Nonlocal Thermoelectric Effects and Nonlocal Onsager relations in a Three-Terminal Proximity-Coupled Superconductor-Ferromagnet Device

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We study thermal and charge transport in a three-terminal setup consisting of one superconducting and two ferromagnetic contacts. We predict that the simultaneous presence of spin filtering and of spin-dependent scattering phase shifts at each of the two interfaces will lead to very large nonlocal thermoelectric effects both in clean and in disordered systems. The symmetries of thermal and electric transport coefficients are related to fundamental thermodynamic principles by the Onsager reciprocity. Our results show that a nonlocal version of the Onsager relations for thermoelectric currents holds in a three-terminal quantum coherent ferromagnet-superconductor heterostructure including a spin-dependent crossed Andreev reflection and coherent electron transfer processes.

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Heterostructures of ferromagnets (F) and superconductors (S) are presently a subject of intense study since they show interesting phenomena based on the singlet-triplet conversion of pairing amplitudes at the interfaces and the resulting spin-dependent proximity effect. Spectacular examples are long-range triplet Josephson currents due to inhomogeneous magnetic order [1] or due to the spin dependence of the interface reflection and transmission amplitudes [2] that were confirmed in a set of pivotal experiments [3–6]. A multitude of coherence phenomena is understood in terms of spin-dependent Andreev bound states [2,7–17], intimately related to spin-mixing [18] and spin-filtering effects at interfaces [19].

A three-terminal superconductor-ferromagnet proximity system also allows us to access nonlocal effects. For example, in Fig. 1, incoming electrons (current I_{I}) can be reflected from the interface (I_{R}) or enter the superconductor, where each builds a Cooper pair with another electron, leaving a hole behind that is retroreflected (a so-called Andreev reflection). These holes can be transmitted back through the same interface (I_{AR}) or reflected to the other interface, where they are either transmitted directly as holes (I_{CAR}) or as electrons via the same conversion process as at the other interface in reversed order (I_{CET}) (part of these electrons can also be reflected back to the first interface, contributing to higher order processes). Nonlocal transport has attracted considerable interest due to the latter two processes, called crossed Andreev reflection (CAR; an electron enters at one terminal, and a hole leaves the other terminal, or vice versa) and coherent electron transfer (CET, sometimes called "elastic cotunneling"; an electron enters one terminal, and an electron leaves the other terminal, or the same for holes) [20–22]. These processes test the internal structure of Cooper pairs and lead to new interesting physics that can be and has been tested experimentally [23–27].

In this Letter, we develop a theory for the hitherto less explored nonlocal *thermal* transport in ferromagnet-superconductor devices and show that a nonlocal version of Onsager relations [28] holds in both the normal and superconducting states. In the superconducting state, we find a strongly enhanced local thermopower and a nonlocal Seebeck effect. These effects do not require noncollinear inhomogeneities in the ferromagnetic regions or at the interfaces (a ubiquitous problem for creating triplet supercurrents [1,2,4–6,29]). Thus, our results should be readily observable in experiments and offer a way to access the microscopic spin-dependent parameters.

In linear response, the transport coefficients relating charge (energy) currents I^q (I^ε) to an applied voltage $\Delta V_j = V_j - V_S$ or temperature difference $\Delta T_j = T_j - T_S$ (throughout this Letter, $j \in \{1, 2\}$ labels the

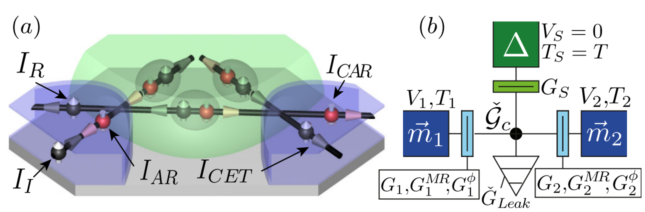

FIG. 1 (color online). (a) The device consisting of two ferromagnets (regions to the left and right in blue) and a superconductor (the green region in the center). Trajectories for electrons (black) and holes (red) illustrate possible transport processes in the ballistic case, as discussed in the text (white arrows denote the spin). (b) Equivalent circuit diagram of the setup shown in (a) for the diffusive limit including the coherence leakage [41]. The interface parameters are discussed in detail beneath Eq. (3).

