

## Entanglement Replication in Driven Dissipative Many-Body systems

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We study the dissipative dynamics of two independent arrays of many-body systems, locally driven by a common entangled field. We show that in the steady state the entanglement of the driving field is reproduced in an arbitrarily large series of inter-array entangled pairs over all distances. Local nonclassical driving thus realizes a scale-free *entanglement replication* and long-distance entanglement distribution mechanism that has immediate bearing on the implementation of quantum communication networks.

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Driving quantum systems to desired target states with very high fidelity is a central goal in quantum sciences and technologies, in order to realize efficient and scalable devices beyond the current state of proof-of-principle demonstrations. In pursuing this end, it has surfaced in recent years that the effects of noise and dissipation do not necessarily have to be detrimental in the realization of quantum coherent structures [1–5]. The possibility of using suitably engineered irreversible dynamics to control quantum many-body systems has been discussed in a variety of settings, including driven dissipative ultracold atoms in optical lattices [6], the asymptotic realization of entangled states and quantum computation in quantum spin models [7,8], the dissipative control of trapped ions [9], and the steady-state entanglement of macroscopic atomic ensembles [10]. On the other hand, ever since the formulation of the proposal for quantum repeaters [11] and the design of schemes for the implementation of remote quantum communication and distributed quantum gates [12], quantum networks have emerged as the strongest viable paradigm for the “quantum internet,” i.e., the implementation of scalable quantum computation and information processing satisfying the combined requirements of robustness, flexibility, multi-tasking and long reach [13]. A key ingredient of a quantum internet is the ability to hybridize, i.e., to interface heterogeneous subsystems in a reliable and reproducible way. The strive toward the realization of such interfaces has been boosted by recent groundbreaking demonstrations of high-efficiency entanglement and state transfer between light and matter systems [14–16] and of light-mediated teleportation between remote nodes of a simple quantum network [17].

In this context, light-matter interfaces for the distribution of entanglement among network nodes that exploit the robustness of irreversible dynamics have been explored in several works [18–20]. There, it was shown that a reservoir of entangled light can drive distant matter systems into entangled states, thereby realizing an efficient

transfer of entanglement from continuous- to discrete-variable systems.

In the present Letter, we show that, when considering independent arrays of many-body quantum systems, this mechanism amounts to the *replication* of the driving entanglement over many pairs of subsystems across the initially independent arrays. Specifically, we address the irreversible dynamics of two noninteracting chains of quantum systems simultaneously driven, on one of their ends, by an entangled two-mode squeezed field (squeezed bath). The constituents in each array are coupled by nearest-neighbor linear interactions whose specific form is introduced below for different models. The competition between the “entanglement pumping” process and the intra-array couplings results in a steady state consisting of a series of inter-array entangled pairs, each involving subsystems occupying corresponding sites in the respective chain (see Fig. 1). Thereby, an arbitrary number of copies of identically entangled states is generated across the two arrays without violating fundamental constraints such as the no-cloning and the no-broadcasting theorems [21].

The replication mechanism works efficiently in different settings such as chains of harmonic oscillators or of spins. For pure harmonic resonators in the stationary state, exactly  $N$  interchain pairs are formed that replicate the

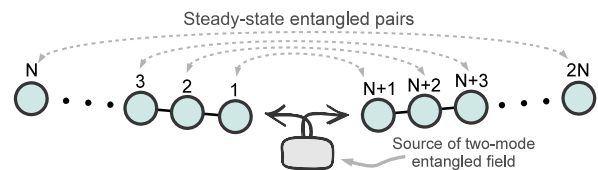


FIG. 1 (color online). A pair of independent arrays of linearly coupled quantum systems is locally driven by a two-mode entangled field. The elements in each array are labeled by the indices  $j \in [1, N]$  (first chain) and  $j \in [N + 1, 2N]$  (second chain). The steady-state inter-array entangled pairs are marked by dashed arrows.

driving state independently of the size of the arrays. For two-level systems, an ideal Einstein-Podolsky-Rosen driving field creates exactly  $N$  Bell states across the two chains.

