

## Scalar Glueball in Radiative $J/\psi$ Decay on the Lattice

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The form factors in the radiative decay of  $J/\psi$  to a scalar glueball are studied within quenched lattice QCD on anisotropic lattices. The continuum extrapolation is carried out by using two different lattice spacings. With the results of these form factors, the partial width of  $J/\psi$  radiatively decaying into the pure gauge scalar glueball is predicted to be 0.35(8) keV, which corresponds to a branching ratio of  $3.8(9) \times 10^{-3}$ . By comparing with experiments, our results indicate that  $f_0(1710)$  has a larger overlap with the pure gauge glueball than other related scalar mesons.

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The existence of glueballs predicted by QCD remains obscure. For a scalar glueball there is some evidence of its existence indicated by the fact that there are ten scalar mesons, such as  $K^*(1430)$ ,  $a_0(1450)$  and three isoscalars  $f_0(1370)$ ,  $f_0(1500)$ , and  $f_0(1710)$ . These mesons are close in mass and can be sorted into a  $SU(3)$  flavor nonet plus a glueball. Recent lattice studies predict that the lightest pure gauge scalar glueball has a similar mass [1–3]. Since the scalar glueball can mix with the nearby  $q\bar{q}$  mesons, the three isoscalars can be the different admixtures of the pure glueball  $G$ , the  $n\bar{n}$  meson, and  $s\bar{s}$  meson. So the key problem is to identify which of the three isoscalars has a dominant glueball component. For this purpose, different mixing scenarios have been proposed by imposing different mass ordering of  $G$ ,  $n\bar{n}$ , and  $s\bar{s}$  along with the known decay branching ratios of scalar mesons [4–11]. However, the resultant mixing patterns are controversial, especially for the status assignment of  $f_0(1500)$  and  $f_0(1710)$ . Obviously, more theoretical information of the scalar glueball is desired for the problem to be finally resolved.

It is well-known that gluons can be copiously produced in  $J/\psi$  decays because of the annihilation of the heavy quark pair. Among all the decays, the radiative decay is of special importance. It is expected that the gluons produced in  $J/\psi$  radiative decays dominantly form a glueball. If the production rate of the scalar glueball in the radiative decay can be reliably obtained from theoretical studies, it will provide important information for identifying the possible candidate for the scalar glueball by comparing the production pattern of scalar mesons in these decay channels.

There have been several studies on this topic based on the tree-level perturbative QCD approach and the dispersion relation method [12–16], but it is difficult to estimate theoretical uncertainties in the used approximations. In contrast, lattice QCD provides the rigorous method to study the radiative decay from first principles. In this Letter, as an exploratory study, we investigate the radiative decay of  $J/\psi$  into a scalar glueball in quenched lattice QCD.

Gauge configurations used in this Letter are generated using the tadpole-improved gauge action [1] on anisotropic lattices with the temporal lattice spacing much finer than the spatial one, say,  $\xi = a_s/a_t = 5$ , where  $a_s$  and  $a_t$  are the spatial and temporal lattice spacing, respectively. Each configuration is separated by 500 heat-bath updating sweeps to avoid the autocorrelation. The much finer lattice in the temporal direction gives a higher resolution to hadron correlation functions, such that masses of heavy particles can be tackled on relatively coarse lattices. The calculations are carried out on two anisotropic lattices, namely  $L^3 \times T = 8^3 \times 96$  and  $12^3 \times 144$ . The relevant input parameters are listed in Table I, where  $a_s$  values are determined from  $r_0^{-1} = 410(20)$  MeV. The spatial extension of both lattices is  $\sim 1.7$  fm, whose finite volume effect was found to be small and negligible for glueballs [3]. On the other hand, this lattice size is large enough for charmonium. For fermions we use the tadpole-improved clover action for anisotropic lattices [17]. The parameters in the action are tuned carefully by requiring that the physical dispersion relations of vector and pseudoscalar

TABLE I. The input parameters for the calculation. Values for the coupling  $\beta$ , anisotropy  $\xi$ , the lattice spacing  $a_s$ , lattice size, and the number of measurements are listed.

$\beta$	$\xi$	$a_s$ (fm)	$La_s$ (fm)	$L^3 \times T$	$N_{\text{conf}}$
2.4	5	0.222(2)	1.78	$8^3 \times 96$	5000
2.8	5	0.138(1)	1.66	$12^3 \times 144$	5000

mesons are correctly reproduced at each bare quark mass [18]. The bare charm quark masses at different  $\beta$  are determined by the physical mass of  $J/\psi$ ,  $m_{J/\psi} = 3.097$  GeV. The ground state masses of  $1S$  and  $1P$  charmonia are also calculated with these two lattices (see Fig. 2 and Table II of Ref. [19] for the details) and the finite  $a_s$  effects are found to be small.