ferromagnet-superconductor contacts and $q = -|e|$ is the electronic charge) of our three-terminal system are

$$\begin{pmatrix} I_1^q \\ I_1^\varepsilon \\ I_2^q \\ I_2^\varepsilon \end{pmatrix} = \underbrace{\begin{pmatrix} L_{11}^{qV} & L_{11}^{qT} & L_{12}^{qV} & L_{12}^{qT} \\ L_{11}^{\varepsilon V} & L_{11}^{\varepsilon T} & L_{12}^{\varepsilon V} & L_{12}^{\varepsilon T} \\ L_{21}^{qV} & L_{21}^{qT} & L_{22}^{qV} & L_{22}^{qT} \\ L_{21}^{\varepsilon V} & L_{21}^{\varepsilon T} & L_{22}^{\varepsilon V} & L_{22}^{\varepsilon T} \end{pmatrix}}_{\hat{L}} \begin{pmatrix} \Delta V_1 \\ \Delta T_1/T_S \\ \Delta V_2 \\ \Delta T_2/T_S \end{pmatrix}. \quad (1)$$

This generalized conduction matrix $\hat{L}$ contains local 2×2 blocks in the diagonal and nonlocal 2×2 blocks in the off diagonal. The local and nonlocal thermoelectric coefficients L_{ij}^{qT} in Eq. (1) give rise to large thermoelectric effects in the superconducting state, as we will show below. In contrast, in the normal state, these coefficients are typically proportional to the asymmetry of the density of states around the chemical potential, which is orders of magnitude smaller. Microscopically, spin-dependent scattering phases at a ferromagnetic contact produce an asymmetry, equal in magnitude and opposite in sign for the two spin species, in the superconducting spectrum of quasiparticles emerging from the contact. Spin filtering, which weights the spin directions differently, can resolve these asymmetric components of the spectrum. Both effects vanish for spin-independent systems. Consequently, this situation is not comparable to the thermoelectric effects related to the supercurrents discussed in the context of normal-metal–superconductor Andreev interferometers [30–33]. The effects we present persist also in the absence of a supercurrent emerging from the superconducting terminal.

We find that the matrix in Eq. (1) (even for noncollinear magnetization configurations) is symmetric, $\hat{L} = \hat{L}^T$, similar to the well-known Onsager symmetries [28], however, for a nonlocal setup that contains ferromagnetic leads and includes supercurrents in the superconducting terminal as well as crossed Andreev reflection and elastic cotunneling processes between the contacts.

We begin our theoretical analysis with the description of the interfaces between the superconductor and the ferromagnets. Each conduction channel n between a superconductor (S) and a ferromagnet (F) (with homogeneous magnetization throughout the interface region) is described by a scattering matrix

$$\hat{S}_{n\sigma} = \begin{pmatrix} r_{n\sigma} e^{i\varphi_{n\sigma}^S} & t_{n\sigma} e^{i\varphi_{n\sigma}^{SF}} \\ t_{n\sigma} e^{i\varphi_{n\sigma}^{FS}} & -r_{n\sigma} e^{i\varphi_{n\sigma}^F} \end{pmatrix}, \quad (2)$$

where $\sigma \in \{\uparrow, \downarrow\}$ and unitarity requires $r_{n\sigma}^2 + t_{n\sigma}^2 = 1$ and $\varphi_{n\sigma}^{SF} + \varphi_{n\sigma}^{FS} = \varphi_{n\sigma}^S + \varphi_{n\sigma}^F$ modulo 2π. This leads, for example, to spin-dependent conductances (spin filtering) characterized by a polarization $\mathcal{P}_n = (t_{n_\uparrow}^2 - t_{n_\downarrow}^2)/(t_{n_\uparrow}^2 + t_{n_\downarrow}^2)$ and a probability for transmission, $\mathcal{T}_n = (t_{n\uparrow}^2 + t_{n\downarrow}^2)/2 \leq (1 + |\mathcal{P}_n|)^{-1}$. Concerning the scattering phases, transport coefficients only depend on the phase shift between the reflections of spin-up and spin-down electrons on the superconducting sides of the contact, $\delta\varphi_n = \varphi_{n_\uparrow}^S - \varphi_{n_\downarrow}^S$, called a spin-mixing angle. Some of the most striking consequences of the spin-dependent scattering phases are triplet pairing [2,34] or subgap resonances in the noise spectral density [11,35]. Finally, the combination of *both* spin-dependent parameters $\mathcal{P}_n$ and $\delta\varphi_n$ leads to thermoelectric effects. We use spin-dependent boundary conditions (SDBCs) [2,7,12,36–38] for quasiclassical Green functions in the setups shown in Fig. 1.