To start, let us consider two chains of resonators, realizing two disjoint Jaynes-Cummings lattices [22,23] that can describe, in limiting cases, the physics of different condensed-matter systems ranging from spin chains to boson or fermion lattice models. The two arrays are assumed equal (deviations from this condition are discussed below), and each consists of  $N$  single-mode cavities with equal resonance frequency and corresponding annihilation (creation) operators  $\hat{a}_j$  ( $\hat{a}_j^\dagger$ ). Cavities belonging to the same array interact via nearest-neighbor linear coupling with strength  $\eta_j$ . Moreover, each cavity can interact resonantly with a two-level system (e.g., an atom in the cavity) with lowering (raising) operator  $\hat{\sigma}_j$  ( $\hat{\sigma}_j^\dagger$ ). As illustrated in Fig. 1, the elements of the first (second) array are labeled by indices  $j \in [1, N]$  ( $j \in [N+1, 2N]$ ). The two end cavities 1 and  $N+1$  are driven by a two-mode squeezed field. Including the dissipation of the cavity fields [24], the master equation describing the system dynamics is  $\dot{\rho} = -i[H_c + H_{cs}, \rho] + \mathcal{L}_D \rho + \mathcal{L}_S \rho$ . The unitary part of the evolution is ruled by the Hamiltonian  $H_c + H_{cs}$  with  $H_c = \sum_{j=1}^{N-1} \eta_j (\hat{a}_j^\dagger \hat{a}_{j+1} + \hat{a}_{N+j}^\dagger \hat{a}_{N+j+1} + \text{H.c.})$  describing the coherent cavity dynamics and  $H_{cs} = \sum_{j=1}^N g_j (\hat{\sigma}_j^\dagger \hat{a}_j + \hat{\sigma}_{N+j}^\dagger \hat{a}_{N+j} + \text{H.c.})$  accounting for the interaction (with coupling  $g_j$ ) between cavity  $j$  and its two-level system. The term  $\mathcal{L}_D$  accounts for the dissipation of the cavities (at rate  $\kappa_j$ ) and reads  $\mathcal{L}_D \rho = \sum_{j=1}^{2N} \kappa_j (2\hat{a}_j \rho \hat{a}_j^\dagger - \{\hat{a}_j^\dagger \hat{a}_j, \rho\})$ . Finally,  $\mathcal{L}_S$  accounts for the driving (at rate  $\zeta$ ) of the first-end pair of cavities (1,  $N+1$ ) by the external two-mode squeezed field [18–20]

$$\begin{aligned} \mathcal{L}_S \rho = & 2\zeta \bar{m} (\hat{a}_1 \rho \hat{a}_{N+1} + \hat{a}_{N+1} \rho \hat{a}_1 \\ & - \hat{a}_1 \hat{a}_{N+1} \rho - \rho \hat{a}_1 \hat{a}_{N+1} + \text{H.c.}) \\ & + \sum_{j=1, \dots, N+1} \zeta [(\bar{n} + 1)(2\hat{a}_j \rho \hat{a}_j^\dagger - \{\hat{a}_j^\dagger \hat{a}_j, \rho\}) \\ & + \bar{n}(2\hat{a}_j^\dagger \rho \hat{a}_j - \{\hat{a}_j \hat{a}_j^\dagger, \rho\})]. \end{aligned}$$

The sum is over indices  $j = 1$  and  $j = N+1$  only, while  $\bar{n}$  and  $\bar{m}$  are related to the statistics of the driving two-mode entangled field:  $\bar{n}$  is the same average photon number for both modes,  $\bar{m}$  accounts for the intermode correlations, and  $\bar{m} \leq \sqrt{\bar{n}(\bar{n} + 1)}$ , with equality holding in the squeezed vacuum. This effective model is based on the elimination of the degrees of freedom of the reservoir (the driving field) in the limit of a large squeezing bandwidth [18–20]. The entanglement in the driving field is the resource to be transferred via the replication mechanism. The state of the driving field is  $\rho_{\text{sq}}^{(\text{in})} = \hat{U}_{\text{in}} \rho_T \hat{U}_{\text{in}}^\dagger$ , with  $\hat{U}_{\text{in}} = e^{\int d\omega r(\omega) (\hat{a}_\omega^\dagger \hat{b}_\omega - \hat{a}_\omega \hat{b}_\omega^\dagger)}$ , where  $\hat{a}_\omega$  and  $\hat{b}_\omega$  are the field mode operators and  $\rho_T$  is a thermal state with  $\bar{n}_T$  average photons. The condition of a large squeezing bandwidth corresponds to an almost constant squeezing parameter,

$r(\omega) \sim r_0$ , over a sufficiently large range of frequencies around the cavity resonance. In this situation, the parameters characterizing the entangled driving field are  $\bar{n} = \bar{n}_T + (2\bar{n}_T + 1) \sinh^2 r_0$ ,  $\bar{m} = (\bar{n}_T + 1/2) \sinh(2r_0)$ . The entanglement is quantified by the logarithmic negativity  $\mathcal{E}_N = \max[0, -\log \nu_-]$  with  $\nu_- = 2\bar{n} + 1 - 2\bar{m}$  the smallest symplectic eigenvalue of the partially transposed covariance matrix for the two-mode field [25]. The state is entangled iff  $\nu_- < 1$ , which implies  $\bar{m} > \bar{n}$ .

