

which is again independent of the size of the ensemble after rescaling with N.

However, this is not the case for $T_1^{0\to 1}$ processes. Both the $|0\rangle$ and the $|W\rangle$ state are strongly affected by such processes, since the existence of any additional spin in the $m_s = 1$ state corresponds to an effective magnetic impurity; this impurity modifies the energy of the collective state, thus tuning it out of resonance with the NV qubit. Additionally, this new state is also no longer an eigenstate of the Hamiltonian; numerical simulations demonstrate that this state dephases very quickly due to dipolar interactions within the ensemble. Thus, since any single spin $T_1^{0\to 1}$ error will immediately decohere the collective state, the effective error rate owing to $p_{T_1}^{0\to 1}$ is enhanced by N and scales with the size of the ensemble.

While the system size scaling of $p_{T_1}^{0\to 1}$ errors might seem unfortunate, in solid-state spin systems, it is often the case that $T_1 \gg T_2$. Our proposed protocol is particularly useful in cases where T_1/N remains longer than T_2, implying that the ensemble's noise is dominated by decoherence as opposed to the enhanced relaxation. The specific example of NV centers highlights this crucial point. The decoherence of the NV originates from fluctuating magnetic fields as neighboring pairs of dipoles flip-flop [30,31]. Even at low temperatures it is impossible to freeze out such magnetic fluctuations and T_2 remains on the order of milliseconds [30,32]. On the other hand, the relaxation of the NV is thought to originate from an Orbach spin-phonon process; such a process has an *exponential* dependence on temperature and implies that even moderate cooling can yield exceedingly long T_1 times ($\gg 1$ s at cryogenic temperatures) [33–35]. By liquid nitrogen temperatures, the errors introduced by the enhanced T_1 processes are already subpercent, enabling us to focus on the effects of decoherence. An alternate approach to combat the enhanced relaxation of the collective state is to utilize conventional dynamical decoupling techniques [e.g., Waugh-Huber-Haeberlen (WAHUHA)] [36] to suppress dipolar interactions within the individual ensembles.

Collective quantum gates.—We now turn to a possible application where isolated NV qubits are interspersed with high-density NV ensembles, forming a regular structure, as depicted in Fig. 4(a). The qubits are used for initialization, single-qubit rotations, and readout. Two-qubit gates between remote spin qubits are mediated by the ensemble between them and thus benefit from collectively enhanced interactions. We would like to point out that such architectures put only modest requirements on the positioning of the NV centers; in particular, the positional disorder within the NV ensembles is essentially irrelevant. The gate time t_g is limited by the SWAP time t_π required to transfer the information from one of the qubits to the ensemble (required four times per gate operation) [37]. The resulting error (assuming $T_1/N \gg T_2$) of the gate is given by $\varepsilon = 1 - \exp[-(4t_\pi/T_2^{\text{eff}})^3]$ in the presence of spin echo decoupling [38]. For an error of $\varepsilon = 10^{-2}$, this translates to a required coherence time of $T_2^{\text{eff}} = 11$ ms, which can be readily realized in isotopically pure diamond samples [30,32] or by using dynamical decoupling pulses [31,39]. The requirements on the coherence time can be further relaxed by increasing the number of spins in the ensemble or by reducing the qubit-ensemble separation.

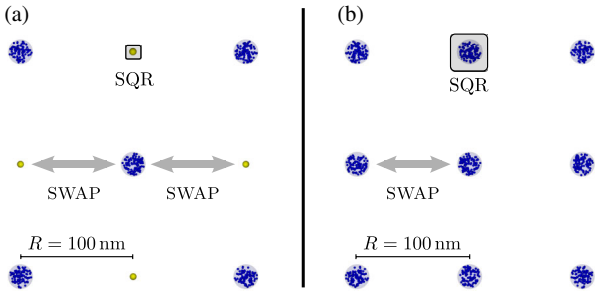

FIG. 4 (color online). Scalable architectures with collectively enhanced interactions, corresponding to a lattice spacing of $R = 100$ nm, compatible with subwavelength optical addressing. (a) Individually addressable NV qubits [light gray (yellow)] are used for single-qubit operations (SQR), whereas the collectively enhanced interaction with an ensemble is used to mediate two-qubit gates via SWAP operations. (b) NV ensembles are used as collective qubits, where also single-qubit operations are performed using the collective $|W\rangle$ state.

An architecture featuring even better gate fidelities can be realized using a collective encoding scheme for the qubits [see Fig. 4(b)]. There, the logical $|0\rangle$ state corresponds to all nuclear spins being polarized, while the logical $|1\rangle$ state is a collective nuclear spin $|W\rangle$ state. This state can be prepared by applying a microwave pulse to map the electronic $|W\rangle$ state onto a nuclear spin $|W\rangle$ state [24]. The time scale for such a single-qubit operation is limited to approximately 100 kHz by the hyperfine splitting of the NV centers in the $m_s = 1$ state ($A_\parallel \approx -2.14$ MHz for ^{14}N) [40]. Operation at cryogenic temperatures allows for resonant readout of the $|W\rangle$ state via the zero phonon line without significant background fluorescence [41]. In this collective qubit architecture, two-qubit gates between ensembles are enhanced by a factor of N instead of $\sqrt{N}$, thus leading to a SWAP time of $t_\pi = 70$ μs. Thus, we find that a gate error of $\varepsilon = 10^{-2}$ requires a coherence time of $T_2^{\text{eff}} = 700$ μs, while for $\varepsilon = 10^{-4}$, a coherence time of $T_2^{\text{eff}} = 3$ ms is needed [30,32,42,43].

In summary, we have shown that collectively enhanced interactions can be realized between a NV qubit and a mesoscopic NV ensemble. Our proposed approach relies upon a transverse magnetic field to inhibit the localization

of a symmetric W eigenstate. Our work enables the realization of collectively enhanced quantum gates with high fidelity and provides an important step towards the realization of scalable quantum information architectures involving solid-state electronic spins.

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