Nonlinear Optics Quantum Computing with Circuit QED

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One approach to quantum information processing is to use photons as quantum bits and rely on linear optical elements for most operations. However, some optical nonlinearity is necessary to enable universal quantum computing. Here, we suggest a circuit-QED approach to nonlinear optics quantum computing in the microwave regime, including a deterministic two-photon phase gate. Our specific example uses a hybrid quantum system comprising a LC resonator coupled to a superconducting flux qubit to implement a nonlinear coupling. Compared to the self-Kerr nonlinearity, we find that our approach has improved tolerance to noise in the qubit while maintaining fast operation.

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Linear optics quantum computing has proven to be one of the conceptually simplest approaches to building novel quantum states and proving the possibility of quantum information processing. It relies on the robustness of linear optical elements, but implicitly requires an optical nonlinearity [1–4]. Unfortunately, progress towards larger scale systems remains challenging due to the limits to optical nonlinearities, such as the measurement of single photons [5,6].

In this Letter, we suggest recent advances in circuit QED in which optical and atomiclike systems in the microwave domain are explored for their novel quantum properties, provides a new paradigm for photon based quantum computing [7–9], which, in contrast to linear optics quantum computing, is deterministic. Specifically, using superconducting nonlinearities in the form of Josephson junctions in flux and phase qubits [10,11], key elements of our approach have been realized: the creation of microwave photon Fock states [9,12–14], controllable beam splitters [9,15], and single microwave photon detection [16,17]. In many cases, photons stored in a transmission line or inductor-capacitor resonator have much better coherence times than the attached superconducting qubit (SQ) [18–20]. This suggests that the main impediment to photon based quantum computing is the realization of appropriate photon nonlinearities to enable two-qubit gates like two-photon phase gates, which are sufficient for universal quantum computation [1,21].

The key element of a two-photon phase gate is a two-photon nonlinear phase shifter. It imparts a π phase on any state consisting of two photons, leaving single photon and vacuum states unaffected. A deterministic approach to such photon nonlinearity is based on the Kerr effect [18,22–24]. In the context of circuit QED, in Ref. [22], a four level N scheme using a coplanar waveguide resonator and a Cooper pair box is used to arrange for electromagnetically induced transparency [25] to generate large Kerr nonlinearities. In this Letter, we explore the possibility of using a dc superconducting quantum interference device (SQUID) [26] to implement a nonlinear coupling between qubit and resonator, which, through an adiabatic scheme, enables a high fidelity, deterministic two-photon nonlinear phase shift in the microwave domain. Along with the nonlinearity, we envision using a dynamically controlled cavity coupling to implement a 50/50 beam splitter operation to construct a two-photon phase gate using dual-rail photon qubits [9,27], in which the logical basis $\{|0\rangle_L = |01\rangle, |1\rangle_L = |10\rangle\}$ corresponds to the existence of a single photon in one of two resonator modes [Fig. 1(d)]. Our approach takes advantage of relatively long coherence times for microwave photons in resonators, and couples only virtually to SQ devices, minimizing noise and loss due to errors in such devices. The combination of the aforementioned techniques for Fock state generation and detection and dynamically controlled beam splitters provides the final element for nonlinear optics quantum computing in the microwave domain.

We now consider photons stored in a high-impedance microwave resonator [28] coupled inductively with strength $0 < \chi < 1$ to a flux SQ in a dc SQUID configuration [Fig. 1(a)]. The resonator loops around the SQUID resulting in a nonlinear cosine dependent interaction with the qubit. In this configuration, we get an effective coupling of the form $V \sim E_J \cos(\hat{\phi} + \phi'_x)\cos\hat{\phi}_r$, where an external flux $\phi'_x \equiv 2\pi\chi\Phi'_x/\Phi_0$ is applied to the resonator which consequently threads the smaller loop of the SQUID, Φ_0 being the superconducting flux quantum. The qubit phase variable and the resonator flux are denoted by $\hat{\phi}$ and $\hat{\phi}_r = 2\pi\hat{\Phi}_r/\Phi_0$, respectively. For $\phi'_x \sim \pi/2$, we see immediately a nonlinear coupling between the qubit and resonator: $V \sim E_J\hat{\phi}\hat{\phi}_r^2$, where two resonator photons can be annihilated to produce one qubit excitation, analogous to parametric up conversion in $\chi^{(2)}$ systems. This results in a coupling of two resonator photons with a single qubit excitation with strength g_2 [Fig. 1(c)]. In essence, in this region, the two-photon state with detuning δ from the