An exact analytical solution for the steady state is obtained if the arrays are driven by a two-mode squeezed vacuum [ $\bar{m} = \sqrt{\bar{n}(\bar{n} + 1)}$ ], and  $\mathcal{L}_D = 0$ . To obtain the steady state in this situation, we exploit the squeezing transformation  $U = \otimes_{j=1}^N U_{j, N+j}$ , with  $U_{j, N+j} = e^{(-1)^j r_0 (\hat{a}_j^\dagger \hat{a}_{N+j}^\dagger - \hat{a}_j \hat{a}_{N+j})}$ , which maps the system into an equivalent one, whose density matrix  $\tilde{\rho} = U^\dagger \rho U$  satisfies the master equation  $\dot{\tilde{\rho}} = -i[H_c + \tilde{H}_{cs}, \tilde{\rho}] + \tilde{\mathcal{L}}_S \tilde{\rho} \equiv \tilde{\mathcal{L}} \tilde{\rho}$ . The new dissipative term reads  $\tilde{\mathcal{L}}_S \tilde{\rho} = \sum_{j=1, \dots, N+1} \zeta [(2\hat{a}_j \tilde{\rho} \hat{a}_j^\dagger - \{\hat{a}_j^\dagger \hat{a}_j, \tilde{\rho}\})]$ , and the transformed Hamiltonian for the cavity-atom interaction is  $\tilde{H}_{cs} = \sum_{j=1}^N g_j [\hat{a}_j^\dagger \hat{C}_j(\bar{n}) + \hat{a}_{N+j}^\dagger \hat{D}_j(\bar{n}) + \text{H.c.}]$ , with  $\hat{C}_j(\bar{n}) = \sqrt{\bar{n} + 1} \hat{\sigma}_j + (-1)^j \sqrt{\bar{n}} \hat{\sigma}_{N+j}^\dagger$  and  $\hat{D}_j(\bar{n}) = \sqrt{\bar{n} + 1} \hat{\sigma}_{j+N} + (-1)^j \sqrt{\bar{n}} \hat{\sigma}_j^\dagger$ . This shows that, in the new representation, the arrays are in contact with a vacuum reservoir and that each field mode interacts with two atoms at sites  $(j, N+j)$ . It turns out that, regardless of the actual values of  $g_j$  and  $\eta_j$ ,  $\forall j \in [1, N]$ , the unique steady state is the pure state (that satisfies  $\tilde{\mathcal{L}}|\varphi\rangle\langle\varphi| = 0$ ) of the form  $|\varphi\rangle = |\phi\rangle \otimes_{j=1}^N |\tilde{0}, \tilde{0}\rangle_{j, N+j}$ , i.e., the tensor product of the transformed modes' vacua with the atomic entangled state

$$|\phi\rangle = \otimes_{j=1}^N \left[ \sqrt{1 - c_{\bar{n}}^2} |1, 1\rangle_{j, N+j} + (-1)^{j+1} c_{\bar{n}} |2, 2\rangle_{j, N+j} \right]. \quad (1)$$

Here,  $|1\rangle$  and  $|2\rangle$  indicate the ground and excited atomic states, and  $c_{\bar{n}} = \sqrt{\bar{n}/(2\bar{n} + 1)}$ . Due to the destructive interference between transition amplitudes involving the atomic pair  $(j, N+j)$  that is coupled to the same mode, state  $|\varphi\rangle$  is such that the atoms are decoupled from the field. Moreover, it is not affected by dissipation because the field modes are in their vacuum state. Therefore, during the dynamics, population accumulates, eventually pumping the system into the entangled state of Eq. (1). Going back to the original representation (by inverting the transformation  $U$ ), the field modes also become entangled in inter-array two-mode squeezed vacua for each pair  $(j, N+j)$ :  $U_{j, N+j} |\tilde{0}, \tilde{0}\rangle_{j, N+j}$ . All inter-array field pairs have the same entanglement of the input driving field, thus realizing a *perfect entanglement replication mechanism*. On the other hand, the entanglement of all inter-array atomic pairs is the same as that discussed in Refs. [18–20] for a single atomic pair but with the essential difference that it is now *exactly* replicated across all the  $N$  pairs. *This is the main result of this Letter*: From an ideal, infinitely entangled state of the driving field, one obtains by engineered dissipation an arbitrary number

of Einstein-Podolsky-Rosen field pairs and Bell states of the atomic pairs. In general, the entanglement of the pairs is limited only by the amount of entanglement of the driving field. Moreover, as will be shown below, this result is rather general, as it holds valid also for spin chains and arrays of harmonic oscillators.