To the lowest order in QED, the amplitude  $M$  for radiative decay  $J/\psi \rightarrow \gamma G$  is given by

$$M_{r,r_\gamma} = \epsilon_\mu^*(\vec{q}, r_\gamma) \langle G(\vec{p}_f) | j^\mu(0) | J/\psi(\vec{p}_i, r) \rangle, \quad (1)$$

where  $\vec{q} = \vec{p}_i - \vec{p}_f$  is the momentum of the real photon,  $r$  and  $r_\gamma$  are the polarization indices of  $J/\psi$  and the photon, respectively.  $\epsilon(\vec{q}, r_\gamma)$  is the polarization vector of the photon and  $j^\mu$  is the electromagnetic current operator. The hadronic matrix element appearing in the above equation can be obtained directly from lattice QCD calculation of corresponding three-point functions.

One of the key issues in our calculation is to construct the interpolating field operator which couples dominantly to the so-called pure gauge scalar glueball, which is defined by using interpolating field operators built from the gauge fields only. For this purpose, we adopt the variational method along with the single-link and double-link smearing schemes [2,3]. More specifically, since the irreducible representation  $A_1^{++}$  of lattice symmetry group  $O$  gives the right quantum number  $J^{PC} = 0^{++}$  in the continuum limit, we construct an  $A_1^{++}$  operator set  $\{\phi_\alpha, \alpha = 1, 2, \dots, 24\}$  of 24 different gluonic operators. Through the Fourier transformation,

TABLE II. Listed in the table are the  $A_1^{++}$  glueball masses  $M_G$ , the renormalization constants  $Z_V^{(s)}$  ( $a_s$ ) of the spatial component of the vector current, and the form factors  $E_1(Q^2 = 0, a_s)$  calculated on the two lattices with  $\beta = 2.4$  and 2.8, respectively. Also shown are the continuum extrapolation of  $E_1(0)$  and the resultant partial width  $\Gamma$ .

$\beta$	$M_G$ (GeV)	$Z_V^{(s)}$ ( $a_s$ )	$E_1(0, a_s)$ (GeV)	$\Gamma$ (keV)
2.4	1.360(9)	1.39(2)	0.0708(43)	...
2.8	1.537(7)	1.11(1)	0.0602(31)	...
$\infty$	1.710(90) [3]	...	0.0536(57)	0.35(8)

$$\phi_\alpha(\vec{p}, t) = \sum_{\vec{x}} \phi_\alpha(\vec{x}, t) e^{-i\vec{p}\cdot\vec{x}}, \quad (2)$$

we obtain the operator set  $\{\phi_\alpha(\vec{p}, t), \alpha = 1, 2, \dots, 24\}$  which couples to an  $A_1^{++}$  glueball state with the definite momentum  $\vec{p}$ . For each  $\vec{p}$ , by solving the generalized eigenvalue problem,

$$\tilde{C}(t_D)\mathbf{v}^{(R)} = e^{-t_D \bar{m}(t_D)} \tilde{C}(0)\mathbf{v}^{(R)}, \quad (3)$$

at  $t_D = 1$ , where  $\tilde{C}(t)$  is the correlation matrix of the operator set,

$$\tilde{C}_{\alpha\beta}(t) = \frac{1}{N_t} \sum_{\tau} \langle 0 | \phi_\alpha(\vec{p}, t + \tau) \phi_\beta^\dagger(\vec{p}, \tau) | 0 \rangle, \quad (4)$$

we obtain an optimal combination of the set of operators,  $\Phi(\vec{p}, t) = \sum v_\alpha \phi_\alpha(\vec{p}, t)$ , which overlaps most to the ground state,

$$C(\vec{p}, t) = \frac{1}{T} \sum_{\tau} \langle \Phi(\vec{p}, t + \tau) \Phi^\dagger(\vec{p}, \tau) \rangle \approx \frac{|\langle 0 | \Phi(\vec{p}, 0) | S(\vec{p}) \rangle|^2}{2E_S V_3} e^{-E_S t} \approx e^{-E_S t}, \quad (5)$$

where the normalization  $C(\vec{p}, 0) = 1$  is also used. This is actually the case that  $C(t)$  can be well described by a single exponential,  $C(t) = W e^{-Et}$ , with  $W$  usually deviating from one by few percents. Figure 1 shows the effective energy plateaus of the  $A_1^{++}$  glueball for typical momentum modes, where one can see that the plateaus start even from  $t = 1$ .