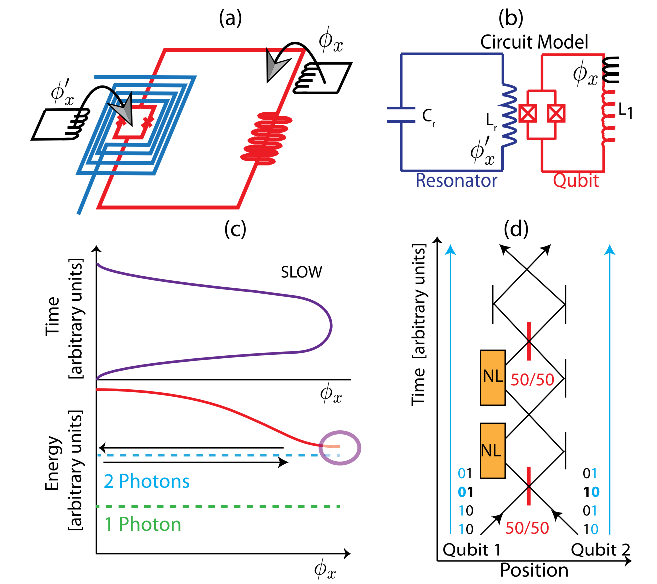

FIG. 1 (color online). (a) Implementation of a high-impedance resonator (blue) coupled to a dc SQUID (red) with an inductive outer loop. (b) A simple circuit model of our physical implementation. (c) Bottom: Energy levels of the coupled system with a sizeable two-photon coupling. Top: Suggested flux bias pulse ϕ_x to implement the nonlinear phase shift; a fast but adiabatic sweep and then a slow variation near the avoided crossing. (d) Use of two nonlinear phase shifters (NL), combined with 50/50 beam splitters, leads to a deterministic two-photon phase gate using dual-rail logic. The two photons in the dual-rail basis $|0\rangle_L|1\rangle_L = |01\rangle_1|10\rangle_2$ of the qubits become bunched into a single mode after passing through the first beam splitter, receiving a π phase from the nonlinear phase shifter. Storage cavities [not shown in (b)] are vertical arrows.

qubit, becomes slightly qubitlike and acquires some nonlinearity. However, the single-photon state, in spite of its coupling to the first SQ excitation with strength g_1, remains mostly photonlike because it is far detuned by Δ. At the end of the procedure, this leads to an additional phase for the two-photon state. The coupling of the two-photon state to other modes arises via linear coupling at $O(g_1)$ and is assumed to be far detuned.

The noise in the SQ, with a decay rate γ of its first excitation, may slightly limit our nonlinear phase accumulation. Although the system is mostly in the photonlike regime with decay rate κ, there will be an additional probability for it to decay due to its coupling to the lossy qubit. In the limit where $|\delta| \gg |g_2|$ and $|\Delta| \gg |g_1|$ with $|\Delta| > |\delta|$, the two-photon nonlinearity goes like g_2^2/δ, and the two-photon state decays approximately at a rate $\gamma g_1^2/\Delta^2 + \gamma g_2^2/\delta^2$. Thus, the losses due to the qubit go like γ/δ provided we allow g_1 to become close to g_2, which is possible by controlling ϕ'_x. Hence, at large detuning, we will then be limited only by κ. In contrast, a Kerr nonlinearity scales like g_1^4/δ^3 and the noise scales like $\gamma g_1^2/\delta^2$, leading to more qubit-induced loss at large detuning.

