

Do Stochastic Inhomogeneities Affect Dark-Energy Precision Measurements?

I. Ben-Dayan,^{1,2} M. Gasperini,^{3,4} G. Marozzi,⁵ F. Nugier,⁶ and G. Veneziano^{5,7}

¹Canadian Institute for Theoretical Astrophysics, 60 St. George Street, Toronto, Ontario M5S 3H8, Canada

²Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2L 2Y5, Canada

³Dipartimento di Fisica, Università di Bari, Via G. Amendola 173, 70126 Bari, Italy

⁴Istituto Nazionale di Fisica Nucleare, Sezione di Bari, Via G. Amendola 173, 70126 Bari, Italy

⁵Collège de France, 11 Place M. Berthelot, 75005 Paris, France

⁶Laboratoire de Physique Théorique de l'École Normale Supérieure, CNRS UMR 8549, 24 Rue Lhomond, 75005 Paris, France

⁷CERN, Theory Unit, Physics Department, CH-1211 Geneva 23, Switzerland

(Received 6 July 2012; published 7 January 2013)

The effect of a stochastic background of cosmological perturbations on the luminosity-redshift relation is computed to second order through a recently proposed covariant and gauge-invariant light-cone averaging procedure. The resulting expressions are free from both ultraviolet and infrared divergences, implying that such perturbations cannot mimic a sizable fraction of dark energy. Different averages are estimated and depend on the particular function of the luminosity distance being averaged. The energy flux being minimally affected by perturbations at large z is proposed as the best choice for precision estimates of dark-energy parameters. Nonetheless, its irreducible (stochastic) variance induces statistical errors on $\Omega_\Lambda(z)$ typically lying in the few-percent range.

DOI: [10.1103/PhysRevLett.110.021301](https://doi.org/10.1103/PhysRevLett.110.021301)

PACS numbers: 98.80.Es, 95.36.+x, 98.62.Py

Establishing the existence of dark energy and determining its parameters is one of the central issues in modern cosmology. Evidence of a sizable dark-energy component in the cosmic fluid comes from different sources: cosmic microwave background anisotropies, models of large-scale-structure formation and, most directly, the luminosity-redshift relation of Type Ia supernovae used as standard candles.

In this latter case, on which we concentrate our attention, the analysis is usually made in the simplified context of a homogeneous and isotropic Friedmann-Lemaître-Robertson-Walker (FLRW) cosmology. The issue has then been raised about whether inhomogeneities may affect the conclusion of such a naive analysis. Inhomogeneous models in which we occupy a privileged position in the Universe, for instance, can mimic dark energy (as first pointed out in Ref. [1]), but look both unrealistic and highly fine-tuned. More interestingly, we should address this question in the presence of stochastically isotropic and homogeneous perturbations of the kind predicted by inflation. We present here the main ideas and results of such a study, while its detailed derivation and discussion are presented in Ref. [2] and in a forthcoming paper [3].

There is by now general agreement that superhorizon perturbations cannot mimic dark-energy effects [4]. By contrast, the impact of subhorizon perturbations is still unsettled [5–7] owing to the appearance of ultraviolet divergences (see Ref. [8] for the possible observational impact of such ultraviolet divergences on the anisotropy of the Hubble flow) while computing their “backreaction” on certain classes of large-scale averages [6,7]. The possibility

that these effects may simulate a substantial fraction of dark energy, or that they may at least play some role in the context of near-future precision cosmology, has to be seriously considered.

In order to address these issues, we have studied the luminosity-redshift relation in a spatially flat Λ plus cold dark matter (Λ CDM) model perturbed by a stochastic background of inhomogeneities. The luminosity distance d_L now depends on the redshift z as well as on the angular coordinates of the sources and must be inserted in an appropriate light cone and ensemble average [9,10]. Unlike the analyses in Refs. [6,7], we find a result always free from ultraviolet divergences and with no significant infrared contributions either. As a consequence, corrections are typically small, certainly too small to mimic a sizeable fraction of dark energy. However, interestingly enough, both their size and their z dependence strongly depend on the particular function of d_L being averaged.