Analogously to the spin-independent theory [39–42], the system properties in the dirty limit (i.e., the elastic mean free path is much shorter than the superconducting coherence length) are fully described by the isotropic matrix Green functions $\check{\mathcal{G}}_c$ of the contact region [see Fig. 1(b)] and $\check{\mathcal{G}}_j$ ($\check{\mathcal{G}}_S$) for the ferromagnets (superconductor) that are 8×8 matrices in Keldysh $\otimes$ Nambu $\otimes$ spin space. $\check{\mathcal{G}}_c$ is determined through a finite element approach, governed by a conservation law for matrix currents [41] (see the Supplemental Material for details [43]): $\sum_j \check{I}_{j,c} + \check{I}_{S,c} + \check{I}_{\rm Leak} = 0$ with the normalization condition $\check{\mathcal{G}}_c^2 = 1$. The leakage current $\check{I}_{\rm Leak}$ describes the decoherence of the superconducting order parameter due to a finite diffusion time in the central region (defining the inverse of the Thouless energy $\varepsilon_{\rm Th}$). The spin-dependent matrix currents $I_{j,c}$ from contact j into the superconducting contact region (denoted as c) are obtained from the SDBC. We introduce the notation $t_{n\sigma} = t_n + \sigma t_n'$ for spin components of the transmission quantized along a magnetization direction $\vec{m}$. Choosing the spinor basis $\hat{\Psi}^\dagger = (\Psi_\uparrow^\dagger, \Psi_\downarrow^\dagger, \Psi_\downarrow, -\Psi_\uparrow)$ and following the line in Ref. [37], we find to leading order in t_n, t_n', and $\delta\varphi_n$ a compact form for the SDBC:

$$\check{I}_{j,c}(\varepsilon) = \frac{q^2}{h} \sum_n [\check{t}_{jn} \check{\mathcal{G}}_j(\varepsilon) \check{t}_{jn} - i\delta\varphi_{jn} \check{\kappa}_j, \check{\mathcal{G}}_c(\varepsilon)], \quad (3)$$

with $\check{t}_{jn} = t_{jn} + t_{jn}' \check{\kappa}_j$ and $\check{\kappa}_j = \check{1} \otimes \check{\tau}_z \otimes (\vec{m}_j \check{\vec{\sigma}})$ ($\check{\tau}$ and $\check{\sigma}$ are Pauli matrices). The t_{jn} and t_{jn}' can be related to the $\mathcal{T}_{jn}$ and $\mathcal{P}_{jn}$ via $(t_{jn} + t_{jn}' \vec{m}_j \check{\vec{\sigma}})^2 = \mathcal{T}_{jn}(1 + \mathcal{P}_{jn} \vec{m}_j \check{\vec{\sigma}})$. Performing the sums over n, only a few parameters remain. In terms of the conductance quantum $G_q \equiv q^2/h$, these are $G_j = 2G_q \sum_n \mathcal{T}_{jn}$, $G_j^{\rm MR} = G_q \sum_n \mathcal{T}_{jn} \mathcal{P}_{jn}$, and $G_j^\phi = 2G_q \sum_n \delta\varphi_{jn}$, as well as $\eta_{\rm Th} \equiv \varepsilon_{\rm Th} G_S/G_q$. Here, G_S is the conductance between the contact region and the bulk superconductor, fulfilling $\check{I}_{S,c} = \frac{G_S}{2}[\check{\mathcal{G}}_S, \check{\mathcal{G}}_c]$. The above procedure is correct for $\delta\varphi_{jn}, \mathcal{T}_{jn} \ll 1$, covering the full range $-1 \leq \mathcal{P}_{jn} \leq 1$. The equations for $\check{\mathcal{G}}_c$ are solved numerically, and the density of states and the currents are calculated as functions of the parameter set introduced above, as described in the Supplemental Material [43].