We now examine a detailed model to support these qualitative arguments. In our case, the second resonator is not coupled to a SQ and is not shown; we focus on the dynamics of the first resonator, which is coupled. The quantum Hamiltonian of the system is [29]

$$H = \left[\frac{\hat{q}_r^2}{2C_r} + \frac{\hat{q}^2}{2C_J} - \frac{\chi}{2C_r}\hat{q}\hat{q}_r\right] + \left(\frac{\Phi_0}{2\pi}\right)^2 \frac{\hat{\phi}_r^2}{2L_r} - E_J[\cos(\hat{\phi} + \chi\hat{\phi}_r + \phi'_x) + \cos\hat{\phi}] + \frac{E_L}{2}(\hat{\phi} + \phi_x)^2, \tag{1}$$

where the last three terms represent the potential energy. In addition to ϕ'_x, an external flux $\phi_x = 2\pi\Phi_x/\Phi_0$ is applied to the outer inductive loop of the squid. The canonical coordinates of the qubit satisfy $[\hat{\phi}, \hat{N}] = i$, where $\hat{N} = \hat{q}(2e)^{-1}$ is the number of Cooper pairs in the junctions. The operators $\hat{\Phi}_r$ and $\hat{q}_r$ represent quantum fluctuations in flux and charge of the resonator satisfying $[\hat{\Phi}_r, \hat{q}_r] = i\hbar$, and χ is the fraction of the flux $\hat{\Phi}_r$ threading the SQUID loop. This inductive coupling causes the effective capacitances of the resonator and qubit to be modified to C_r and C_J, respectively. E_J is the Josephson energy of each junction, while $E_L = \Phi_0^2/(4\pi^2 L_1)$ represents the inductive energy of the qubit due to the bigger loop. We define an effective charging energy of the junction to be $E_C = (2e)^2 C_J^{-1}$ and introduce another dimensionless parameter $\mu = 2\pi\Phi_0^{-1}\Phi_r^0$, where $\Phi_r^0 = \sqrt{L_r\omega\hbar/2}$ is the width of quantum fluctuations in the resonator flux. In terms of the quantum of conductance $G_0 = 2e^2/h$ and the characteristic impedance of the resonator $Z = (L_r/C_r)^{1/2}$, we can write $\mu = \sqrt{2\pi G_0 Z}$. Since $\mu \ll 1$, we can expand V in powers of $\chi\hat{\phi}_r \propto \mu$. Performing the expansion to second order, we get $H = H_r + H_q + V_I$ with

$$H_r = \frac{\hat{q}_r^2}{2C_r} + \frac{\hat{\Phi}_r^2}{2L_r},$$
$$H_q = \frac{\hat{q}^2}{2C_J} - 2E_J\cos\left[\frac{\phi'_x}{2}\right]\cos\left[\hat{\phi} + \frac{\phi'_x}{2}\right] + \frac{E_L}{2}(\hat{\phi} + \phi_x)^2,$$
$$V_I = \chi E_J\left[\hat{\phi}_r\sin(\hat{\phi} + \phi'_x) + \frac{\chi\hat{\phi}_r^2}{2}\cos(\hat{\phi} + \phi'_x)\right] - \frac{\chi\hat{q}\hat{q}_r}{2C_r}, \tag{2}$$

corresponding to the resonator, qubit, and interaction terms. We remark that asymmetry in the Josephson junctions leads to additional terms, but our general linearization approach described below remains valid, and provides qualitatively similar results.

In the regime $E_L \gg E_J$ we can linearize the potential term in (1) around the classical values of the resonator reduced flux $\phi_{\rm cl}$ and the qubit phase $\beta_{\rm cl} = -\phi_x + f$, with quantum fluctuations $\hat{\varphi}_r$ and $\hat{\varphi}$ around them. Any nonlinearity can then be treated perturbatively. We note that $\phi_{\rm cl}$, f, r, s, t, and u are all known functions of ϕ_x and

