Self-Trapping in an Array of Coupled 1D Bose Gases

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We study the transverse expansion of arrays of ultracold ^{87}Rb atoms weakly confined in tubes created by a 2D optical lattice and observe that transverse expansion is delayed because of mutual atom interactions. A mean-field model of a coupled array shows that atoms become localized within a roughly square fortlike self-trapping barrier with time-evolving edges. But the observed dynamics are poorly described by the mean-field model. The theoretical introduction of random phase fluctuations among tubes improves the agreement with experiment but does not correctly predict the density at which the atoms start to expand with larger lattice depths. Our results suggest a new type of self-trapping, where quantum correlations suppress tunneling even when there are no density gradients.

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Ultracold atomic gases trapped by light are well-characterized many-body quantum systems. Without the disorder that is common in condensed matter systems, theoretical analyses of cold gases in equilibrium can be extremely accurate, although high powered numerical techniques must sometimes be employed [1]. Cold gas experiments are also well suited to studying out-of-equilibrium dynamics, such as the evolution of many-body correlations, since experimental time scales are relatively slow yet faster than typical relaxation and decoherence rates [2–4]. Significant progress in the description of the out-of-equilibrium dynamics of 1D bosonic and fermionic systems has been achieved thanks to the existence of exact solutions [5] and the development of numerical methods such as the time-dependent density-matrix renormalization group [6] and time-evolving block decimation [7] methods. Those approaches fail in 2D and higher dimensions, where most out-of-equilibrium calculations rely on approximate analytical techniques that are generally restricted to the weak interaction regime [8]. The experiment-theory comparisons in this Letter illustrate the limitations of these approximate approaches. The difficulty of improving on these approximations highlights the need for more computationally tractable methods for dealing with intermediate coupling. The particular way the theory deviates from the experiment strongly suggests a qualitatively new nonequilibrium effect.

Macroscopic self-trapping in quantum degenerate Bose gases, where mean-field energy gradients suppress tunneling, presents an interesting set of nonequilibrium phenomena, with analogs in nonlinear photon optics [9,10] and Josephson junction arrays [11]. Self-trapping has been studied theoretically in all dimensions [12–20] and experimentally in double-well systems [21] and in arrays of 2D pancakelike Bose-Einstein condensates (BECs) created by a deep 1D lattice potential [22]. These self-trapping experiments are in the weak coupling limit, where mean-field theory clearly applies. In this Letter, we experimentally and theoretically investigate self-trapping behavior in an array of coupled quasi-1D tubes created by a 2D optical lattice. Atoms freely expand along the axes of the tubes, so that their densities decrease with time, until they eventually become too dilute for self-trapping and expand ballistically transversely to the tubes. In contrast to previous self-trapping work, our quasi-1D gases are in the intermediate coupling regime [23]. To provide a baseline theoretical description of our experiments, we build a mean-field model of expanding coupled 1D gases. We then incorporate fluctuations into our mean-field treatment via an approximation to the so-called truncated Wigner approximation (TWA) [8,24] by introducing random tube-to-tube phase fluctuations with a tunable magnitude at the initial time of the evolution. At low lattice depths, small phase fluctuations improve the agreement with the experiment, while the dynamics in deeper lattices are better described when the phases are maximally randomized. Still, the self-trapping seen in the experiment is more widespread across the array and persists to at least 3 times lower densities than even maximally randomized mean-field-based models predict. We postulate a new kind of self-trapping mechanism, based on correlation-suppressed tunneling, that does not depend on density or phase gradients.

It is instructive to first describe the pure mean-field theory evolution qualitatively. In an initially 2D Thomas-Fermi profile, there is a critical distance along the lattice directions at which the mean-field imbalance between adjacent tubes is too large for tunneling to conserve energy. Atoms at larger radii will not tunnel radially outward, while atoms at smaller radii will start to expand, at some point reflecting from the self-trapped edge. Atoms at 45° to the lattice axes are first self-trapped at larger radii since their density gradients are smaller in the lattice directions. The distributions therefore develop a density depression in the middle with a roughly square fortlike barrier around the edges. Since all atoms tunnel in some lattice direction, the self-trapped edges evolve with time. In the absence of axial trapping, densities, and hence tube-to-tube density gradients, drop. Self-trapped edges are eventually lost, leaving the cloud to expand ballistically in the (x, y) plane.