In the clean limit (i.e., the elastic mean free path is much longer than the superconducting coherence length), we apply the theory developed in Refs. [10,12,44]. In this case, the current density at one particular contact can be decomposed into local (depending on the distribution function of the ferromagnet at the same contact) and nonlocal (depending on the distribution function of the ferromagnet at the other contact) contributions: incoming ($I_{\rm I}$), reflected ($I_{\rm R}$), Andreev reflected ($I_{\rm AR}$), crossed Andreev reflected ($I_{\rm CAR}$), and coherent electron transfer $I_{\rm CET}$ (see Fig. 1). The total current through contact j into the superconductor is given by

$$I_j^\alpha = I_{j,{\rm I}}^\alpha - I_{j,{\rm R}}^\alpha + I_{j,{\rm AR}}^\alpha - I_{j,{\rm CET}}^\alpha + I_{j,{\rm CAR}}^\alpha, \quad (4)$$

with $\alpha \in \{q, \varepsilon\}$ and contact index $j \in \{1, 2\}$. We consider two contacts of diameter that are small compared to the superconducting coherence length ξ_0 and to the intercontact distance L. Then, quasiclassical trajectories connect the two contacts, with contact i seen from contact j under a solid angle $\delta\Omega_j = \mathcal{A}_i^z/L^2$, where $\mathcal{A}_i^z$ is the area of contact i projected onto the plane normal to the line connecting the two contacts (here, the z axis). The current through contact j is proportional to $\mathcal{A}_j^z$, and its nonlocal part is proportional to $\mathcal{A}_1^z\mathcal{A}_2^z/L^2$, as is the nonlocal part of the current through contact i. Nonlocal contributions also enter $I_{\rm R}$ and $I_{\rm AR}$; however, they are the only contributions to $I_{\rm CAR}$ and $I_{\rm CET}$. Only *nonlocal* contributions, via the trajectory connecting the two contacts, give rise to thermopower and the Seebeck effect in the ballistic limit.

We write nonlocal current contributions as

$$I_j^\alpha = \frac{\delta^2 p}{\delta\Omega}\bigg|_{p_{j\to i}} \frac{\mathcal{A}_1^z\mathcal{A}_2^z}{(2\pi\hbar)^3 L^2}\int_{-\infty}^{\infty} \alpha[j_j(\varepsilon) + \tilde{j}_j(\varepsilon)]d\varepsilon, \quad (5)$$

with $(\delta^2 p/\delta\Omega)|_{p_{1\to 2}} = (\delta^2 p/\delta\Omega)|_{p_{2\to 1}}$ being the differential fraction of the Fermi surface of the superconductor with Fermi momentum such that the corresponding Fermi velocity $\vec{v}_{\rm F}$ connects the two contacts, per solid angle Ω. With the deviations of the distribution functions from that in the superconductor, for particles δf_p and holes δf_h, the contributions to $j_j = j_{j,{\rm I}} - j_{j,{\rm R}} + j_{j,{\rm AR}} - j_{j,{\rm CET}} + j_{j,{\rm CAR}}$ are, e.g., for contact $j = 1$, $j_{1,{\rm I}}(\varepsilon) = 2\delta f_{1,p}$,

$$j_{1,{\rm R}}(\varepsilon) = 2|r_{1\uparrow} - v_1 t_{1\uparrow}^2 r_{1\downarrow} e^{i\delta\varphi_1}\gamma_0\gamma_1|^2 \delta f_{1,p}, \quad (6)$$

$$j_{1,{\rm AR}}(\varepsilon) = (t_{1\uparrow}t_{1\downarrow})^2|v_1|^2(|\gamma_1|^2 + |\gamma_0|^2)\delta f_{1,h}, \quad (7)$$

$$j_{1,{\rm CET}}(\varepsilon) = (t_{1\uparrow}t_{2\uparrow})^2|v_1 u_{12}|^2(1 + |\gamma_0|^4 r_{1\downarrow}^2 r_{2\downarrow}^2)\delta f_{2,p}, \quad (8)$$

$$j_{1,{\rm CAR}}(\varepsilon) = (t_{1\uparrow}t_{2\downarrow})^2|v_1 u_{12}|^2|\gamma_0|^2(r_{2\uparrow}^2 + r_{1\downarrow}^2)\delta f_{2,h}, \quad (9)$$