ϕ'_x which arise from linearization, and will not be mentioned explicitly. With the effective inductance of resonator $\tilde{L}_r^{-1} = L_r^{-1} + (2\pi/\Phi_0)^2 E_J \chi^2 u$ and $\hat{\Phi}_r = \Phi_0/(2\pi)\hat{\varphi}_r$, the resonator and qubit Hamiltonians can now be written as

$$H_r = \frac{\hat{q}_r^2}{2C_r} + \frac{\hat{\Phi}_r^2}{2\tilde{L}_r}; \qquad H_q = \frac{\hat{q}^2}{2C_J} + \frac{E_L + E_J(t+u)}{2}\hat{\varphi}^2, \tag{3}$$

with respective frequencies $\omega = (\tilde{L}_r C_r)^{-1/2}$ and $\omega_q = \sqrt{\omega_C[\omega_L + \omega_J(t+u)]}$. We see immediately that changing the external fluxes changes these frequencies, and hence, the qubit-resonator detuning $\Delta = \omega_q - \omega$. Introducing creation and annihilation operators for the resonator and qubit satisfying $[\hat{a}, \hat{a}^\dagger] = 1 = [\hat{b}, \hat{b}^\dagger]$ with

$$\hat{\varphi} = \sqrt{\frac{\omega_C}{2\omega_q}}(\hat{b} + \hat{b}^\dagger); \quad \hat{N} = -i\sqrt{\frac{\omega_q}{2\omega_C}}(\hat{b} - \hat{b}^\dagger),$$
$$\hat{\Phi}_r = \sqrt{\frac{\tilde{L}_r \omega \hbar}{2}}(\hat{a} + \hat{a}^\dagger); \quad \hat{q}_r = -i\sqrt{\frac{\hbar}{2\tilde{L}_r\omega}}(\hat{a} - \hat{a}^\dagger). \tag{4}$$

the resonator and qubit Hamiltonians become $H_r = \omega\hat{a}^\dagger\hat{a}$ and $H_q = \omega_q \hat{b}^\dagger\hat{b}$. When the qubit is not linearized as in (2), the potential energy terms can be written as $V_1 = \eta_1(\hat{a} + \hat{a}^\dagger)\sin(\hat{\phi} + \phi'_x)$, $V_2 = \eta_2(\hat{a} + \hat{a}^\dagger)^2\cos(\hat{\phi} + \phi'_x)$, $V_3 = i\eta_3(\hat{a} - \hat{a}^\dagger)\hat{N}$, with coupling coefficients $\eta_1 = \chi E_J \mu$, $\eta_2 = \eta_1^2/(2E_J)$, and $\eta_3 = (\eta_1\hbar\omega)/(2E_J)$. The potential that is of relevance is V_2 from which the nonlinear coupling is seen to be $g_2 = \sqrt{2}\eta_2\langle 0_q|\cos(\hat{\phi} + \phi'_x)|1_q\rangle$, where the matrix element is between the ground and first excited qubit states. The nonlinear coupling coefficient η_2 depends on the characteristic impedance Z of the LC circuit implicit in the parameter μ. Therefore, we have to implement a high-impedance resonator to make g_2 sizeable.

The linearized flux dependent Hamiltonian of the system is

$$H_L = H_r + H_q - \frac{\chi}{2C_r}\hat{q}\hat{q}_r + \chi E_J u \hat{\varphi}_r \hat{\varphi}. \tag{5}$$

We neglect all higher order nonlinear terms and only consider the perturbative $\chi^{(2)}$ type nonlinearity given by $V_2 = -E_J\chi^2 s/2\hat{\varphi}\hat{\varphi}_r^2$. We can make a rotating wave approximation and write H_L in terms of creation and annihilation operators $H_L = \omega\hat{a}^\dagger\hat{a} + \omega_q\hat{b}^\dagger\hat{b} + g_1(\hat{a}\hat{b}^\dagger + \hat{a}^\dagger\hat{b})$, where the linear coupling $g_1 = \eta_1 u\sqrt{\omega_C/(2\omega_q)} - \eta_3\sqrt{\omega_q/(2\omega_C)}$.