Details of the experimental setup are given in Ref. [25], and the experimental geometry is depicted in Fig. 1(a). A BEC with a barely detectable impurity fraction and $N \approx 3.5 \times 10^5$ ^{87}Rb atoms in the $|F = 1, m_F = 1\rangle$ state is produced in a crossed-optical-dipole trap, created by the intersection of two 1.06 μm wavelength horizontal laser beams. The trapping frequencies experienced by the BEC are $\omega_{x,y} = 2\pi \times 38$ Hz in the horizontal plane and $\omega_z = 2\pi \times 94$ Hz vertically. A vertical magnetic field gradient of 30.5 G/cm levitates the atoms. A two-dimensional square optical lattice is created using two slightly different frequency pairs of linearly polarized retroreflected beams with $1/e^2$ widths of 650 and 705 μm, blue-detuned by 5.2 THz from the $D2$ transition in ^{87}Rb. The 2D lattice is ramped up in time according to $I(t) \propto [1 - (t/\tau)]^{-2}$, where I is the lattice intensity and $\tau = 4.15$ ms is the time constant. Nonadiabaticity in the lattice turn-on, measured by turning off the lattice with the reverse ramp, adds less than 4 nK $\cdot\, k_B$ of energy, which is small compared to the 1D chemical potential (150–250 nK $\cdot\, k_B$).

After the lattice is turned on, the crossed-dipole trap beams are suddenly turned off so that the atomic distribution evolves in the 2D lattice alone. After a time t, we image the *in situ* spatial distribution of the cloud using high-intensity absorption imaging [26]. This technique involves illuminating the atoms with a resonant probe beam with intensity $I \gg I_{\rm sat}$, where $I_{\rm sat}$ is the saturation intensity. After the beam passes through the atomic cloud, it is blocked by a 400 μm diameter dark spot in the Fourier plane of a one-to-one imaging system. Using Babinet's principle, one can show that the detected signal is proportional to n^2, or the square of the atomic density distribution integrated along the line of sight [26]. We use this technique because it is less sensitive than most to lensing by dense atomic clouds. Still, we see a 10% root-mean-square (rms) width change due to lensing at our highest densities, for which we correct by observing the axial expansion in a $35E_R$ lattice, where there is no transverse expansion. $E_R = \hbar^2k^2/2M$ is the recoil energy and M is the atomic mass.

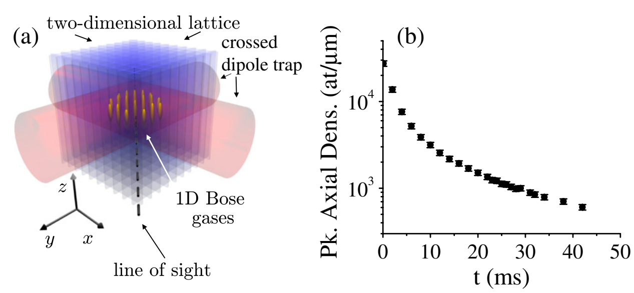

FIG. 1 (color online). (a) Experimental setup (not to scale): Atom clouds are confined in a crossed-dipole trap and partitioned by a blue-detuned optical lattice into a coupled array of vertical tubes. (b) Peak transversely integrated axial density as a function of time for $V_0 = 13E_R$. The linear density drops rapidly with time as the atoms expand axially, mostly driven initially by their mutual interaction energy.

Images along the z axis show no self-trapping depressions, although our imaging system could resolve them if they were there. Instead, when viewed from along z or transversely 45° from the lattice axes, the integrated density-squared profiles are featureless and well fitted by Gaussians. All data displayed below were taken from the side view. We integrate the distributions over the central $\pm$10% of the vertical Thomas-Fermi radius so that the atoms we consider within each tube have approximately the same density. We fit the resulting 1D distribution to a Gaussian and extract σ_{n^2}, the transverse rms width. In Fig. 1(b), we plot an example of how the transversely integrated axial peak density varies with time, which we derive from measuring the axial expansion of atoms. In Fig. 2, we display σ_{n^2} as a function of evolution time in the 2D lattice for lattice depths of $V_0 = (7.25, 9.25, 11, 13)E_R$ (uncertainty $\pm$2.5%).