We will now study the effects of a non-negligible thermal nature of the driving field and of other sources of dissipation and noise. We consider first the limit in which the model reduces to two chains of harmonic oscillators, i.e., when the atoms are not present ( $g_j = 0 \forall j$ ). In this case, an exact analytical solution is found also if the external field is not perfectly squeezed,  $\bar{m} \leq \sqrt{\bar{n}(\bar{n} + 1)}$ . We still assume that  $\mathcal{L}_D = 0$  and we find that, in the squeezed representation, the steady state of each cavity is thermal,  $\tilde{\rho}_T^{(j)}$ , with mean occupation number  $\bar{n}_T$ . In the antitransformed representation, this corresponds to a two-mode squeezed thermal state for each pair of field modes ( $j, N + j$ ) that reads  $U_{j,j+N} \tilde{\rho}_T^{(j)} \otimes \tilde{\rho}_T^{(j+N)} U_{j,j+N}^\dagger$ . The corresponding steady-state entanglement is *the same* as that of the driving field, regardless of  $j, N, \bar{n}$ , and  $\bar{m}$ . Therefore, the exact replication of the driving field entanglement also takes place in this case. When the other sources of dissipation described by  $\mathcal{L}_D$  are included, the steady state of the system can be determined numerically, and the logarithmic negativity  $\mathcal{E}_N[j, k]$  of any pair ( $j, k$ ) of cavity fields is obtained from the corresponding covariance matrix [25]. Quantitatively, we study the logarithmic negativity normalized to unity, defined as  $E_N^{(\text{cav})}[j, k] = \mathcal{E}_N[j, k]/(1 + \mathcal{E}_N[j, k])$ .

Most of the results to follow are obtained for a reservoir with  $\bar{n} = 1$ , such that the corresponding entanglement is relatively small. Remarkably, even in this strongly non-ideal situation, the replication mechanism is significantly resilient to the added noise. As shown in Fig. 2(a), the entanglement decreases with the decay rate of the cavities. At a fixed decay rate, the largest  $E_N^{(\text{cav})}$  is achieved by the pair  $(1, N + 1)$  that is directly coupled to the driving field. The entanglement of the other pairs decreases moderately with the distance from the driven pair and exhibits a weak

revival for a few pairs at the opposite end of the arrays. Figure 2(b) illustrates how the entanglement mildly decays with the size of the arrays, remaining nonvanishing up to large values of  $N$ . Hence, the entanglement replication mechanism exhibits a notable robustness in the presence of losses. The dependence of the entanglement on the statistics of the input field is shown in Figs. 2(c) and 2(d). When the driving is a squeezed vacuum, its entanglement increases with  $\bar{n}$  [gray line in Fig. 2(c)] and reaches unity asymptotically as  $\bar{n} \rightarrow \infty$ . For lossy cavities, the entanglement saturates to a value smaller than unity that depends on the pair being considered. The entanglement distributed through a squeezed thermal state is reported in Fig. 2(d), showing that  $E_N^{(\text{cav})}$  is nonvanishing for all values of  $\bar{m}$  for which the driving field is entangled ( $\bar{m} > \bar{n}$ ). When only the end cavities are open ( $\kappa_{j \neq N, 2N} = 0$ ), the pairwise entanglement is minimum at  $\kappa_N = \kappa_{2N} \simeq \eta$  for all pairs  $(j, N + j)$ , except for the pair  $(N, 2N)$  whose entanglement instead decreases monotonically with  $\kappa_N$  (see Fig. 2(e)). As  $\kappa_N$  increases, the coherent coupling between the last cavity of each array and the neighboring one is progressively inhibited. At large values of  $\kappa_N$ , each of them is effectively decoupled from the rest of the system, whose entanglement is thus restored to the value of the nondissipative case. Moreover, the field leaking out of the last pair of cavities is entangled as well [26] and even equal to that of the driving field for some frequencies [26]. This feature allows for the reusability of the transferred entanglement for networking protocols. So far, we have discussed results obtained with homogeneous couplings  $\eta_j \equiv \eta$ . Analogous results hold even with intra-array patterns of inhomogeneous couplings, as long as the two arrays remain equal. Asymmetries between the arrays reduce the inter-array entanglement, but the replication mechanism remains valid as long as they are not too strong. This is shown in Fig. 3(a), obtained for random couplings  $\eta_j = \eta_0 + \xi_j$ , with  $j \in [1, 2N]$ , where  $\xi_j$  are zero-mean random variables uniformly distributed in a range  $\Delta\xi$ .