By introducing new operators $\hat{c}$ and $\hat{d}$ which preserve the commutation relations $[\hat{c}, \hat{c}^\dagger] = 1 = [\hat{d}, \hat{d}^\dagger]$, the normal mode Hamiltonian can be shown to be $H_N = \Omega_1\hat{c}^\dagger\hat{c} + \Omega_2\hat{d}^\dagger\hat{d}$, with energies $\Omega_{1,2} = \omega + \Delta/2(1 \mp \sqrt{1 + 4g_1^2/\Delta^2})$. We assume $\Delta > 0$. The bare basis states of the system is denoted by $|n\rangle \otimes |q\rangle \equiv |nq\rangle$, where the first and second labels refer to the quantum numbers of the resonator and qubit. The relevant eigenstates of the Hamiltonian in the new basis are number excitations of $\hat{c}^\dagger\hat{c}$ and $\hat{d}^\dagger\hat{d}$. Denoting these kets as $|\bar{C}\bar{D}\rangle$, we can write down three important eigenstates with energies Ω_1, Ω_2, and $2\Omega_1$. They are $|\bar{1}\,\bar{0}\rangle = \cos\theta|10\rangle + \sin\theta|01\rangle$, $|\bar{0}\,\bar{1}\rangle = -\sin\theta|10\rangle + \cos\theta|01\rangle$, and $|\bar{2}\,\bar{0}\rangle = \cos^2\theta|20\rangle + \sqrt{2}\cos\theta\sin\theta|11\rangle + \sin^2\theta|02\rangle$. The parameter θ satisfies $\tan 2\theta = -2g_1\Delta^{-1}$. For $\Delta \gg |g_1|$, $|\bar{1}\,\bar{0}\rangle \to |10\rangle$, $|\bar{0}\,\bar{1}\rangle \to |01\rangle$, $|\bar{2}\,\bar{0}\rangle \to |20\rangle$, $\Omega_1 \to \omega$, and $\Omega_2 \to \omega_q$.

The nonlinear coupling V_2 couples the states $|\bar{2}\,\bar{0}\rangle$ and $|\bar{0}\,\bar{1}\rangle$ leading to a sizeable avoided crossing in Fig. 1(c) between the two-photon and qubit. Working in the truncated subspace spanned by the states $\{|0\rangle \equiv |\bar{0}\,\bar{0}\rangle, |a\rangle \equiv |\bar{1}\,\bar{0}\rangle, |b\rangle \equiv |\bar{2}\,\bar{0}\rangle, |c\rangle \equiv |\bar{0}\,\bar{1}\rangle\}$, we write the relevant Hamiltonian as $H = H_0 + V$ where $H_0 = \Omega_1|a\rangle\times\langle a| + 2\Omega_1|b\rangle\langle b| + \Omega_2|c\rangle\langle c|$ and the coupling $V = \lambda_1(|a\rangle\langle b| + |b\rangle\langle a|) + \lambda_2(|b\rangle\langle c| + |c\rangle\langle b|)$. The parameters $\lambda_1 = \sqrt{2}\eta'_2\cos^2\theta\sin\theta \equiv r_1\eta'_2$ and $\lambda_2 = -\sqrt{2}\eta'_2\cos^3\theta \equiv r_2\eta'_2$ with $\eta'_2 = \eta_2 s/\sqrt{2}$. We can adiabatically eliminate the state $|a\rangle$ to find an effective Hamiltonian $H_e = (\Omega_1 - r_1^2\eta_2'^2/\Omega_1)|a\rangle\langle a| + (2\Omega_1 + r_1^2\eta_2'^2/\Omega_1)|b\rangle\langle b| + \Omega_2|c\rangle\langle c| + r_2\eta'_2(|b\rangle\langle c| + |c\rangle\langle b|)$.

We can use this Hamiltonian to calculate the two-photon nonlinearity N_l. For $|\delta'| \equiv |\Omega_2 - 2\Omega_1| \gg |\eta'_2 r_2|$, we have $N_l = -(\eta'_2 r_2)^2/\delta' \equiv -g_2^2/\delta' \approx -g_2^2/\delta$, where we have associated the nonlinear coupling g_2 with $\eta'_2 r_2$. The nonlinear phase-shift protocol requires initializing the system in the states $|10\rangle \approx |\bar{1}\,\bar{0}\rangle$ and $|20\rangle \approx |\bar{2}\,\bar{0}\rangle$ with errors that go like g_1^2/Δ^2. Then the external fluxes are varied adiabatically so that the state $|\bar{2}\,\bar{0}\rangle$ becomes slightly qubitlike, mostly because of $|11\rangle$. After accumulating the desired phase, the process is reversed to retrieve the photons. For some integer n, we require for a total time τ_g, $\int_0^{\tau_g} N_l(t)dt = (2n+1)\pi$. The final outcome is then $\frac{1}{\sqrt{3}}(|00\rangle + |10\rangle + |20\rangle) \to \frac{1}{\sqrt{3}}(|00\rangle + |10\rangle - |20\rangle)$.