The behavior of σ_{n^2} is qualitatively different for $V_0 = 7.25E_R$ compared to larger lattice depths. For $V_0 = 7.25E_R$, the rms width of the cloud increases quickly from $t = 0$ and slows near $t = 5$ ms, where the curve changes its concavity. For $V_0 = 9.25$, 11, and $13E_R$, the rms width remains constant to within experimental uncertainty for $t < t_c$ or until the density drops by a factor of $\sim$4, 8, and 10, respectively. For $t > t_c$, the rms width increases linearly in time. We estimate that $t_c = \{4.5 \pm 1.0, 8.5 \pm 1.5, 11 \pm 2\}$ ms for $V_0/E_R = \{9.25, 11, 13\}$ and plot the mean value of the $t < t_c$ data as a horizontal red line in Figs. 2(b)–2(d). We can associate with the time t_c a 3D density at the center of the cloud, $\rho(t_c)$. We find that the ratio $\rho(t_c)/J = \{1190 \pm 310, 860 \pm 190, 910 \pm 210\}$ $\mu\text{m}^{-3}E_R^{-1}$, where J is the tunneling matrix element. The rough constancy of this ratio is in accord with the general concept of a self-trapping threshold [15], but the absence of an initial internal expansion shows that the pure mean-field description is incomplete.

The starting point for our mean-field theory is the Gross-Pitaevskii equation (GPE),

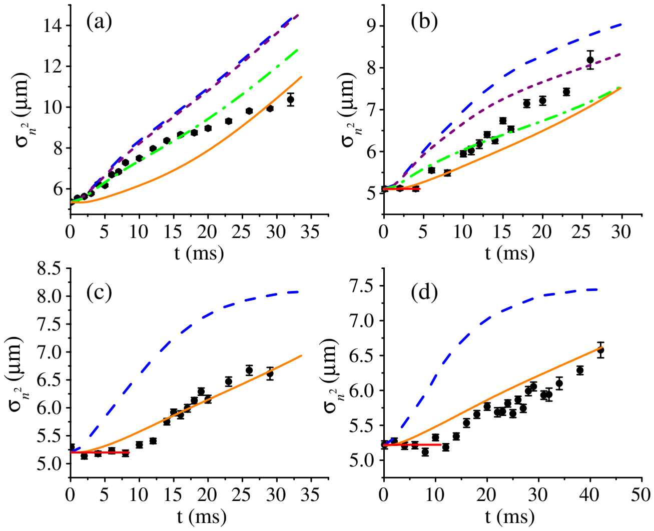

FIG. 2 (color online). Transverse rms widths of the directly measured density-squared distributions as a function of time for (a) $V_0 = 7.25E_R$, (b) $9.25E_R$, (c) $11E_R$, and (d) $13E_R$. Each data point is the average of 10 measurements. The error bars represent our random uncertainty and are the quadrature sum of the statistical noise and the uncertainty from the density dependence of the absorption beam lensing. There is an overall systematic uncertainty of 0.5 μm associated with the imaging resolution; comparable shifts of the initial size simply shift the theoretical curves at early times (below 10 ms), followed by a gradual divergence of the curves at longer times. The dashed blue line is the result of a mean-field calculation of the dynamics with no random phase between the tubes ($\eta = 0$). The dotted purple line is for $\eta = 0.2$, the dash-dotted green lines are for (a) $\eta = 0.4$ and (b) $\eta = 0.5$, and the solid orange line is for $\eta = 1$. The horizontal red line is the mean of the $t < t_c$ data.

$$i\hbar\frac{\partial\Psi}{\partial t} = \left(-\frac{\hbar^2}{2M}\nabla^2 + V(\vec{x}) + g|\Psi|^2\right)\Psi. \tag{1}$$

Here, $\Psi = \Psi(\vec{x}, t)$ is the matter wave field, $g = \frac{4\pi a_s\hbar^2}{M}$, a_s is the s-wave scattering length, $V = V_0[\sin^2(\pi x/d) + \sin^2(\pi y/d)]$ is the optical lattice potential, and d is the lattice spacing.

The wave field $\Psi(\vec{x}, t)$ can be expanded in terms of the lowest band Wannier functions along the optical lattice directions:

$$\Psi(\vec{x}, t) = \sum_{n,m} W_{n,m}(x, y)\Phi_{n,m}(z, t). \tag{2}$$

$N_{n,m}(t) = \int dz|\Phi_{n,m}(z, t)|^2$ is the number of atoms at the site (n, m), satisfying $\sum_{n,m} N_{n,m} = N$, with N the total number of atoms. $W_{n,m}(x, y)$ is the lowest band Wannier orbital localized at site (n, m).