When each cavity interacts with a two-level atom, we can study the entanglement properties of the atoms by

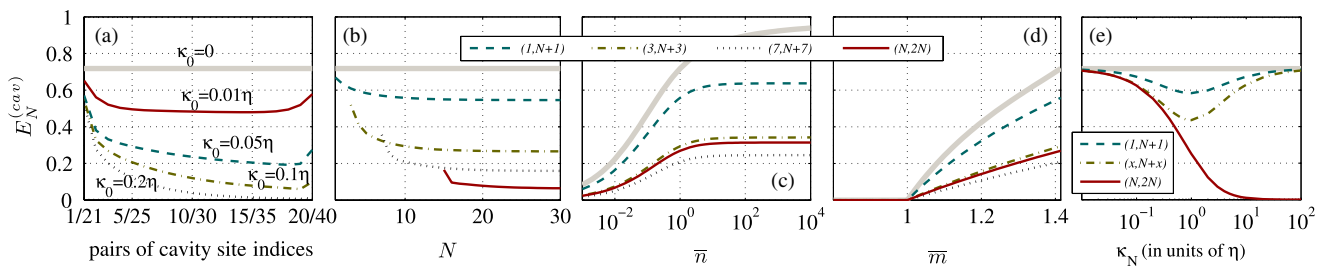


FIG. 2 (color online).  $E_N^{(\text{cav})}$  as a function of (a) the pair-site label ( $j, N + j$ ) (with  $N = 20$  and  $\kappa_j \equiv \kappa_0 \forall j$ ), (b)  $N$  (with  $\kappa_j = 0.1\eta$ ,  $\forall j$ ), (c)  $\bar{n}$  [with  $\bar{m} = \sqrt{\bar{n}(\bar{n} + 1)}$ ], (d)  $\bar{m}$  (with  $\bar{n} = 1$ ), and (e)  $\kappa_N = \kappa_{2N}$  (with  $\kappa_{j \neq N, 2N} = 0$ ). In all cases,  $\eta_j = \eta$ ,  $\forall j$ , and  $\zeta = \eta$ . The remaining parameters in (a), (b), and (e) are  $\bar{n} = 1$  and  $\bar{m} = \sqrt{\bar{n}(\bar{n} + 1)}$ ; in (c) and (d), they are  $\kappa_j = 0.1\eta$  and  $N = 10$ . The insets indicate the pair  $(j, N + j)$  corresponding to each line. In (e), the dash-dotted curve corresponds to all pairs  $(j, N + j)$  for  $j \in [2, N - 1]$ . These results are independent of  $N$  and have been verified numerically for arrays of size up to  $N = 30$ . The solid thick (gray) lines report the entanglement of the driving field that is equal to the entanglement of each pair when  $\kappa_j = 0$ ,  $\forall j$ .

approximating the system with an effective spin model. We focus on the weak coupling limit, such that the couplings  $g_j$  between the atoms and the cavities are sufficiently small [26] and we can adiabatically eliminate the cavity fields to find a closed equation for the atoms. The resulting spin model exhibits nontrivial long-range interactions and collective decay of the spins, as reported in detail in the Supplemental Material [26]. Here, we discuss the results relevant for the corresponding steady state. Let us consider the logarithmic negativity  $E_N^{(\text{at})}[j, k] = \log_2 \|\rho_{jk}^{\text{PT}}\|_1$  of the state  $\rho_{jk}$  of the atomic pair  $(j, k)$ , where  $\|\cdot\|_1$  is the trace norm and PT stands for partial transposition. The entanglement properties of the atoms are similar to those of the free cavity fields. However, at variance with the latter case,  $E_N^{(\text{at})}[j, k]$  is sensitive to the statistics of the driving entangled field and decreases more rapidly with decreasing  $\bar{m}$ , as illustrated in Fig. 3(b).