In addition to our analytical model, we also diagonalize the Hamiltonian numerically in the tensor product space $H = H_r \otimes H_q$ of the resonator and qubit using the Hamiltonian (2). The basis states in the resonator space are number excitations $|n\rangle$. The qubit space is written in the basis of qubit wave functions $\psi_q(\phi) = \langle\phi|q\rangle$. We let $\hbar = 1$ and choose $\omega_C/(2\pi) = 1$ GHz, $\omega_J/(2\pi) = 5$ GHz, $\omega_L = 3\omega_J$, and $\omega/(2\pi) = 2.225$ GHz. The impedance $Z \approx 449\ \Omega$. We choose a $\chi = 0.17$, representing an easily achievable mutual inductance, from which follow $\eta_1/(2\pi) = 400$ MHz, $\eta_2/(2\pi) = 16$ MHz, and $\eta_3/(2\pi) = 89$ MHz. Some results of our numerical analysis are depicted in Fig. 2.

We now discuss the effect of loss on our gate. Since throughout the gate operation the system remains photonlike, loss is dominated by the cavity with a decay rate κ. For the photonlike state $|\bar{2}\,\bar{0}\rangle$, there are two other decay channels due to the cavity-qubit coupling. The linear

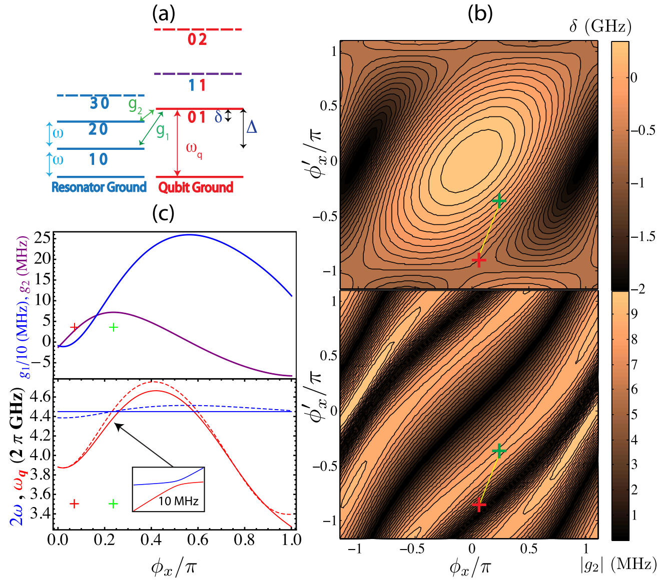

FIG. 2 (color online). (a) Schematic of the system bare energy levels and couplings. (b) Contour plots of detuning δ and $|g_2|$ with the on and off points marked in green and red. The on point is chosen such that the g_2 is maximized. (c) Top: The coupling $g_1/10$ and g_2. Bottom: The analytical (dashed line) and numerical (solid line) results of the bare frequencies $2\omega/(2\pi)$ and $\omega_q/(2\pi)$. The overall qubit-resonator interaction leads to a roughly 10 MHz splitting.

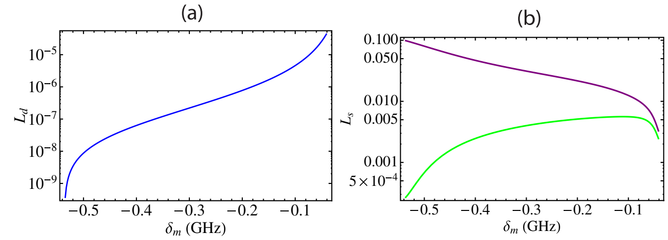

FIG. 3 (color online). (a) A plot of the dimensionless dynamic loss L_d for $\kappa = 1$ kHz, $\gamma = 100\kappa$, and $\epsilon^2 = 0.01$. The detuning -536 MHz $\leq \delta_m \leq -41$ MHz. (b) The static loss L_s (top) and the static loss without the effect of the cavity decay rate (bottom).

coupling g_1 in the limit $\Delta \gg |g_1|$ leads to $\gamma_1 \equiv \gamma g_1^2/\Delta^2 = \gamma g_1^2/(\delta + \omega)^2$. Similarly the nonlinear coupling leads to $\gamma_2 \equiv \gamma g_2^2/\delta^2$ for $|\delta| \gg |g_2|$. Thus, the total decay rate of the two-photonlike state becomes $\Gamma(\delta) = \kappa + \gamma_1 + \gamma_2$.