Using the ansatz in Eq. (2) and integrating within each tube over the lattice directions (x and y), we get a set of coupled 1D GPEs:

$$i\hbar\dot{\Phi}_{n,m} = \left(-\frac{\hbar^2}{2M}\frac{\partial^2}{\partial^2 z} + U * d|\Phi_{n,m}|^2\right)\Phi_{n,m} - J[\Phi_{n+1,m} + \Phi_{n-1,m} + \Phi_{n,m+1} + \Phi_{n,m-1}], \tag{3}$$

where $\Phi_{n,m} = \Phi_{n,m}(z, t)$, $J = \int d^2x W^*_{n,m}[\frac{\hbar^2}{2M}\nabla^2 - V]\times W_{n+1,m}$ is the nearest-neighbor tunneling, and $U = \frac{4\pi a_s\hbar^2}{dM}\int d^2x|W_{n,m}|^4$ is the on site interaction energy. We numerically solve the GPEs by discretizing them along z and calculating the kinetic energy term using a Fourier transform method. The widths obtained by assuming full adiabaticity during the lattice turn-on are larger than the experimentally observed widths. We therefore match the initial transverse width to the experimentally observed value and adjust the initial axial size so that the energy released in the axial expansion matches the experimental value.

In the mean-field simulation [27], the central part of the atom cloud starts to expand at $t = 0$. Within several ms, a roughly square fortlike structure forms [see Fig. 3(a)]. Outwardly moving atoms cannot pass the steep density gradient and they reflect back. The self-trapped edges evolve, leading to a complicated and ever-changing density distribution near the edges and a slow spreading of the outer atoms. With axial expansion, the interaction energy differences between adjacent tubes eventually become smaller than the tunneling energy, and the cloud expands ballistically at a constant rate in the transverse direction.

In order to compare the simulation with the experimentally measured rms widths, we extract widths from the simulated distributions in a way that mimics the measurement process (i.e., we square the density distribution and convolve it with a Gaussian of rms width 1.7 μm, the imaging system resolution). This process removes fine features from the theoretical curves that are not resolvable in the experiment, but it does not guarantee that the theoretical distributions are well fitted by Gaussians. In fact, the pure mean-field theory calculations retain clear evidence of self-trapped squares even after this processing [27]. The theoretical rms widths are shown as dashed blue lines in Fig. 2. For all lattice depths presented, the agreement is poor. For $V_0 = 7.25E_R$, the mean-field calculation captures the qualitative dynamics but overestimates the expansion rate at early times. For $V_0 = 9.25$, 11, and $13E_R$, the calculation does not capture the qualitative dynamics. It greatly overestimates the initial expansion rates and predicts that the rms width increases much more slowly during the later ballistic expansion than is observed.

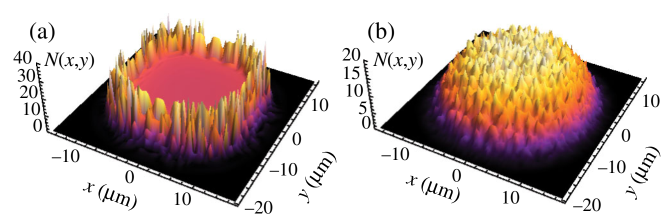

FIG. 3 (color online). Transverse density distributions 12.4 ms after release for $V_0 = 13E_R$. (a) $\eta = 0$; (b) $\eta = 1$.

The mean-field treatment neglects thermal and quantum fluctuations, which can play large roles in the intermediate coupling limit. We note that it is hard to adiabatically transform a 3D gas into a 1D gas in theory [28], and there is some irreversibility in turning on the lattice. Although the system is therefore not initially in its ground state, the observed dynamics are fairly insensitive to the lattice turn-on and the purity of the initial BEC, making quantum fluctuations the better candidate for the missing piece of the model. If we ignore tunneling and calculate the initial coupling strength, γ [29], of atoms near where atoms are predicted by mean-field theory to have a self-trapped edge, it varies from 0.2 to 0.31 for $V_0 = 7.25E_R$ and from 0.32 to 0.5 for $13E_R$. Higher coupling implies larger phase variation along a tube. For small V_0, coherence among the tubes is maintained by tunneling. But, as V_0 is increased and the tubes become more 1D, phase fluctuations along each tube will give rise to phase fluctuations from tube to tube.