The effective spin model with long-range interactions can be compared with the case in which two independent spin chains with  $XX$  short-range interactions are coupled on one end to the driving field. As shown in Figs. 3(b) and 3(c), one obtains very similar results. The master equation for this case reads  $\dot{\rho} = -i[H_s, \rho] + \mathcal{L}_S \rho$ , with  $H_s = \frac{1}{2} \sum_{j=1}^{N-1} \sum_{k=0, \dots, N} J_j (\hat{\sigma}_{k+j}^x \hat{\sigma}_{j+k+1}^x + \hat{\sigma}_{k+j}^y \hat{\sigma}_{j+k+1}^y)$ , where  $J_j$  is the spin-spin coupling and  $\hat{\sigma}_j^{x,y}$  are the Pauli spin operators. The effect of the driving field is described by

$$\begin{aligned} \frac{\mathcal{L}_S \rho}{\gamma} = & 2\bar{m}(\hat{\sigma}_1 \rho \hat{\sigma}_{N+1} + \hat{\sigma}_{N+1} \rho \hat{\sigma}_1 \\ & - \hat{\sigma}_1 \hat{\sigma}_{N+1} \rho - \rho \hat{\sigma}_1 \hat{\sigma}_{N+1} + \text{H.c.}) \\ & + \sum_{j=1, \dots, N+1} [(\bar{n} + 1)(2\hat{\sigma}_j \rho \hat{\sigma}_j^+ \\ & - \{\hat{\sigma}_j^+ \hat{\sigma}_j, \rho\}) + \bar{n}(2\hat{\sigma}_j^+ \rho \hat{\sigma}_j - \{\hat{\sigma}_j \hat{\sigma}_j^+, \rho\})], \end{aligned}$$

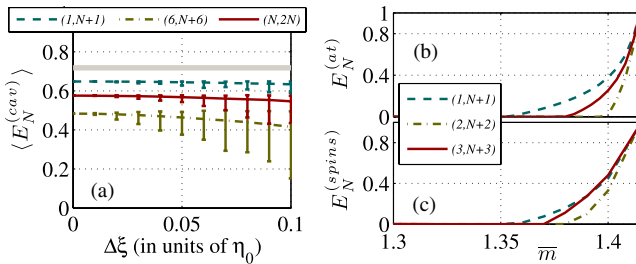


FIG. 3 (color online). (a)  $E_N^{\text{cav}}$  for a model with random couplings as specified in the text. The curves are obtained by averaging the result over 500 realizations. For each value of  $\Delta\xi$ , the vertical bars represent the interval between the realizations of maximum and minimum entanglement. The other parameters are  $N = 10$ ,  $\bar{n} = 1$ ,  $\bar{m} = \sqrt{\bar{n}(\bar{n} + 1)}$ ,  $\zeta = \eta_0$ , and  $\kappa_j = 0.02\eta_0 \forall j$ . (b), (c) Comparison between the logarithmic negativity for atoms in cavity arrays, (b)  $E_N^{(\text{at})}$ , and for spins in  $XX$  spin chains, (c)  $E_N^{(\text{spins})}$ , as functions of  $\bar{m}$  for  $\bar{n} = 1$ . The remaining parameters are  $\kappa_j = 0 \forall j$ ,  $N = 3$ ,  $\zeta = \eta$ , and  $g = 0.01\eta$  for the atoms and  $N = 3$ , and  $J_j = \gamma \forall j$  for the spins. The insets specify the correspondence between curves and pairs  $(j, j + N)$ .

with  $\sigma_j$  ( $\sigma_j^+$ ) the spin lowering (raising) operator. While in the cavity-atom system the effective spin-spin interactions are long range [26], here we deal only with local ones. Nevertheless, entanglement replication continues to hold. Indeed, the stationary state of the system for  $\bar{m} = \sqrt{\bar{n}(\bar{n} + 1)}$  can be evaluated analytically and coincides with that of Eq. (1), where  $|1\rangle$  and  $|2\rangle$  now denote, respectively, the spin-up and spin-down states. Finally, we observe that the similarity of the steady-state entanglement properties in the two systems holds even when the driving field has a nonvanishing thermal component, as shown in Figs. 3(b) and 3(c). This result shows the generality of the entanglement replication mechanism that is largely independent of the specific physical realization.

In conclusion, we have discussed a scheme realizing the replication of entanglement, based on the interface of a driving two-mode entangled field with two distant and independent dissipative many-body systems. The replication mechanism works efficiently both for arrays of discrete- and continuous-variable systems. Since the phenomenon occurs in the steady state of the irreversible driven dissipative dynamics, it exhibits an intrinsic robustness against the detrimental effects of noise. We have highlighted the roles played by quantum interference and the competition between dissipation, driving, and interactions in producing such a steady state. The corresponding entanglement is robust against deviations from ideal conditions including a nonvanishing thermal component of the driving field, asymmetries between the arrays, and decay of the cavity fields. Ideally, the replication mechanism yields an arbitrary number of maximally entangled pairs and is scale-free in the sense that it is independent of the actual length of the arrays. Thus, it is a potentially valuable resource for remote quantum communication and distributed quantum computation [12,13] that could be combined with other driven dissipative strategies for the realization of scalable quantum networks [27]. Seen from a different viewpoint, this scheme implements a protocol of long-distance entanglement distribution [28,29] and nested entangled-pair production [30], two key tasks for quantum networking, achieved via the interactions intrinsic in many-body systems.