Assuming g_2 is time independent, adiabaticity requires $g_2^2|\dot{\delta}|^2(\delta^2 + 4g_2^2)^{-3} \ll 1$. Setting this equal to some $\epsilon^2 \ll 1$, we solve for $\tau_h(\delta_m) = \epsilon^{-1}\int_{\delta_m}^{\delta_i} |g_2|/(\delta^2 + 4g_2^2)^{3/2} d\delta$, which is the time taken to go from $|\delta_i| \gg |g_2|$ at $t = 0$ to smaller values of detuning with a minimum δ_m. The total dynamic loss during the process is given by $L_d(\delta_m) = 2\epsilon^{-1}\int_{\delta_m}^{\delta_i} \Gamma(\delta)|g_2|/(\delta^2 + 4g_2^2)^{3/2} d\delta$. When the detuning is held at δ_m for a time $\tau_s = \pi\delta_m/g_2^2$, the static loss is $L_s = \tau_s\Gamma(\delta_m)$. More explicitly,

$$L_s(\delta_m) = \pi\left[\frac{\kappa\delta_m}{g_2^2} + \frac{\gamma\delta_m}{(\delta_m + \omega)^2}\left(\frac{g_1}{g_2}\right)^2 + \frac{\gamma}{\delta_m}\right], \quad (6)$$

and the total time of the protocol is $\tau_g = 2\tau_h + \tau_s$. Assuming $\delta_m \ll \omega$, $L_s(\delta_m)$ is minimized when $\delta_m \approx g_2\sqrt{\gamma/\kappa}$. However, the on-off ratio of the photon nonlinearity goes like $|\delta_i/\delta_m|$, and a value of δ_m that makes this ratio at least a hundred is desirable. For $\delta \sim \omega$, we can make $g_1 \approx g_2$ so that $L_s(\delta) < \kappa\delta/g_2^2 + 2\gamma/\delta$. In this regime L_s is limited by κ, as can be verified from Fig. 3(b). Thus, we optimize our protocol so that the loss $L = L_d + L_s \ll 1$. We note that our protection is only against qubit noise and loss, and comes at the cost of increased reliance on the cavity quality factor.

The protocol might also be limited by dephasing of the qubit due to flux noise [30–32]. The average slopes of the single and two-photon energy levels with respect to the reduced flux ϕ_x are approximately 50 MHz and 100 MHz, respectively, while the slope of the qubit energy level is at most 1 GHz for the parameters chosen. However, the exact loss due to dephasing depends on the flux noise amplitude [33,34].

In conclusion, we have demonstrated that by appropriately tuning the two control fluxes, the nonlinear coupling enables a two-photon nonlinear phase shift operation with loss at large detuning limited only by the cavity quality factor. This is highly desirable compared to the self-Kerr nonlinearity which leads to more qubit-induced loss at large detunings. Furthermore, our approach may be adaptable to recent ultra-high quality factor resonators enabling nonlinear optics quantum computing in a fully engineered system [20].

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[1] E. Knill, R. Laflamme, and G. J. Milburn, Nature (London) **409**, 46 (2001).
[2] P. Kok *et al.*, Rev. Mod. Phys. **79**, 135174 (2007).
[3] M. A. Nielsen, Phys. Rev. Lett. **93**, 040503 (2004).
[4] D. E. Browne and T. Rudolph, Phys. Rev. Lett. **95**, 010501 (2005).
[5] G. S. Buller and R. J. Collins, Meas. Sci. Technol., **21**, 012002 (2010).
[6] D. I. Schuster *et al.*, Nature (London) **445**, 515 (2007).
[7] A. Blais, R. S. Huang, A. Wallraff, S. M. Girvin, and R. J. Schoelkopf, Phys. Rev. A **69**, 062320 (2004).
[8] A. Wallraff, D. I. Schuster, A. Blais, L. Frunzio, R.-S. Huang, J. Majer, S. Kumar, S. M. Girvin, and R. J. Schoelkopf, Nature (London) **431**, 162 (2004).

