

## Modulated Floquet Topological Insulators

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The application of spatially uniform light on conventional insulators can induce Floquet spectra with characteristics akin to those of topological insulators. We demonstrate that spatial modulation of light allows for remarkable control of the properties in these systems. We provide configurations to generate one-dimensional bulk modes, photoinduced currents, as well as fractionalized excitations. We show a close analogy to  $p$ -wave superconductors and use this analogy to explain our results.

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The quantum Hall effect [1] led to the discovery of a close connection between topology and certain physical properties of condensed matter systems [2–4]. Our understanding of the role of topology has greatly expanded following the recent discovery of new classes of topological phases and of new materials displaying topological properties [5–12]. Topological phases are characterized by integer-valued numbers that are invariant to small changes of their Hamiltonian. This makes intriguing effects, such as quantized Hall conductivity and non-Abelian excitations, robust properties of these systems [1,13–15].

Recently, it has been shown that topological properties can be induced in conventional insulators by the application of time-periodic perturbations [16–22]. Proposals for these so-called “Floquet topological insulators” (FTIs) include a wide range of physical solid state and atomic realizations, driven both at resonance and off-resonance. These systems display metallic conduction enabled by quasistationary states at the edges [16,17,20], Dirac cones in three dimensional systems [21], and even Floquet Majorana fermions [22].

In this Letter, we demonstrate dramatic effects that arise in FTIs when light is modulated in space. Nonuniform light can give rise to controlled one-dimensional modes in the bulk, to fractionalized excitations, and to photoinduced electric currents. We establish these results both numerically and analytically. We show that the Floquet spectrum resembles that of a  $p$ -wave superconductor with a spatially modulated order parameter. This analogy provides a simple description of the mechanism behind our results. We propose setups by which the properties of light-induced topological phases can be controlled. For example, by modifying the angle of incident light on a system one can set the density of one-dimensional modes in its bulk.

We begin by building a description of FTIs in a generic zinc blende lattice model. The unperturbed system is given by the Bloch Hamiltonian

$$H_k = \begin{pmatrix} \tilde{H}_k & 0 \\ 0 & \tilde{H}_{-k}^* \end{pmatrix}. \quad (1)$$

This can describe, for example, HgTe quantum wells, in which case  $\tilde{H}_k$  [ $\tilde{H}_{-k}^*$ ] is a  $2 \times 2$  Hamiltonian acting on

the subspace spanned by the  $J_z = (\frac{1}{2}, \frac{3}{2})$  [ $J_z = (-\frac{1}{2}, -\frac{3}{2})$ ] states, respectively. Thus, the two blocks in Eq. (1) are related to each other by time reversal symmetry. Most generally, one can write

$$\tilde{H}_k = \vec{d}_k \cdot \vec{\sigma} + \varepsilon_k I_{2 \times 2}. \quad (2)$$

We consider time-dependent perturbations that do not connect the two Hamiltonian blocks, and perform the analysis on a single block. For example, we will study the  $2 \times 2$  Hamiltonian

$$\tilde{H}_{\text{lin}}(t) = \vec{d}_k \cdot \vec{\sigma} + \varepsilon_k I_{2 \times 2} + \vec{V}_k \cdot \vec{\sigma} \cos(\omega t + \alpha), \quad (3)$$

where  $\vec{V}_k$  can depict the effect of shining linearly polarized light on the sample [20].

The solutions of the Schrödinger equation for a time-dependent system evolve according to  $\psi(t') = U(t', t)\psi(t)$ , where  $U$  is the time evolution operator

$$U(t', t) = T \left\{ \exp \left[ -i \int_t^{t'} H(t'') dt'' \right] \right\}. \quad (4)$$