We find, in particular, that the energy flux $\Phi \sim d_L^{-2}$ is practically unaffected by inhomogeneities, while the most commonly used variables (like the distance modulus $\mu \sim 5\log_{10}d_L$) may receive much larger corrections. This creates (at least in principle) intrinsic ambiguities in the measure of the dark-energy parameters, unless the backreaction of stochastic inhomogeneities is properly taken into account. Actually, the advantages of flux averaging for minimizing biases on dark-energy parameters was first pointed out in Ref. [11], where it was shown how the binning of data in appropriate redshift intervals can reduce the bias due to systematic effects such as weak lensing. It is intriguing that the preferred role played by the flux variable also comes out in this Letter where we perform a

completely different averaging procedure at fixed redshift. Our conclusions are not due to a binning of data, but to an application of our covariant space-time average to different functions of the luminosity distance.

Let us start by recalling the standard expression for the luminosity distance in an unperturbed flat Λ CDM model, with present fractions of critical density Ω_Λ and $\Omega_m = 1 - \Omega_\Lambda$,

$$d_L^{\text{FLRW}}(z) = \frac{1+z}{H_0} \int_0^z \frac{dz'}{[\Omega_\Lambda + \Omega_m(1+z')^3]^{1/2}}. \quad (1)$$

Consider now the expression for d_L in the corresponding perturbed geometry. Combining light-cone and ensemble averages (denoted, respectively, by brackets and overbars), we can write the averaged result in the form

$$\overline{\langle d_L \rangle}(z) = d_L^{\text{FLRW}}[1 + f_d(z)], \quad (2)$$

where $f_d(z)$ represents the ‘‘backreaction’’ on d_L due to inhomogeneities. For consistency, d_L has to be computed (at least) up to the second perturbative order since ensemble averages of first-order quantities are vanishing for stochastic perturbations. In particular, backreaction terms arise also from correlations between the inhomogeneities present in the averaged variable and in the covariant integration measure. Therefore, a consistent perturbative calculation requires the inclusion of linear second-order contributions, since they are of the same order as the above quadratic first-order terms (see also Ref. [10], Sec. 4). A detailed computation of $f_d(z)$ would thus enable us to extract the ‘‘true’’ value of the dark-energy parameters from the measurement of $\overline{\langle d_L \rangle}(z)$ after taking the correction into account.

However, as already stressed in Ref. [10], given the covariant (light-cone) average of a perturbed (inhomogeneous) observable S , the average of a generic function of this observable differs in general from the function of its average, i.e., $\overline{\langle F(S) \rangle} \neq F(\overline{\langle S \rangle})$. Expanding the observable to second order as $S = S_0 + S_1 + S_2 + \dots$, one finds

$$\overline{\langle F(S) \rangle} = F(S_0) + F'(S_0)\overline{\langle S_1 \rangle} + F''(S_0)\overline{\langle S_1^2 \rangle/2}, \quad (3)$$

where $\overline{\langle S_1 \rangle} \neq 0$ as a consequence of the ‘‘induced backreaction’’ terms (see Ref. [10], Sec. 4). Thus, different functions of the luminosity distance are differently affected by the inhomogeneities and require different ‘‘subtraction’’ procedures. Finding the function that minimizes the backreaction will help of course for a precision estimate of the cosmological parameters. One of the main claims of this Letter is the identification of such an optimal observable with the energy flux $\Phi = L/(4\pi d_L^2)$ received from a standard candle of luminosity L located on the observer’s past light cone. We now illustrate how we performed such a calculation.