Glueballs are noisy objects and large statistics is usually required. In this Letter, we generated 5000 configurations for both lattice systems. In order to increase the statistics additionally, for each configuration we calculate  $T$  charm quark propagators  $S_F(\vec{x}, t; \vec{0}, \tau)$  by setting a point source on each time slice  $\tau$ , which permits us to average over the temporal direction when calculating the three-point functions. Therefore, the three-point functions we calculate in this Letter are

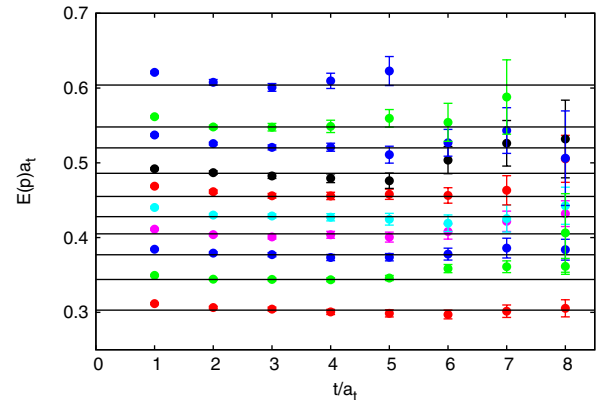


FIG. 1 (color online). The effective energy plot for the  $A_1^{++}$  glueball with different spatial momenta. From top to bottom are the plateaus for momentum modes,  $\vec{p} = 2\pi\vec{n}/L$ , with  $\vec{n} = (2, 2, 2), (2, 2, 1), (2, 2, 0), (2, 1, 1), (2, 1, 0), (2, 0, 0), (1, 1, 1), (1, 1, 0), (1, 0, 0),$  and  $(0, 0, 0)$ .

$$\begin{aligned}
\Gamma_{\mu,j}^{(3)}(\vec{p}_f, \vec{q}; t_f, t) &= \frac{1}{T} \sum_{\tau=0}^{T-1} \sum_{\vec{y}} e^{-i\vec{q}\cdot\vec{y}} \langle \Phi(\vec{p}_f, t_f + \tau) J_\mu(\vec{y}, t + \tau) O_{V,j}(\vec{0}, \tau) \rangle \\
&= \frac{1}{T} \sum_{\tau=0}^{T-1} \sum_{\vec{y}} e^{-i\vec{q}\cdot\vec{y}} \langle \Phi(\vec{p}_f, t_f + \tau) \text{Tr}[\gamma_\mu S_F(\vec{y}, t + \tau; \vec{0}, \tau) \gamma_j \gamma_5 S_F^\dagger(\vec{y}, t + \tau; \vec{0}, \tau) \gamma_5] \rangle \\
&= \sum_{S,V,r} \frac{e^{-E_S(t_f-t)} e^{-E_V t}}{2E_S(\vec{p}_f) V_3 2E_V(\vec{p}_i)} \langle 0 | \Phi(\vec{p}_f, 0) | S(\vec{p}_f) \rangle \langle S(\vec{p}_f) | J_\mu(0) | V(\vec{p}_i, r) \rangle \langle V(\vec{p}_i, r) | O_{V,j}^\dagger(0) | 0 \rangle, \quad (6)
\end{aligned}$$

where  $J_\mu(x) = \bar{c}(x) \gamma_\mu c(x)$  is the vector current operator,  $O_{V,j} = \bar{c} \gamma_j c$  the conventional interpolation field for  $J/\psi$ , and the summation in the last equality is over all the possible states and vector polarizations,  $\vec{p}_i$  is the spatial momentum of the initial vector charmonium and satisfies the relation  $\vec{p}_i = \vec{p}_f + \vec{q}$ . The vector current  $J_\mu(x)$ , which is conserved in the continuum limit, is no longer conserved on the lattice and requires a multiplicative renormalization. In this Letter, we adopt the nonperturbative strategy proposed by Ref. [20] to define the renormalization constant,

$$Z_V^\mu(t) = \frac{p^\mu}{2E(p)} \frac{\Gamma_{\eta_c \eta_c}^{(2)}(\vec{p}, t_f = T/2)}{\Gamma_{\eta_c \gamma_\mu \eta_c}^{(3)}(\vec{p}_f = \vec{p}_i = \vec{p}, t_f = T/2, t)}, \quad (7)$$

where  $\Gamma_{\eta_c \eta_c}^{(2)}$  is the two-point function of the pseudoscalar charmonium  $\eta_c$ ,  $\Gamma_{\eta_c \gamma_\mu \eta_c}^{(3)}$  is the corresponding three point function with the vector current insertion on one of the quark lines. It should be remarked that the possible disconnected diagrams due to the charm quark-antiquark annihilation are neglected in this Letter.