For a time-periodic system, Floquet’s theorem states that these solutions can be written as  $\psi(t) = \sum_a e^{i\varepsilon_a t} \varphi_a(t)$ , where  $\varphi_a(t) = \varphi_a(t + \tau)$  and  $\tau = \frac{2\pi}{\omega}$  [23]. The  $\varepsilon_a$  are called quasienergies, and are only defined modulo  $\omega$ ; the  $\varphi_a(t)$  satisfy the eigenvalue problem  $H_F \varphi_a(t) = \varepsilon_a \varphi_a(t)$ , where  $H_F$  is the Floquet Hamiltonian, obtained from the time evolution operator over a full cycle

$$e^{-iH_F \tau} \equiv U(\tau, 0). \quad (5)$$

For a  $2 \times 2$  block,  $H_F$  can be written most generally as

$$H_F = \vec{n}_k \cdot \vec{\sigma} + \varepsilon_k I_{2 \times 2}. \quad (6)$$

This has the structure of a gapped system provided that  $\vec{n}_k$  does not vanish on the Brillouin zone. We then introduce a topological invariant [7,9]

$$C_F = \frac{1}{4\pi} \iint_{\text{BZ}} d^2k (\partial_{k_x} \hat{n}_k \times \partial_{k_y} \hat{n}_k) \cdot \hat{n}_k, \quad (7)$$

where  $\hat{n}_k = \vec{n}_k / |\vec{n}_k|$ .  $C_F$  can be nontrivial even when the unperturbed system is topologically trivial [20]. As a

consequence, the time-dependent perturbation can give rise to topologically protected edge states in  $H_F$ .

The Floquet spectrum is independent of the value of  $\alpha \in [0, 2\pi]$ . To see this, note that a shift in  $\alpha$  results in  $U_\alpha(\tau, 0) = W^\dagger U_{\alpha=0}(\tau, 0)W$ , where  $W = U_{\alpha=0}^\dagger(\frac{\alpha}{\omega}, 0)$ . Thus, changing  $\alpha$  is equivalent to a similarity transformation of  $H_F$ ,  $H_F(\alpha) = W^\dagger H_F(0)W$ . The spectrum of  $H_F$  is therefore independent of  $\alpha$ , even if its eigenstates are not. In particular, varying  $\alpha$  cannot induce a gap closure, nor change  $C_F$ . Naively, this implies that at the interface between two portions of the sample with different values of  $\alpha$ , no edge modes are expected in the Floquet spectrum. However, we'll show below configurations in which localized modes appear at such an interface. At first glance this is remarkable, since the interface connects two systems with identical topological classification.

*Lattice model.*—We first demonstrate the existence of quasistationary interface modes in a lattice model. Consider the time-independent Hamiltonian, Eq. (2), with

$$\vec{d}_k = (A \sin k_x, A \sin k_y, M + 2B(\cos k_x + \cos k_y - 2)). \quad (8)$$

We choose  $\varepsilon_k = 0$ . Then Eq. (2) has particle-hole symmetry (PHS) in which the valence and conduction bands are interchanged. We add a time-dependent perturbation

$$V = V_0 \sigma_z \cos(\omega t + \alpha). \quad (9)$$

We evaluate the time evolution operator numerically, Eq. (4), by discretizing the time interval between  $t = 0$  and  $t' = \tau \equiv \frac{2\pi}{\omega}$ . We then obtain the Floquet spectrum by diagonalizing  $U$ ; see Eq (5). When  $\alpha$  is a constant, the system has translational invariance and  $\vec{n}_k$  can be calculated for each  $\vec{k}$  separately. Fig. 1 shows that  $\hat{n}_k$  wraps exactly one time around the unit sphere over the first Brillouin zone. The system is therefore topologically non-trivial with  $C_F = 1$ , and we expect one localized chiral mode at each edge of the system. To demonstrate this, we choose cylindrical geometry with open boundaries in the  $y$  direction and periodic boundaries in the  $x$  direction.

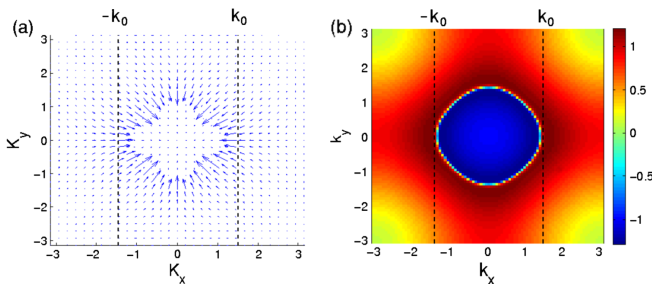


FIG. 1 (color online).  $\vec{n}_k$ , defined in Eq. (6), for the lattice model, Eqs. (8) and (9), with  $\alpha = 0$ . (a)  $x$  and  $y$  components of  $\vec{n}_k$ . (b)  $n_k^z$ . Note that  $\vec{n}_k$  is in a hedgehog configuration, as it wraps the unit sphere exactly once. This corresponds to  $C_F = 1$ . The dashed lines depict the range  $[-k_0, k_0]$  over which  $C_{k_x}' \neq 0$ .