The average value of Φ , obviously controlled by the average of d_L^{-2} , has to be carried out on the past light cone

of the observer at a fixed redshift z using the gauge-invariant prescription introduced in Ref. [9]. This is most conveniently done [9,10] in the so-called geodesic light-cone gauge (GLC), where the metric depends on six arbitrary functions ($Y, U^a, \gamma_{ab}, a, b = 1, 2$), and the line element takes the form (with $\tilde{\theta}^1 = \tilde{\theta}, \tilde{\theta}^2 = \tilde{\phi}$),

$$ds^2 = Y^2 dw^2 - 2Y dw d\tau + \gamma_{ab}(d\tilde{\theta}^a - U^a dw)(d\tilde{\theta}^b - U^b dw). \quad (4)$$

The correspondence between the GLC gauge and the spatially flat FLRW geometry is [9] $\tau = t, w = r + \eta, Y = a(t), U^a = 0$, and $\gamma_{ab}d\tilde{\theta}^a d\tilde{\theta}^b = a^2 r^2 (d\tilde{\theta}^2 + \sin^2 \tilde{\theta} d\tilde{\phi}^2)$, where η is the conformal-time coordinate ($d\eta = dt/a$).

In the GLC gauge, the past light cone is defined by the condition $w = w_0 = \text{const}$, and the redshift is given by

$$1+z = Y(w_0, \tau_0, \tilde{\theta}^a)/Y(w_0, \tau, \tilde{\theta}^a). \quad (5)$$

Furthermore, the luminosity distance of the source is simply expressed as [10] $d_L = (1+z)^2 \gamma^{1/4} (\sin \tilde{\theta})^{-1/2}$ yielding the following exact result [2]:

$$\langle d_L^{-2} \rangle(z, w_0) = \frac{4\pi(1+z)^{-4}}{\int d^2 \tilde{\theta}^a \sqrt{\gamma[w_0, \tau(z, \tilde{\theta}^a), \tilde{\theta}^b]}}, \quad (6)$$

where $\gamma = \det \gamma_{ab}$, and $\tau(z, \tilde{\theta}^a)$ is obtained by solving Eq. (5). The above expression has a simple physical interpretation: the averaged flux for a given z is inversely proportional to the proper area [computed with respect to the metric (4)] of the surface lying on our past light cone at the given value of z . Flux conservation is probably at the basis of the particular simplicity of this average and of its minimal deviation from the homogeneous value.

To compute this quantity in the perturbed geometry of our interest, we need to express it in a gauge where the stochastic background of cosmological perturbations is explicitly known up to second order. To this purpose, we can use the standard Poisson gauge where we include first- and second-order scalar perturbations, neglecting their tensor and vector counterparts (see Ref. [2] for a discussion of this point). Performing the relevant transformations to second order, we arrive at the following analogue of Eq. (2):

$$\overline{\langle d_L^{-2} \rangle} = (d_L^{\text{FLRW}})^{-2} \overline{\langle I_\Phi(z) \rangle}^{-1} \equiv (d_L^{\text{FLRW}})^{-2} [1 + f_\Phi(z)], \quad (7)$$

where I_Φ has in general the following structure:

$$I_\Phi(z) = \int \frac{d\tilde{\phi} d\tilde{\theta} \sin \tilde{\theta}}{4\pi} [1 + I_1 + I_{1,1} + I_2](\tilde{\theta}, \tilde{\phi}, z). \quad (8)$$

Here $I_1, I_{1,1}, I_2$ are, respectively, the first-order, quadratic first-order, and genuine second-order contributions of our stochastic fluctuations. After solving the relevant perturbation equations [12] they can all be expressed in terms of the first-order Bardeen potential $\Psi(x, \eta)$. Using the stochastic properties of this perturbation and expanding in Fourier

modes $\Psi_k(\eta)$, we can then obtain an expression for $\overline{(I_\Phi)^{-1}}$ where first-order contributions drop out because of the ensemble average, and the scalar perturbations only appear through the so-called dimensionless power spectrum, $\mathcal{P}(k, \eta) = (k^3/2\pi^2)|\Psi_k(\eta)|^2$.