The parameters  $E_S$ ,  $E_V$ , the matrix elements  $\langle 0 | \Phi(\vec{p}_f, 0) | S(\vec{p}_f) \rangle$  and  $\langle 0 | O_{V,j} | V(\vec{p}_i, r) \rangle$  can be derived from the relevant two-point functions of glueballs and  $J/\psi$ . Specifically, from Eq. (5) we have

$$\langle 0 | \Phi(\vec{p}_f, 0) | S(\vec{p}_f) \rangle \approx \sqrt{2E_S(\vec{p}_f) V_3}. \quad (8)$$

For the vector meson we take the following convention,

$$\langle 0 | O_{V,j}(0) | V(\vec{p}, r) \rangle = f_V \epsilon_j(\vec{p}, r), \quad (9)$$

where  $f_V$  is a parameter independent of  $\vec{p}$ , and  $\epsilon_j(\vec{p}, r)$  the polarization vector of the vector meson, whose concrete expression depends on reference frames and is irrelevant to the calculation in this Letter. By using the multipole decomposition, the matrix elements  $\langle S(\vec{p}_f) | J_\mu(0) | V(\vec{p}_i, r) \rangle$  can be written as [20],

$$\begin{aligned}
&\sum_r \langle S(\vec{p}_f) | J_\mu(0) | V(\vec{p}_i, r) \rangle \epsilon_j(\vec{p}_i, r) \\
&= \alpha_{\mu,j} E_1(Q^2) + \beta_{\mu,j} C_1(Q^2), \quad (10)
\end{aligned}$$

where  $\alpha_{\mu,j}$  and  $\beta_{\mu,j}$  are known functions of  $p_f$  and  $p_i$  (their explicit expressions are neglected here),  $E_1(Q^2)$  and  $C_1(Q^2)$  are the two form factors which depend only on  $Q^2 = -(p_i - p_f)^2$ . Only the form factor  $E_1(Q^2)$  will be needed to determine the decay width with

$$\Gamma(J/\psi \rightarrow \gamma G_{0^{++}}) = \frac{4}{27} \alpha \frac{|\vec{p}_\gamma|}{M_{J/\psi}^2} |E_1(0)|^2, \quad (11)$$

where  $\alpha$  is the fine structure constant,  $p_\gamma$  the photon momentum with  $|\vec{p}_\gamma| = (M_{J/\psi}^2 - M_G^2)/(2M_{J/\psi})$ . Therefore, we will only focus on the extraction of  $E_1(Q^2)$ .

To measure  $E_1(Q^2)$  with different  $Q^2$  on the lattice, we create  $J/\psi$  on lattices with the momentum  $\vec{p}_i = \vec{0}$  or  $|\vec{p}_i| = 2\pi/La_s$ , and the scalar glueball with the momentum  $\vec{p}_f = 2\pi\vec{n}/La_s$ , where  $\vec{n}$  is ranged from (0, 0, 0) to (2, 2, 2). Among all the combinations of the vector current index  $\mu$ , the polarization index  $j$ , the glueball momentum  $p_f$ , and the  $J/\psi$  momentum  $p_i$ , it is found that there are specific combinations which give  $\alpha_{\mu i}(p_f, p_i) = 1$  and  $\beta_{\mu i}(p_f, p_i) = 0$ . Hereafter, we will only select these combinations for our practical data analysis. An additional benefit of this selection is that in these combinations one has  $\sum_r \epsilon_j^*(\vec{p}_i, r) \epsilon_j(\vec{p}_i, r) = 1$ .