Figure 2 shows that indeed, one zero quasienergy mode exists at each boundary of the system. This corresponds to the edge state found in Ref. [20].

We now allow  $\alpha$  to be position dependent. As a first example, we consider a domain wall configuration, across which the external perturbation changes sign,  $\alpha(y) = \pi\theta(y)$ . This captures the phase shift across the nodes of a standing wave created by two interfering light rays incident on the sample. By adjusting the incidence angle of the rays, one can control the periodicity of the standing wave such that these nodes are well separated. We choose cylindrical geometry. Since the system remains translationally invariant along  $x$ , we work in the hybrid coordinate basis  $(k_x, y)$  and diagonalize  $U$  for each  $k_x$  value. Figure 2 shows the resulting quasienergy spectrum. Note that, in addition to the edge modes, the spectrum now includes two counter-propagating zero quasi-energy modes localized near the domain wall at  $y = 0$ .

As a second example, we consider a vortex configuration, in which the phase  $\alpha$  winds by  $2\pi$  about a point,  $\alpha(\vec{r}) = \arctan(y/x)$ . A lattice of such vortices can be created by interfering three lasers, and an isolated vortex can be created using a phase mask. We set open boundaries and diagonalize  $U$  in real space to obtain the Floquet spectrum. In addition to the edge modes, we find a zero quasienergy state bound to the vortex. Figure 3 shows the wave function of the bound state. This state hybridizes with an edge state and opens a small gap. However, this is a finite-size effect, and the gap energy decays exponentially with system size.

*Analogy to  $p_x + ip_y$  superconductors.*—In order to explain these results, we establish an analogy between our system and a  $p_x + ip_y$  superconductor ( $p$ SC). We then relate the domain wall and vortex core states to the well-known zero energy states of  $p$ SCs.

We derive an approximate analytic expression for  $H_F$ . The derivation is first carried out for constant  $\alpha$ . We start by omitting the component of  $\vec{V}_k$  parallel to  $\hat{d}_k$ , since it only affects the dynamics weakly when averaged over a full cycle and is known not to influence the topological properties of the system [20]. The remaining perpendicular

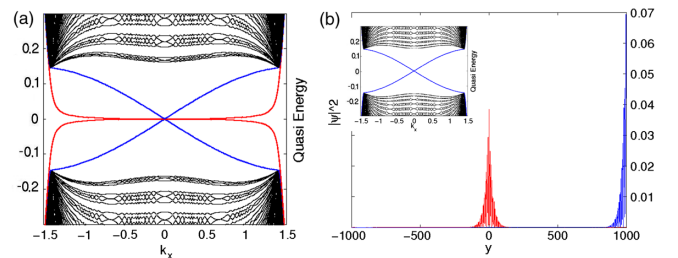


FIG. 2 (color online). (a) Floquet spectrum of Eq. (3) for a domain wall configuration. The plotted range corresponds approximately to  $[-k_0, k_0]$ . (b) Amplitude of the localized bulk (red) and edge (blue) states. Inset: Floquet spectrum for constant  $\alpha$ . Results are for  $V_0 = A = -B = 0.2M$ ,  $\omega = 2.7M$ .

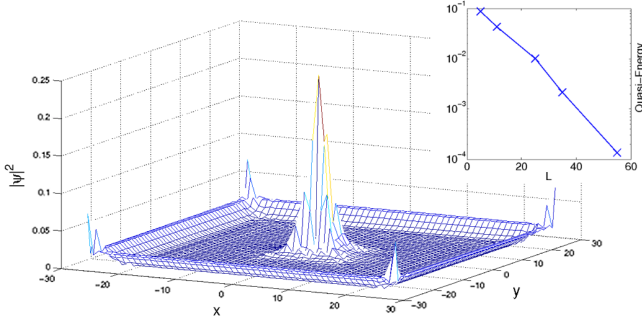


FIG. 3 (color online). Amplitude of the vortex bound state. Inset: quasienergy of the state as a function of the system size  $L$ . Results are for  $V_0 = 1.5M$ ,  $A = 0.2M$ ,  $B = -0.1M$ ,  $\omega = 2.7M$ .