Unfortunately, $\overline{(I_\Phi)^{-1}}$ contains integrals over null geodesics lying on the past light cone of the given observer (see Ref. [10], Sec. 3.2), which get intertwined with the time dependence of \mathcal{P} , forcing us to proceed with an approximate numerical integration. This will be done below after inserting (as an instructive example) an illustration of the limiting CDM case, where all integrals but the one over k can be done analytically thanks to the time independence of \mathcal{P} ([10], Sec. 5).

In that case, the result can be written in the form

$$f_\Phi(z) = \int_0^\infty \frac{dk}{k} \mathcal{P}(k) [f_{1,1}(k, z) + f_2(k, z)], \quad (9)$$

where $f_{1,1}$ and f_2 are complicated—but known—analytic functions of their arguments [3]. Furthermore, the leading contribution in the region of z relevant for dark-energy phenomenology comes from terms of the type $f(k, z) \sim (k/\mathcal{H}_0)^2 \tilde{f}(z)$, where \mathcal{H}_0 is the present Hubble scale. We can then write to a very good accuracy,

$$f_\Phi(z) \approx [\tilde{f}_{1,1}(z) + \tilde{f}_2(z)] \int_0^\infty \frac{dk}{k} \left(\frac{k}{\mathcal{H}_0}\right)^2 \mathcal{P}(k), \quad (10)$$

where an explicit calculation gives [3]

$$\tilde{f}_{1,1}(z) = \frac{10 - 12\sqrt{1+z} + 5z(2 + \sqrt{1+z})}{27(1+z)(\sqrt{1+z} - 1)^2}, \quad (11)$$

$$\tilde{f}_2(z) = -\frac{1}{189} \left[\frac{2 - 2\sqrt{1+z} + z(9 - 2\sqrt{1+z})}{(1+z)(\sqrt{1+z} - 1)} \right]. \quad (12)$$

The absolute value (and sign) of $f_\Phi(z)$ are illustrated in Fig. 1 showing the accuracy of the leading order terms [Eq. (10)], and confirming that the backreaction of a realistic spectrum of stochastic perturbations induces negligible corrections to the averaged flux at large z (the larger corrections at small z due to “Doppler terms” has been discussed in Ref. [10]). In addition, it shows that in any case, such corrections have the wrong z dependence (in particular, they change sign at some z) to simulate even a tiny dark-energy component. For the considered spectrum (behaving as $k^{n_s-5} \log^2 k$ at large k , see Ref. [13]) the spectral integral is convergent and very weakly sensitive to the chosen value of the UV cutoff [10] representing here the limit of validity of our perturbative approach.

We now come to the more realistic Λ CDM case, where the f_Φ correction should be obtained by a full numerical integration of Eqs. (7) and (8). For simplicity, we will only take into account those terms giving the leading (k^2 -enhanced) contributions in the CDM case. For Λ CDM we can generally expect a smaller correction due

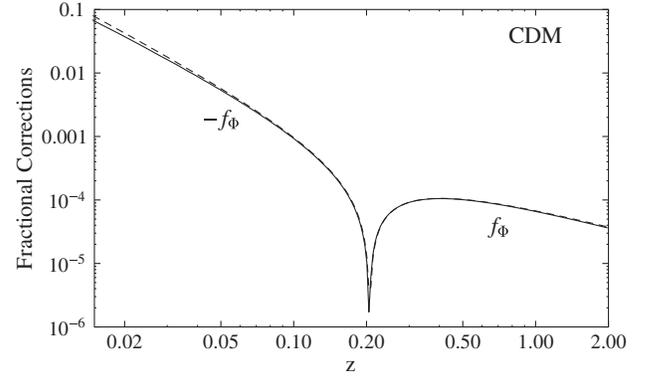


FIG. 1. The fractional correction f_Φ of Eq. (9) (solid curve) compared with the same quantity given to leading order by Eq. (10) (dashed curve) in the context of an inhomogeneous CDM model. We have used for $\mathcal{P}(k)$ the inflationary scalar spectrum with the WMAP parameters [17] and the transfer function given in Ref. [13] (see also Ref. [10]). The plotted curve refers, as an illustrative example, to an UV cutoff $k_{UV} = 1 \text{ Mpc}^{-1}$.

to the fact that the spectrum is now suppressed at large k by a lower value of the equality scale k_{eq} [13]. This is confirmed by the explicit numerical result for $|f_\Phi|$ presented in Fig. 2. The small value of $|f_\Phi|$ at large z leads us to conclude that the averaged flux is a particularly appropriate quantity for extracting from the observational data the “true” cosmological parameters. As we are going to see now, the situation is somewhat different for other functions of d_L .