with $\gamma_0(\varepsilon) = -\Delta/(\varepsilon + i\omega)$, $\omega(\varepsilon) = \sqrt{\Delta^2 - \varepsilon^2}$, $\Gamma_j(\varepsilon) = \gamma_0 r_{j\uparrow} r_{j\downarrow} e^{i\delta\varphi_j}$, $u_{12}(\varepsilon) = [c - is(\varepsilon + \Gamma_2\Delta)/\omega]^{-1}$, $\gamma_1(\varepsilon) = u_{12}[\Gamma_2 c + is(\Delta + \Gamma_2\varepsilon)/\omega]$, and $v_1(\varepsilon) = (1 - \gamma_1\Gamma_1)^{-1}$, with $c(\varepsilon) = \cosh(\omega L/\hbar v_{\rm F})$ and $s(\varepsilon) = \sinh(\omega L/\hbar v_{\rm F})$. Finally, $\tilde{j}_j(\varepsilon)$ in Eq. (5) is obtained by interchanging $\uparrow\leftrightarrow\downarrow$ and $\delta\varphi_j \to -\delta\varphi_j$ for both contacts in the expressions above. The distribution functions are

$$\delta f_{j,p}(\varepsilon) = \frac{q\Delta V_j + \varepsilon\Delta T_j/T_S}{4k_{\rm B}T_S\cosh^2(\varepsilon/2k_{\rm B}T_S)} = \delta f_{j,h}(-\varepsilon). \quad (10)$$

Equations (4)–(10) are valid for arbitrary transparencies and spin polarizations. Nonlocal effects decay when L exceeds the scale of the superconducting coherence length ($\xi_0 = \hbar v_{\rm F}/k_{\rm B}T_c$ in the clean limit). See the Supplemental Material [43] for examples.

The temperature dependence of the superconducting pair potential Δ is taken into account by solving self-consistently the gap equation in weak coupling BCS theory (with its zero temperature value denoted as Δ_0).

As shown in the Supplemental Material [43], in ballistic systems, only processes that involve the opposite contact contribute to the local thermoelectric coefficients L_{jj}^{qT} and $L_{jj}^{\varepsilon V}$. The term $I_{j,{\rm AR}}^\alpha$ does not contribute because $j_{1,{\rm AR}}(-\varepsilon)$ cancels the corresponding term for $\tilde{j}_{1,{\rm AR}}(\varepsilon)$ in the expressions for the thermoelectric coefficients [both have the same prefactor $(t_{1\uparrow}t_{1\downarrow})^2$; i.e., spin filtering is not active here]. In contrast, the expression for $I_{j,{\rm R}}^\alpha$ does not show such a cancellation when contact 1 is spin polarized, due to the asymmetric combination of transmission and reflection coefficients in $j_{1,{\rm R}}(\varepsilon)$ (i.e., spin filtering is active) and the presence of spin mixing ($\delta\varphi_1$). It does, however, require in addition that $r_{2\uparrow}r_{2\downarrow}e^{i\delta\varphi_2} \neq 1$ (which means the presence of a second contact) in order for it to cause nonzero thermoelectric effects. When the impurity mean free path or the dimension of the superconducting terminal shrinks below ξ_0, direct backscattering due to impurities or surfaces contributes and leads to a local thermopower even in a two-terminal device.

As the mechanism behind the thermoelectric effects can be understood from the density of states (DOS) in the contact region, we discuss first this quantity. In the dirty limit (see Fig. 2) for $G^\phi = 0$, the DOS displays peaks at $\varepsilon = \Delta$ resulting from the superconducting leads and the proximity induced minigap. The magnetization directions are chosen parallel. Increasing G^ϕ simultaneously in both terminals leads to a Zeeman splitting of the minigap in spin-up and down parts and consequently breaks the symmetry of the spin-projected DOS (SDOS) around the Fermi energy $\varepsilon_{\rm F}$ (see Fig. 2). Hence, we expect a nonvanishing thermopower if a spin-filtering term $G^{\rm MR}$ is present simultaneously. An equivalent discussion of the SDOS depending on the spin-mixing angle $\delta\varphi$ for a ballistic system is done in Ref. [13]. The subgap peaks there are much sharper

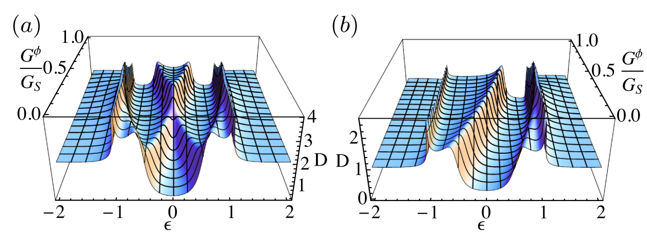

FIG. 2 (color online). Density of states D in the contact region for $G_1 = G_2 = 0.1G_S$, $G_1^{\rm MR} = G_2^{\rm MR} = 0.005G_S$ (10% polarization), and $\eta_{\rm Th} \equiv \varepsilon_{\rm Th}G_S/G_q = 0.5\Delta_0$ (with the Thouless energy $\varepsilon_{\rm Th}$ of the contact region). (a) Total DOS depending on the spin-mixing term G^ϕ for equal ferromagnets. The G^ϕ term splits the pseudogap into the different spin directions. (b) shows the asymmetry in the SDOS for spin-down (the spin-up SDOS looks equal but mirrored at the $\varepsilon = 0$ axis).