The TWA gives leading order quantum corrections to classical dynamics by adding fluctuations to the initial conditions, distributed according to the Wigner function. Since our initial state has moderately strong interactions, the Wigner distribution is difficult to find. We therefore attempt to model emerging intertube fluctuations with an approximate TWA. Specifically, we introduce phase fluctuations into the initial conditions by adding random phases among the tubes: $\phi_{n,m}(t=0) \rightarrow 2\pi\eta\Lambda$, where $\phi_{n,m}(0)$ is the initial phase of the tube at lattice site (n, m), Λ is a uniformly distributed random variable between 0 and 1, and η parametrizes the strength of the phase fluctuations. Within each tube, the gas is still described by mean-field theory, so this approximate TWA ignores correlations within a tube, except in so far as these lead to intertube phase fluctuations. When $\eta = 0$, the initial conditions correspond to a fully coherent array of 1D gases and, when $\eta = 1$, we have a totally randomized, initially incoherent array.

As long as $\eta \gtrsim 0.4$, the squared and convolved theoretical distributions fit fairly well to Gaussians. We average the results from 20 randomized phase implementations and display the rms widths in Fig. 2 for various η values. For $V_0 = 7.25E_R$, $\eta \simeq 0.4$ best fits the early evolution, when there is self-trapping. For larger V_0, $\eta = 1$ clearly fits the early evolution best. Randomized phases cause initially random tunneling current directions throughout the tube array. Site-to-site density fluctuations rapidly develop, as seen in Fig. 3(b). Since there is no reflection from relatively sharp self-trapped edges, the acceleration of the cloud does not go negative (as it briefly does for $\eta = 0$) but instead asymptotically approaches zero from above. Self-trapping is still lost when the density drops below a V_0-dependent critical value.

Although the modeled curves with the appropriate η are never too far from the observed ones for $V_0 = 11E_R$ and $13E_R$, the experiment shows a more sudden and more delayed onset of ballistic expansion than the model. Adding fluctuations in the initial atom number barely changes the curves. Assuming a normal error distribution, the p value is only 2×10^{-7} (6×10^{-19}) that the points between 0 and 8 (12) ms in the $V_0 = 11E_R$ $(13E_R)$ curves are consistent with mean-field theory with maximal fluctuations. In contrast, the p value is 0.45 (0.15) for them to lie on a horizontal line. The data suggest that self-trapping occurs throughout the array and persists until the central density has dropped by at least a factor of 8, which is a factor of 3 lower than the density at which the modified mean-field theory predicts expansion would be visible.

Detailed two-body correlations within each tube may be responsible. As the 2D lattice is turned on, correlations develop that make atoms avoid each other, at a kinetic energy cost, in order to avoid a larger mean-field energy cost. Since the wave function of a tunneling atom will not in general be appropriately correlated with the atoms in an adjacent tube, it would have to pay much of the mean-field energy cost that the correlations avoid. If that additional energy exceeds the tunneling energy, tunneling cannot proceed. In the strong coupling limit, correlation-suppressed tunneling is similar to Pauli blocking. It does not require density gradients, which is why the expansion of the central part of the tube bundle is fully suppressed until the overall density, and hence the mean-field cost of tunneling, gets sufficiently small. We speculate that such a process might tend to suppress mass transport whenever there is intermediate or strong coupling, regardless of the dimension [30,31].

In conclusion, we have measured the transverse expansion of quasi-1D arrays of atoms in the intermediate coupling regime. We find coarse agreement with a mean-field model when we introduce intertube phase fluctuations to the initial conditions. Modest phase fluctuations make the theory agree with experiment for $V_0 = 7.25E_R$ for short times. For larger V_0, agreement with experiment is best with maximal initial phase fluctuations among tubes, which slow down the initial transverse expansion. However, the experiment shows even less initial expansion, which suggests that there is strong self-trapping of a qualitatively different type, perhaps due to microscopic quantum correlations. This behavior could in principle be captured by a full TWA calculation that includes intratube quantum correlations [19]. However, it may be difficult to obtain the complete initial 3D Wigner distribution, and it is not clear that high order corrections will not limit the calculation before the ballistic regime is reached. More work is needed to quantitatively model tunneling behavior in this quantum many-body regime.

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