Indeed, let us apply the general result of Eq. (3) to the flux variable $S = \Phi$ and consider two important examples: $F(\Phi) = \Phi^{-1/2} \sim d_L$ and $F(\Phi) = -2.5 \log_{10} \Phi + \text{const} \sim \mu$ (the distance modulus). For the luminosity distance, following the notations of Eq. (2) and using the general result from Eq. (3), we obtain

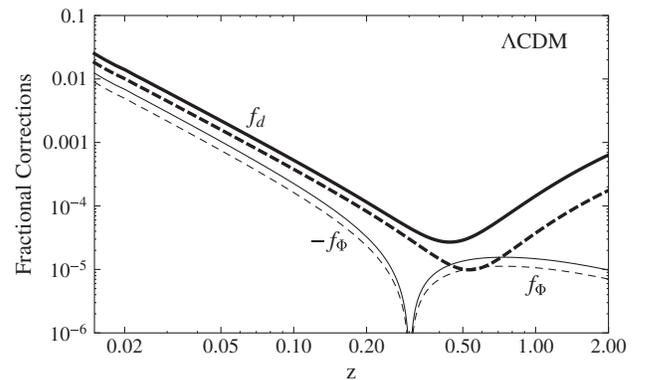


FIG. 2. The fractional correction to the flux f_Φ of Eq. (7) (thin curves) is compared with the fractional correction to the luminosity distance f_d of Eq. (13) (thick curves) for a Λ CDM model with $\Omega_\Lambda = 0.73$. We have used two different cutoff values: $k_{UV} = 0.1 \text{ Mpc}^{-1}$ (dashed curves) and $k_{UV} = 1 \text{ Mpc}^{-1}$ (solid curves). The spectrum is the same as that of Fig. 1 adapted to Λ CDM.

$$f_d = -(1/2)f_\Phi + (3/8)\overline{\langle(\Phi_1/\Phi_0)^2\rangle}. \quad (13)$$

Similarly, for the distance modulus we obtain

$$\overline{\langle\mu\rangle} - \mu^{\text{FLRW}} = -1.25(\log_{10}e)[2f_\Phi - \overline{\langle(\Phi_1/\Phi_0)^2\rangle}], \quad (14)$$

where f_Φ is defined in Eq. (7).

As clearly shown by the two above equations, the corrections to the averaged values of d_L and μ are qualitatively different from those of the flux (represented by f_Φ) because of the extra contribution (inevitable for any nonlinear function of the flux) proportional to the square of the first-order fluctuations. As mentioned before, the averaged flux corrections have leading spectral contributions of the type $k^2\mathcal{P}(k)$. On the contrary, the new corrections to d_L and μ are due to the so-called ‘‘lensing effect’’; they dominate at large z and have leading spectral contributions of the type $k^3\mathcal{P}(k)$ (as already discussed in Ref. [10]). The explicit numerical integration reported in Fig. 2 confirms that as a result, $|f_\Phi| \ll f_d$ at large z . We stress that even the k^3 -enhanced contributions are UV finite for the case under consideration.

We also stress that our results concerning the effects of lensing are in good agreement with previous estimates of the bias on supernova observables [14] and other cosmological parameters [15] induced by weak-lensing magnification effects. Unlike in those papers, however, our general approach automatically includes (and estimates the effects of) all possible corrections due to the stochastic fluctuations of the cosmological background to second order for all given functions of the flux (or of d_L). In fact, as discussed in detail in Refs. [2,3], the fractional correction f_d also includes, besides the lensing effect, Doppler, Sachs-Wolfe, integrated Sachs-Wolfe, frame-dragging effects, etc.