The outlined scheme is general and flexible enough to find application in many systems that effectively realize chains of harmonic oscillators or spins, such as cavity or circuit QED [31,32], arrays of optomechanical systems, trapped ions, or ultracold atoms in optical lattices. The mechanism could be verified with arrays of coupled resonators, recently produced in photonic crystals [33,34], that realize chains of linearly coupled harmonic oscillators. In Ref. [33], the cavities are almost resonant and they interact with nearest-neighbor couplings of strength within the range  $\sim 60\text{--}2000$  GHz. These values can be tailored by selecting the distance between the cavities. The reported cavity linewidth is of the order of  $\sim 1$  GHz. These parameters are consistent with those discussed in our analysis. However, the broadest squeezing at the wavelength of the



resonators of Ref. [33] ( $\sim 1.5 \mu\text{m}$ ) has a bandwidth of about  $\sim 2$  GHz [35]. This value is still relatively small and does not well satisfy the broadband condition assumed throughout our work. Nevertheless, larger squeezing bandwidths and photonic-crystal nanocavities with weaker decay rates are expected to be realizable in the near future [35,36], thus matching the required condition. On the other hand, the currently available experimental situation might already suffice for testing the entanglement replication mechanism. Indeed, a relevant theoretical question that deserves further investigation is whether entanglement replication holds also for driving squeezed fields of finite bandwidth.