compared to the washed-out peak in the dirty limit. This is associated with the fact that only trajectories connecting the two contacts contribute to the nonlocal transport, in which case it is governed by a single length L. This is not the case in diffusive structures, where quasiparticles take random paths of various length between the contacts (and back to the same contact). Nevertheless, both ways lead to an asymmetry in the SDOS and consequently to the astonishing prediction of giant thermoelectric effects for spin-polarized interfaces.

We now turn to the experimentally relevant question of how to define a nonlocal thermopower $\mathcal{S}_{12} = -\Delta V_1/\Delta T_2$, which is not unique in contrast to the local thermopower $\mathcal{S}_j = -\Delta V_j/\Delta T_j = L_{jj}^{qT}/(T_S L_{jj}^{qV})$. In the Supplemental Material [43], we discuss several possibilities to relate voltage and temperature differences between the two ferromagnets and the superconductor, avoiding a control of energy currents. In this Letter, we chose to define the thermopower at contact 1 via $\mathcal{S}_{12} = L_{12}^{qT}/(T_S L_{11}^{qV})$, which is caused by a temperature difference ΔT_2 at contact 2 under the conditions $\Delta V_2 = 0$, $\Delta T_1 = 0$, and $I_1^q = 0$.

In Fig. 3, we show the dependence of $\mathcal{S} \equiv \mathcal{S}_{12}$ on the polarization and spin mixing for $T/T_c \ll 1$, assuming equal ferromagnets. The clean and the diffusive limits show similar behavior, in particular, for weak polarizations. For large polarization, values of more than 100 μV/K are achievable in both limits. Both limits exhibit the same point symmetry with respect to the origin and vanish if one of the spin-dependent parameters vanishes. This behavior is understood from the SDOS as follows. The symmetry of $\mathcal{S}$ with respect to the origin is, according to Eq. (3), a consequence of a π rotation in spin space. The trace in the current formula (shown in the Supplemental Material [43]) is invariant under such a unitary transformation. The sign change with respect to the axes can be understood by Fig. 2. The two spin projections produce thermoelectric effects with opposite signs.

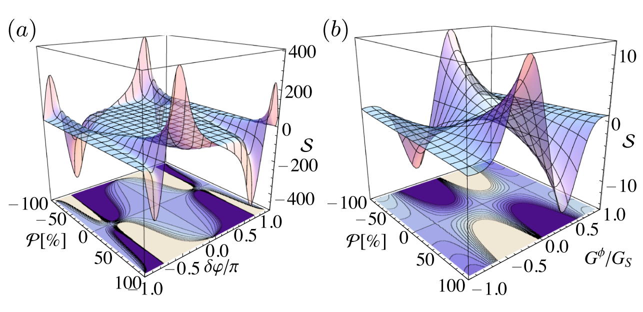

FIG. 3 (color online). Nonlocal thermopower $\mathcal{S} = L_{12}^{qT}/T_S L_{11}^{qV}$ for a symmetric setup as a function of polarization $\mathcal{P}$ and a spin-mixing parameter in the (a) clean and (b) dirty limits for $T = T_S = 0.1T_c$. We assume equally polarized channels, $\mathcal{P}_n \equiv \mathcal{P}$. In (a), $\mathcal{T}_{n1} \equiv \mathcal{T}_1 = 0.1 = \mathcal{T}_2 \equiv \mathcal{T}_{n2}$, $L = 0.5\xi_0$, $\delta\Omega_1 = \delta\Omega_2 = \pi/20$; in (b), $G_1 = G_2 = 0.1G_S$ and $\eta_{\mathrm{Th}} = 0.5\Delta_0$. $\mathcal{S}$ is plotted in units of $gk_{\mathrm{B}}/|q|$, where $g = -\mathcal{T}_2(1+\mathcal{P}^2)\delta\Omega_2/2\pi$ in the clean limit and $g = -G_2/(G_2+G_S)$ in the dirty limit.