Let us now briefly discuss to what extent the enhanced corrections due to the squared first-order fluctuations can affect the determination of the dark-energy parameters if quantities other than the flux are used in the fits. To this purpose, we consider the much used (average of the) distance modulus given in Eq. (14), referred to as usual to the homogeneous Milne model with $\mu^M = 5\log_{10}[(2+z)z/(2H_0)]$. In Fig. 3 we compare the averaged value $\overline{\langle\mu\rangle} - \mu^M$ with the corresponding expression in a homogeneous Λ CDM model with different values of Ω_Λ . We also show the expected dispersion around the averaged result represented by the square root of the variance [10]. The latter is given by

$$\sqrt{\langle\mu^2\rangle} - \langle\mu\rangle^2 = \pm 2.5(\log_{10}e)\sqrt{\langle(\Phi_1/\Phi_0)^2\rangle}, \quad (15)$$

while for the flux we simply find

$$\sqrt{\langle(\Phi/\Phi_0)^2\rangle} - \langle\Phi/\Phi_0\rangle^2 = \pm\sqrt{\langle(\Phi_1/\Phi_0)^2\rangle}. \quad (16)$$

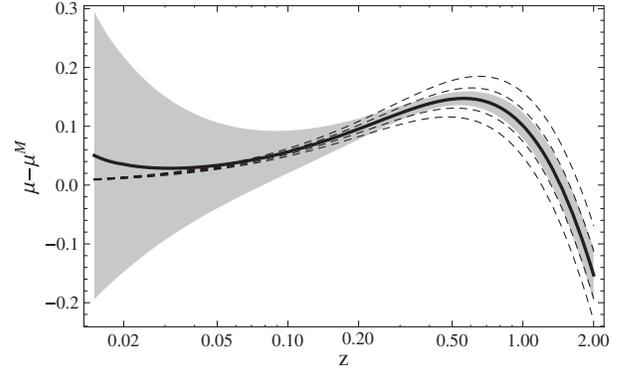


FIG. 3. The averaged distance modulus $\overline{\langle\mu\rangle} - \mu^M$ (thick solid curve) and its dispersion of Eq. (15) (shaded region) are computed for $\Omega_\Lambda = 0.73$ and compared with the homogeneous value for the unperturbed Λ CDM models with $\Omega_\Lambda = 0.69, 0.71, 0.73, 0.75, 0.77$ (dashed curves). We have used $k_{\text{UV}} = 1 \text{ Mpc}^{-1}$ and the same spectrum as in Fig. 2.

As illustrated in Fig. 3, we find that even for the distance modulus, the effect of inhomogeneities on the average only affects the determination of Ω_Λ at the third decimal figure (see also Fig. 2), at least for the inflationary power spectrum with the Λ CDM transfer function of Ref. [13]. In that case, the curves for $\overline{\langle\mu\rangle}$ and μ^{FLRW} are practically coincident at large z . We have considered other spectra which take into account nonlinear effects and have more power at short scales, like those obtained following [16]. Using such spectra only affects very mildly the k^2 -enhanced terms (hence the flux) while they increase the corrections wherever the k^3 -enhanced lensing terms play a major role. In particular, the variance due to the fluctuations, which is already at the few-percent level at large z for the power spectrum of Ref. [13] (see Fig. 3), can be further increased [3]. Note that even for these improved spectra, all our integrals are still free of UV divergences since, in any case, \mathcal{P} falls faster than k^{-3} (i.e., the matter density contrast spectrum grows slower than k).

Our main conclusions can be summarized as follows:

(1) Dealing directly with the experimentally measured luminosity-redshift relation within a gauge-independent approach leads to results for the fractional corrections to the averaged variables and the corresponding variances which are automatically free from UV (and IR) divergences for any function of the luminosity distance. This can be contrasted with the case of more formal spacelike averages [6,7] for which the physical interpretation of the results may have no direct relation with the observed cosmic acceleration (first reference in [5]) and, as shown in Ref. [7], the accidental cancellation of UV divergences is strongly dependent on the observable considered.