component is  $\vec{V}_{k,\perp} = \vec{V}_k - (\vec{V}_k \cdot \hat{d}_k)\hat{d}_k$ . We define  $V_{k,\perp} \equiv |\vec{V}_{k,\perp}|$  and introduce  $\hat{v}_k = \vec{V}_{k,\perp}/V_{k,\perp}$  and  $\hat{w}_k = \hat{d}_k \times \hat{v}_k$ , such that  $\hat{d}_k$ ,  $\hat{v}_k$  and  $\hat{w}_k$  form a right-handed triad. The perturbation can be decomposed into terms that rotate and counterrotate about  $\hat{d}_k$ ,

$$\begin{aligned} \tilde{H}_k \approx & \vec{d}_k \cdot \vec{\sigma} + \frac{1}{2} V_{k,\perp} (\hat{v}_k \cos \omega t + \hat{w}_k \sin \omega t) \cdot \vec{\sigma} \\ & + \frac{1}{2} V_{k,\perp} (\hat{v}_k \cos \omega t - \hat{w}_k \sin \omega t) \cdot \vec{\sigma}. \end{aligned}$$

We go to a rotating frame through the time-dependent unitary transformation  $R(t) = \exp(-i\vec{d}_k \cdot \vec{\sigma} \frac{\omega t}{2})$ . The resulting states  $|\psi(t)\rangle_r = R(t)|\psi(t)\rangle$  satisfy  $i\partial_t |\psi(t)\rangle_r = \tilde{H}' |\psi(t)\rangle_r$ , where  $\tilde{H}' = R^\dagger (\tilde{H} - i\partial_t) R$ . We then find

$$\begin{aligned} \tilde{H}'_k = & \left[ \vec{d}_k - \frac{\omega}{2} \hat{d}_k + \frac{1}{2} V_{k,\perp} (\hat{v}_k \cos \alpha + \hat{w}_k \sin \alpha) \right] \cdot \vec{\sigma} \\ & + \frac{1}{2} V_{k,\perp} [\hat{v}_k \cos(2\omega t + \alpha) - \hat{w}_k \sin(2\omega t + \alpha)] \cdot \vec{\sigma}. \end{aligned}$$

The rotating wave approximation (RWA) consists of neglecting the  $2\omega$  term in  $\tilde{H}'_k$ . This is valid near a resonance,  $|\omega - \Delta E| \ll \omega + \Delta E$ , where  $\Delta E$  is the energy difference between states in the lower and upper bands. This procedure yields a time-independent operator

$$H_{F,\alpha}^{\text{RWA}} = \left[ \left(1 - \frac{\omega}{2d_k}\right) \vec{d}_k + \frac{1}{2} V_{k,\perp} (\hat{v}_k \cos \alpha + \hat{w}_k \sin \alpha) \right] \cdot \vec{\sigma}, \quad (10)$$

which is the Floquet Hamiltonian in the RWA.

The analogy to a  $p$ SC can be seen explicitly by performing a  $k$ -dependent unitary transformation that rotates  $(\hat{d}_k, \hat{v}_k, \hat{w}_k) \rightarrow (\hat{z}, \hat{k}, \hat{z} \times \hat{k})$ , where  $\vec{k} = (k_x, k_y)$ . This leads to a Hamiltonian of the form

$$H'_F = \begin{pmatrix} \zeta_k & \Delta_k e^{-i\alpha}(k_x - ik_y) \\ \Delta_k e^{i\alpha}(k_x + ik_y) & -\zeta_k \end{pmatrix}, \quad (11)$$

where  $\zeta_k = d_k - \omega/2$  and  $\Delta_k = V_{k,\perp}/2k$  are real. Equation (11) resembles the Hamiltonian of a  $p$ SC with complex order parameter  $\Delta_k e^{-i\alpha}$ . The analogy to the  $p$ SC

can be seen graphically in Fig. 1, where panel (a) depicts the normal dispersion and panel (b) the superconducting order parameter, which is seen to have  $p_x + ip_y$  symmetry. Note that, unlike an actual superconductor in which the Nambu basis describes particle and hole states, here Eq. (11) acts on two particlelike states corresponding to valence and conduction bands of the Floquet problem. Hence, the spectrum of Eq. (11) matches the corresponding  $p$ SC, but the nature of the wave functions in the two cases is related by a particle-hole transformation.