With these prescriptions, the form factor  $E_1(Q^2)$  can be derived as,

$$\begin{aligned}
\tilde{E}_1(Q^2, t_f, t) &\approx \frac{Z_V^{(s)} \Gamma^{(3)}(\vec{p}_f, \vec{p}_i; t_f, t)}{C(\vec{p}_f, t_f - t) \Gamma^{(2)}(\vec{p}_i, t)} f_V \sqrt{2E_S(\vec{p}_f) V_3} \\
&\times (t, t_f - t \gg 0), \quad (12)
\end{aligned}$$

where  $Q^2$  can be given by  $\vec{p}_i$  and  $\vec{p}_f$ , the indices of the three-point function  $\Gamma^{(3)}$  and the related two-point functions  $\Gamma^{(2)}$  are omitted here, and  $Z_V^{(s)}$  is the renormalization constant of the spatial components of the vector current. In practice, the symmetric indices and momentum combinations which give the same  $Q^2$  are averaged to increase the statistics. Traditionally, the time separation  $t$  and  $t_f - t$  should be kept large enough for the ground states to contribute dominantly to the three point function. Even with this large statistics, we find that the signal of the glueball damps rapidly with respect to  $t_f - t$ . However, this is not a real disaster since the optimal glueball operators we use couple almost exclusively to the ground state, as is mentioned before. So we fix  $t_f - t = 1$  with varying  $t$  and extract  $E_1(Q^2)$  from the plateaus of  $\tilde{E}_1(Q^2, t_f, t)$ . With the very high statistics in this Letter, the hadron parameters, such as the energies of the glueball and  $J/\psi$ , the constant  $f_V$  in Eq. (9), and the matrix elements

$\langle 0|\Phi(\vec{p}, 0)|S(\vec{p})\rangle$  can be determined very precisely and are treated as known parameters.

$E_1(Q^2)$  for different  $Q^2$  are extracted from the same configuration ensemble and are therefore highly correlated. In the data analysis we fit them through the correlated data fitting. For each lattice system, the 5000 configurations are divided into 100 bins with 50 configurations in each bin. The measurements in each bin are averaged and the average is taken as an independent measurement. After that, all  $E_1(Q^2)$ s are extracted simultaneously through the jackknife method. In order to get the form factor at  $Q^2 = 0$ , we carry out a correlated polynomial fit to the  $E_1(Q^2)$  from  $Q^2 = -1.0 \text{ GeV}^2$  to  $2.5 \text{ GeV}^2$ ,

$$E_1(Q^2) = E_1(0) + aQ^2 + bQ^4. \quad (13)$$

Figure 2 shows the final results of  $E_1(Q^2)$  for  $\beta = 2.4$  (the upper panel) and  $\beta = 2.8$  (the lower panel), where the red points are the calculated value with jackknife errors, and the red curves are the polynomial fit with jackknife error bands, the black points label the interpolated  $E_1(0, a_s)$ .

The last step is the continuum extrapolation using the two lattice systems. The continuum limit of  $E_1(0, a_s)$  is determined to be  $E_1(0) = 0.0536(57) \text{ GeV}$  by performing a linear extrapolation in  $a_s^2$ . For the continuum value of the scalar glueball mass, we take  $M_G = 1.710(90) \text{ GeV}$  from Ref. [3]. Thus, according to Eq. (11), we finally get

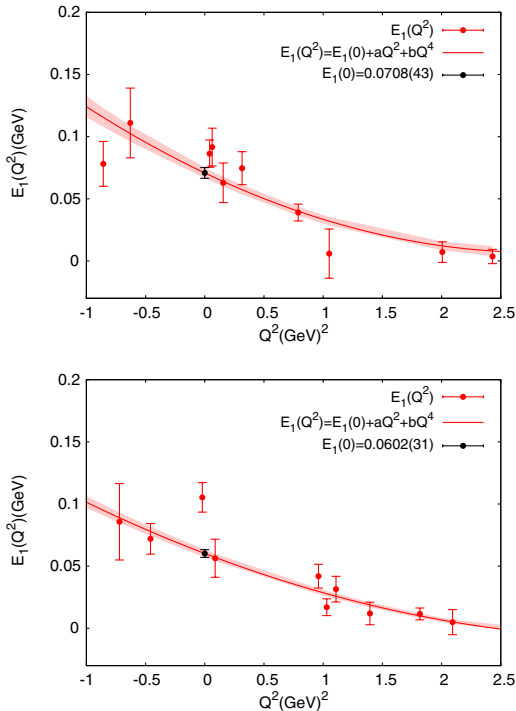


FIG. 2 (color online). The extracted form factors  $E_1(Q^2)$  in the physical units. The upper panel is for  $\beta = 2.4$  and the lower one for  $\beta = 2.8$ . The curves with error bands show the polynomial fit with  $E_1(Q^2) = E_1(0) + aQ^2 + bQ^4$ , as the black dot is the interpolated value  $E_1(0)$  at  $Q^2 = 0$ .