[9] E. Zakka-Bajjani, F. Nguyen, M. Lee, L. R. Vale, R. W. Simmonds, and J. Aumentado, Nat. Phys. **7**, 599 (2011).
[10] J. M. Martinis, arXiv:cond-mat/0402415v1.
[11] M. Boissonneault, J. M. Gambetta, and A. Blais, Phys. Rev. A **86**, 022326 (2012).
[12] C. Eichler, D. Bozyigit, C. Lang, L. Steffen, J. Fink, and A. Wallraff, Phys. Rev. Lett. **106**, 220503 (2011).
[13] A. A. Houck *et al.*, Nature (London) **449**, 328 (2007).
[14] M. Hofheinz, E. M. Weig, M. Ansmann, R. C. Bialczak, E. Lucero, M. Neeley, A. D. O'Connell, H. Wang, J. M. Martinis, and A. N. Cleland, Nature (London) **454**, 310 (2008).
[15] Y. Xiao, M. Klein, M. Hohensee, L. Jiang, D. Phillips, M. Lukin, and R. Walsworth, Phys. Rev. Lett. **101**, 043601 (2008).
[16] B. R. Johnson *et al.*, Nat. Phys. **6**, 663 (2010).
[17] G. Romero, J. J. García-Ripoll, and E. Solano, Phys. Rev. Lett. **102**, 173602 (2009).
[18] Y. Yin *et al.*, Phys. Rev. A **85**, 023826 (2012).
[19] A. A. Abdumalikov, O. Astafiev, Y. Nakamura, Y. A. Pashkin, and J. S. Tsai, Phys. Rev. B **78**, 180502(R) (2008).
[20] H. Paik *et al.*, Phys. Rev. Lett. **107**, 240501 (2011).
[21] D. P. DiVincenzo, Phys. Rev. A **51**, 1015 (1995).
[22] S. Rebić, J. Twamley, and G. J. Milburn, Phys. Rev. Lett. **103**, 150503 (2009).
[23] K. Nemoto and W. J. Munro, Phys. Rev. Lett. **93**, 250502 (2004).
[24] I.-C. Hoi *et al.*, arXiv:1207.1203v1.
[25] M. Fleischhauer, A. Imamŏglu, and J. P. Marangos, Rev. Mod. Phys. **77**, 633 (2005).
[26] J. Clarke and A. I. Braginsky, *The Squid HandBook: Applications of Squids and Squid Systems* (Wiley-VCH, Weinheim, 2007), Vol. 2.
[27] M. Nielsen and I. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, England, 2000).
[28] J. D. Teufel, D. Li, M. S. Allman, K. Cicak, A. J. Sirois, J. D. Whittaker, and R. W. Simmonds, Nature (London) **471**, 204 (2011).
[29] M. H. Devoret, *Quantum Fluctuations, Les Houches Session LXIII* (Elsevier, New York, 1995), pp. 351–386.
[30] J. M. Martinis, S. Nam, J. Aumentdo, K. M. Lang, and C. Urbina, Phys. Rev. B **67**, 094510 (2003).
[31] R. C. Bialczak *et al.*, Phys. Rev. Lett. **99**, 187006 (2007).
[32] K. Kakuyanagi, T. Meno, S. Saito, H. Nakano, K. Semba, H. Takayanagi, F. Deppe, and A. Shnirman, Phys. Rev. Lett. **98**, 047004 (2007).
[33] M. Steffen, S. Kumar, D. DiVincenzo, J. Rozen, G. Keefe, M. Rothwell, and M. Ketchen, Phys. Rev. Lett. **105**, 100502 (2010).
[34] S. M. Anton *et al.*, Phys. Rev. B **85**, 224505 (2012).