Depending on positive or negative G^{MR}, one or the other of the two contributions will be weighted more. Thus, a sign change in G^{MR} changes the sign of the thermopower. On the other hand, a sign change in G^ϕ interchanges the roles of spin-up and spin-down contributions to the DOS and hence changes the sign of the thermopower, too. Similar arguments explain the zero crossing of the thermopower when both spin-polarized peak positions in Fig. 2(a) cross the Fermi level. The same mechanism leads to a sign change in the clean limit, when the spin-split Andreev levels cross at the Fermi energy. Here, the effect is even more drastic since the width of the crossing peaks is determined solely by the transmission to the ferromagnets.

We determine the coefficient matrix $\hat{L}$ in Eq. (1) for temperatures across T_c. We concentrate on the parameters L_{11}^{qT} and L_{12}^{qT}, as they are representative for local and nonlocal thermoelectric properties. In Fig. 4, we plot these parameters for different spin-mixing angles and 10% polarization. Remarkably, we obtain qualitatively comparable behaviors of both limits although they are based on very different assumptions. The quantitative differences are related to the different shifting mechanisms of the subgap peaks already pointed out above. Hence, the best comparison is found for small values of $\delta\varphi$ (ballistic) and G^ϕ (diffusive). We find a zero crossing at a finite temperature in both cases. The similarity of local and nonlocal parameters for small temperatures can be understand from the thermally insulating behavior of superconductors at small temperatures.

We observe that the coefficients in Eq. (1) fulfill a generalized Onsager symmetry. Onsager's symmetry for local currents was originally derived from microscopic reversibility [28]. Generalizations of Onsager's reciprocity theorem have been recently discussed using statistical arguments [45–47]. Here, we find a generalization for

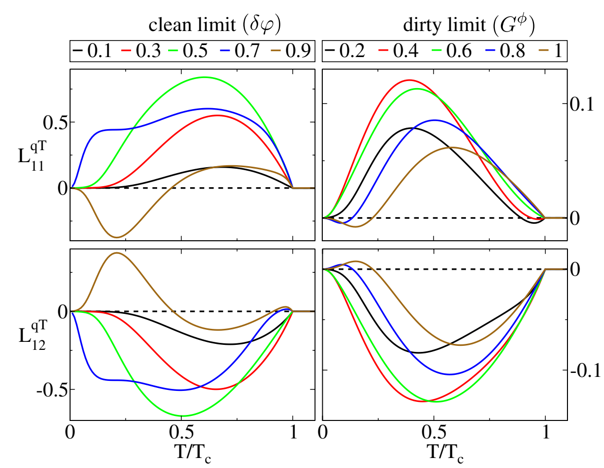

FIG. 4 (color online). Temperature dependence of local and nonlocal thermoelectric coefficients for a symmetric setup in the clean and dirty limits for various spin-mixing parameters $\delta\varphi$ and G^ϕ. Both coefficients are normalized to the normal state value of the nonlocal conductance $(L_{12}^{qV})_{T>T_c}$ and are plotted in units of $k_{\mathrm{B}}T_c/|q|$. Here, $\mathcal{P}_n \equiv \mathcal{P} = 0.1$, $\eta_{\mathrm{Th}} = \Delta_0$, and all other parameters are the same as in Fig. 3.

nonlocal superconductor-ferromagnet three-terminal devices that include supercurrents as well as crossed Andreev reflection processes. This follows directly from the analytical formulas (5)–(10) in the clean limit using relations like $\Gamma_j(\varepsilon; -\delta\varphi) = -\Gamma_j^*(-\varepsilon; \delta\varphi)$ (an example is given in the Supplemental Material [43]) and is also verified numerically for the diffusive case. This Onsager symmetry holds for any relative angle between the magnetization axes of the two ferromagnets.

In conclusion, we have opened a way of utilizing thermoelectric effects in superconducting spintronics. This possibility of controlling energy flow in superconducting heterostructures with spin polarized electrodes allows for a multitude of novel applications. Particularly interesting for applications is our finding of a zero crossing in the Seebeck coefficients as a function of temperature, spin polarization, and the relative angle of the magnetization axes. This not only would give a possibility to measure spin-filtering parameters and the so far experimentally inaccessible spin-mixing parameters but would also allow for sensitive and controllable thermal elements in superconducting circuits.

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