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- [1] J. F. Poyatos, J. I. Cirac, and P. Zoller, *Phys. Rev. Lett.* **77**, 4728 (1996).
- [2] M. B. Plenio and S. F. Huelga, *Phys. Rev. Lett.* **88**, 197901 (2002).
- [3] F. Benatti, R. Floreanini, and M. Piani, *Phys. Rev. Lett.* **91**, 070402 (2003).
- [4] S. Pielawa, G. Morigi, D. Vitali, and L. Davidovich, *Phys. Rev. Lett.* **98**, 240401 (2007).
- [5] R. Schmidt, A. Negretti, J. Ankerhold, T. Calarco, and J. T. Stockburger, *Phys. Rev. Lett.* **107**, 130404 (2011).
- [6] S. Diehl, A. Micheli, A. Kantian, B. Kraus, H. P. Büchler, and P. Zoller, *Nat. Phys.* **4**, 878 (2008); S. Diehl, E. Rico, M. A. Baranov, and P. Zoller, *Nat. Phys.* **7**, 971 (2011).
- [7] B. Kraus, H. P. Büchler, S. Diehl, A. Kantian, A. Micheli, and P. Zoller, *Phys. Rev. A* **78**, 042307 (2008).
- [8] F. Verstraete, M. M. Wolf, and J. I. Cirac, *Nat. Phys.* **5**, 633 (2009).
- [9] J. T. Barreiro, M. Müller, P. Schindler, D. Nigg, T. Monz, M. Chwalla, M. Hennrich, C. F. Roos, P. Zoller, and R. Blatt, *Nature (London)* **470**, 486 (2011).
- [10] H. Krauter, C. A. Muschik, K. Jensen, W. Wasilewski, J. M. Petersen, J. I. Cirac, and E. S. Polzik, *Phys. Rev. Lett.* **107**, 080503 (2011).
- [11] H.-J. Briegel, W. Dür, J. I. Cirac, and P. Zoller, *Phys. Rev. Lett.* **81**, 5932 (1998).
- [12] D. Gottesman and I. Chuang, *Nature (London)* **402**, 390 (1999); J. Eisert, K. Jacobs, P. Papadopoulos, and M. B. Plenio, *Phys. Rev. A* **62**, 052317 (2000); D. Collins, N. Linden, and S. Popescu, *ibid.* **64**, 032302 (2001); S. F. Huelga, M. B. Plenio, and J. A. Vaccaro, *ibid.* **65**, 042316 (2002); M. Paternostro, M. S. Kim, and G. M. Palma, *J. Mod. Opt.* **50**, 2075 (2003).
- [13] H. J. Kimble, *Nature (London)* **453**, 1023 (2008); L.-M. Duan and C. Monroe, *Rev. Mod. Phys.* **82**, 1209 (2010).
- [14] A. I. Lvovsky, B. C. Sanders, and W. Tittel, *Nat. Photonics* **3**, 706 (2009) and references therein.
- [15] B. B. Blinov, D. L. Moehring, L.-M. Duan, and C. Monroe, *Nature (London)* **428**, 153 (2004).
- [16] E. Togan, Y. Chu, A. S. Trifonov, L. Jiang, J. Maze, L. Childress, M. V. G. Dutt, A. S. Sørensen, P. R. Hemmer, A. S. Zibrov, and M. D. Lukin, *Nature (London)* **466**, 730 (2010).
- [17] S. Olmschenk, D. N. Matsukevich, P. Maunz, D. Hayes, L.-M. Duan, and C. Monroe, *Science* **323**, 486 (2009).
- [18] B. Kraus and J. I. Cirac, *Phys. Rev. Lett.* **92**, 013602 (2004).
- [19] M. Paternostro, W. Son, and M. S. Kim, *Phys. Rev. Lett.* **92**, 197901 (2004).
- [20] G. Adesso, S. Campbell, F. Illuminati, and M. Paternostro, *Phys. Rev. Lett.* **104**, 240501 (2010).
- [21] W. K. Wootters and W. H. Zurek, *Nature (London)* **299**, 802 (1982); H. Barnum, C. M. Caves, C. A. Fuchs, R. Jozsa, and B. Schumacher, *Phys. Rev. Lett.* **76**, 2818 (1996).
- [22] M. J. Hartmann, F. G. S. L. Brandão, and M. B. Plenio, *Nat. Phys.* **2**, 849 (2006); A. D. Greentree, C. Tahan, J. H. Cole, and L. C. L. Hollenberg, *Nat. Phys.* **2**, 856 (2006); F. Illuminati, *Nat. Phys.* **2**, 803 (2006); D. G. Angelakis, M. F. Santos, and S. Bose, *Phys. Rev. A* **76**, 031805 (2007).
- [23] M. J. Hartmann, F. G. S. L. Brandão, and M. B. Plenio, *Laser Photonics Rev.* **2**, 527 (2008); A. Tomadin and R. Fazio, *J. Opt. Soc. Am. B* **27**, A130 (2010); A. A. Houck, H. E. Türeci, and J. Koch, *Nat. Phys.* **8**, 292 (2012) and references therein.
- [24] Spontaneous decay of the two-level systems is neglected. The effect of its inclusion in similar settings is discussed in Refs. [18–20]. It amounts to straightforward quantitative effects that do not modify the essential qualitative aspects of the model.
- [25] G. Adesso and F. Illuminati, *J. Phys. A* **40**, 7821 (2007).
- [26] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevLett.110.040503> for an additional analysis on the properties of the system.
- [27] K. G. H. Vollbrecht, C. A. Muschik, and J. I. Cirac, *Phys. Rev. Lett.* **107**, 120502 (2011).
- [28] L. C. Venuti, C. D. E. Boschi, and M. Roncaglia, *Phys. Rev. Lett.* **96**, 247206 (2006); L. C. Venuti, S. M. Giampaolo, F. Illuminati, and P. Zanardi, *Phys. Rev. A* **76**, 052328 (2007).
- [29] S. M. Giampaolo and F. Illuminati, *Phys. Rev. A* **80**, 050301 (R) (2009); S. M. Giampaolo and F. Illuminati, *New J. Phys.* **12**, 025019 (2010); G. Gualdi, S. M. Giampaolo, and F. Illuminati, *Phys. Rev. Lett.* **106**, 050501 (2011).
- [30] C. Di Franco, M. Paternostro, and M. S. Kim, *Phys. Rev. A* **77**, 020303 (2008).
- [31] G. Lepert, M. Trupke, M. J. Hartmann, M. B. Plenio, and E. A. Hinds, *New J. Phys.* **13**, 113002 (2011).
- [32] R. J. Schölkopf and S. M. Girvin, *Nature (London)* **451**, 664 (2008).
- [33] M. Notomi, E. Kuramochi, and T. Tanabe, *Nat. Photonics* **2**, 741 (2008).
- [34] Y. Sato, Y. Tanaka, J. Upham, Y. Takahashi, T. Asano, and S. Noda, *Nat. Photonics* **6**, 56 (2012).
- [35] S. Ast, A. Samblowski, M. Mehmet, S. Steinlechner, T. Eberle, and R. Schnabel, *Opt. Lett.* **37**, 2367 (2012).
- [36] Y. Taguchi, Y. Takahashi, Y. Sato, T. Asano, and S. Noda, *Opt. Express* **19**, 11916 (2011).