the decay width  $\Gamma(J/\psi \rightarrow \gamma G_{0^{++}}) = 0.35(8) \text{ keV}$ . Using the reported total width of  $J/\psi$ ,  $\Gamma_{\text{tot}} = 92.9(2.8) \text{ keV}$ , the corresponding branching ratio is

$$\Gamma(J/\psi \rightarrow \gamma G_{0^{++}})/\Gamma_{\text{tot}} = 3.8(9) \times 10^{-3}. \quad (14)$$

By comparing our result with their production rates in the radiative decay of  $J/\psi$ , we can get some useful information for the glueball components of the scalar mesons  $f_0(1710)$ ,  $f_0(1500)$ , and  $f_0(1370)$ . From PDG2010 [21], the branching ratios of the observed radiative decay modes of  $J/\psi$  to  $f_0(1710)$  are  $\text{Br}(J/\psi \rightarrow \gamma f_0(1710) \rightarrow \gamma K\bar{K}) = 8.5_{-0.9}^{+1.2} \times 10^{-4}$ ,  $\text{Br}(J/\psi \rightarrow \gamma f_0(1710) \rightarrow \gamma \pi\pi) = (4.0 \pm 1.0) \times 10^{-4}$ ,  $\text{Br}(J/\psi \rightarrow \gamma f_0(1710) \rightarrow \gamma \omega\omega) = (3.1 \pm 1.0) \times 10^{-4}$ , which add up to about  $1.5 \times 10^{-3}$ . With the measured branching ratio  $\text{Br}(f_0(1710) \rightarrow K\bar{K}) = 0.36 \pm 0.12$  [22], and the ratio  $\Gamma(f_0(1710) \rightarrow \pi\pi)/\Gamma(f_0(1710) \rightarrow K\bar{K}) = 0.41_{-0.17}^{+0.11}$  [23] (Ref. [22] also predicts this ratio to be  $0.32 \pm 0.14$  from a coupled channel study of meson-meson  $S$ -waves), one can estimate the production rate of  $f_0(1710)$  to be  $(2.4 \pm 0.8) \times 10^{-3}$  or  $(2.7 \pm 1.3) \times 10^{-3}$ . This is compatible with our lattice result. For the  $f_0(1500)$ , PDG2010 gives a lower bound to its production rate in  $J/\psi$  radiative decay,  $\text{Br}(J/\psi \rightarrow \gamma f_0(1500)) > 5.7(8) \times 10^{-4}$  [21]. On the other hand, with the BESII result  $\text{Br}(J/\psi \rightarrow \gamma f_0(1500) \rightarrow \gamma \pi\pi) = (1.01 \pm 0.32) \times 10^{-4}$  [23], and the branching ratio  $\text{Br}(f_0(1500) \rightarrow \pi\pi) = 0.349 \pm 0.023$  [21],  $\text{Br}(J/\psi \rightarrow \gamma f_0(1500))$  is estimated to be  $2.9(9) \times 10^{-4}$ . Both are much smaller than our prediction. Finally, there is no evidence of the production of  $f_0(1370)$  in the  $J/\psi$  radiative decays. Based on this comparison,  $f_0(1710)$  seems to have scalar glueballs as its dominant components, while for the other two scalar mesons, this does not seem to be the case.

To summarize, we have carried out the first lattice study on the  $E_1$  amplitude of  $J/\psi$  radiatively decays into the pure gauge scalar glueball  $G_{0^{++}}$  in the quenched approximation. With two different lattice spacings, the decay amplitude is extrapolated to the continuum limit with a value  $E_1(Q^2 = 0) = 0.0536(57) \text{ GeV}$ . Thus, the partial decay width  $\Gamma(J/\psi \rightarrow \gamma G_{0^{++}})$  is predicted to be  $0.35(8) \text{ keV}$ , which gives the branching ratio  $\Gamma/\Gamma_{\text{tot}} = 3.8(9) \times 10^{-3}$ . We admit that the systematic uncertainties due to the quenched approximation are not under control in this Letter; however, we are pleased to see that a recent  $2 + 1$ -flavor dynamical lattice study on the glueball spectrum claims that there are not large unquenching effects observed, especially for the scalar and tensor glueballs [24]. Anyway, our results are helpful in the sense that  $f_0(1710)$  appears to have the largest overlap to the pure gauge glueball among the relevant scalar mesons. We hope this fact sheds some light on the long-lasting puzzle of the identification of the scalar glueball.

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