(2) The actual value of the backreaction strongly depends on the quantity being averaged. It turns out to be minimal for the flux Φ , which is also practically insensitive to the short-distance behavior of the power spectrum.

Therefore, the flux stands out as the safest observable for precision cosmology. For other observables, such as the distance modulus, the backreaction is considerably larger and is more sensitive to the spectrum used.

(3) The dispersion due to stochastic fluctuations is much larger than the backreaction itself, implying an irreducible scatter of the data that may limit to the percent level (see Fig. 3) the precision attainable on cosmological parameters because of the present limited statistics.

(4) We calculated here the full second-order effect of stochastic perturbations and concluded that they cannot simulate a substantial fraction of dark energy. Possible contributions coming from the nonperturbative regime on length scales much smaller than 1 Mpc have still to be taken into consideration before final conclusions can be drawn.

We wish to thank Ruth Durrer, Valerio Marra, Slava Mukhanov, Misao Sasaki, and Roman Scoccimarro for stimulating discussions. The research of I.B.D. at Perimeter Institute is supported by the government of Canada through Industry Canada and by the Province of Ontario through the Ministry of Research and Innovation.

[1] M.-N. Celerier, *Astron. Astrophys.* **353**, 63 (2000).

[2] I. Ben-Dayan, G. Marozzi, F. Nugier, and G. Veneziano, *J. Cosmol. Astropart. Phys.* **11** (2012) 045.

[3] I. Ben-Dayan, M. Gasperini, G. Marozzi, F. Nugier, and G. Veneziano (to be published).

- [4] E. E. Flanagan, *Phys. Rev. D* **71**, 103521 (2005); G. Geshnizjani, D. J. H. Chung, and N. Afshordi, *Phys. Rev. D* **72**, 023517 (2005); C. M. Hirata and U. Seljak, *Phys. Rev. D* **72**, 083501 (2005).
- [5] A. Ishibashi and R. M. Wald, *Classical Quantum Gravity* **23**, 235 (2006); A. Paranjape and T. P. Singh, *Phys. Rev. Lett.* **101**, 181101 (2008).
- [6] E. W. Kolb, *Classical Quantum Gravity* **28**, 164009 (2011).
- [7] C. Clarkson and O. Umeh, *Classical Quantum Gravity* **28**, 164010 (2011).
- [8] G. Marozzi and J.-P. Uzan, *Phys. Rev. D* **86**, 063528 (2012).
- [9] M. Gasperini, G. Marozzi, F. Nugier, and G. Veneziano, *J. Cosmol. Astropart. Phys.* **07** (2011) 008.
- [10] I. Ben-Dayan, M. Gasperini, G. Marozzi, F. Nugier, and G. Veneziano, *J. Cosmol. Astropart. Phys.* **04** (2012) 036.
- [11] Y. Wang, *Astrophys. J.* **536**, 531 (2000).
- [12] N. Bartolo, S. Matarrese, and A. Riotto, *J. Cosmol. Astropart. Phys.* **05** (2006) 010.
- [13] D. J. Eisenstein and W. Hu, *Astrophys. J.* **496**, 605 (1998).
- [14] Y. Wang, D. E. Holz, and D. Munshi, *Astrophys. J.* **572**, L15 (2002).
- [15] D. Sarkar, A. Amblard, D. F. Holz, and A. Cooray, *Astrophys. J.* **678**, 1 (2008).
- [16] R. E. Smith, J. A. Peacock, A. Jenkins, S. D. M. White, C. S. Frenk, F. R. Pearce, P. A. Thomas, G. Efstathiou, and H. M. P. Couchman, *Mon. Not. R. Astron. Soc.* **341**, 1311 (2003).
- [17] E. Komatsu *et al.*, *Astrophys. J. Suppl. Ser.* **192**, 18 (2011).