We apply these results to our system. The unperturbed Hamiltonian for small  $\vec{k}$  is  $\tilde{H}_k = A\vec{k} \cdot \vec{\sigma} + M\sigma_z$ . The Floquet Hamiltonian in the RWA is then

$$H_{F,\alpha}^{\text{RWA}} = \left[ \eta \left( \hat{z} + \frac{A}{M} \vec{k} \right) + \Delta_0 (\vec{k} \cos \alpha + \hat{z} \times \vec{k} \sin \alpha) \right] \cdot \vec{\sigma},$$

where  $\eta = (M - \frac{\omega}{2})$  and  $\Delta_0 = \frac{AV_0}{2M}$ . When  $\alpha$  varies in space, this is generalized to a Bogoliubov—de Gennes (BdG) equation [here  $\psi = (u, v)^T$ ]

$$\begin{aligned} (\varepsilon - \eta)u &= \left[ \frac{A}{M} \eta - \Delta(\vec{r}) \right] * (-i\partial_x + \partial_y)v, \\ (\varepsilon + \eta)v &= (-i\partial_x - \partial_y) * \left[ \frac{A}{M} \eta - \Delta^*(\vec{r}) \right] u, \end{aligned} \quad (12)$$

where  $*$  denotes the symmetric product  $a * b = \frac{1}{2}(ab + ba)$ . In theories of superconductivity the gap function  $\Delta(\vec{r})$  is calculated self-consistently. Here,  $\Delta(\vec{r})$  is directly determined by the external perturbation.

When  $\alpha(\vec{r})$  describes a vortex, the BdG equation has a zero energy solution bound to the vortex core. This state is analogous to the well-known  $p$ SC vortex core modes, and just as in the case of a  $p$ SC, it is topologically protected provided particle-hole symmetry is present [24,25]. This gives a fractional charge of  $\pm 1/2$  at the vortex, depending on whether the state is occupied or not [26]. In the full  $4 \times 4$  system, the vortex is felt in both subspaces, yielding two zero energy states which can be independently occupied. This leads to fractionalized excitations ( $\pm 1/2, \pm 1/2$ ), where the indices denote the charge in each subspace. The scenario is closely analogous to fractionalization in polyacetylene, where fractional charge excitations in the spin-up and spin-down channels combine into spin  $1/2$  excitations with integer charge [27].

When  $\alpha(\vec{r})$  describes a domain wall, the BdG equation has two zero quasienergy solutions, localized around  $y = 0$  with a localization length of  $\xi = \frac{\eta}{\Delta_0}$ . This configuration is analogous to an interface between two  $p$ SCs with a relative phase difference of  $\pi$ —a “ $\pi$  junction”—which is known to have zero energy bound Andreev states [25,28]. To understand these modes, let us first consider a system with no disorder and no interactions. Then  $k_x$  is a good quantum number and the system reduces to a 1D  $p$ SC for each  $k_x$ . We can then define a  $k_x$ -dependent topological invariant,  $C'_{k_x}$ , as the winding number of  $\hat{n}_k$  in the  $(n_k^y, n_k^z)$

plane. As can be seen from Fig. 1, when  $\alpha = 0$  the 1D  $p$ SC is topological with  $C'_{k_x} = 1$  for  $k_x \in [-k_0, k_0]$  and it is trivial otherwise. On the other side of the domain wall,  $\alpha = \pi$ , and the sign of  $C'_{k_x}$  is reversed. For  $k_x \in [-k_0, k_0]$  the change in sign in  $C'_{k_x}$  implies a pair of localized states exists at the interface, which disperse in opposite directions as a function of  $k_x$ . In particular, at  $k_x = 0$  these modes cross with zero quasienergy. When PHS is present, one of these states is even under the PH transformation, whereas the other is odd, preventing them from mixing and opening up a gap. This protection is robust even in the presence of disorder and interactions, provided these do not break PHS.

Note that the domain wall and vortex configurations are closely related. To see this, we write  $\alpha(\vec{r}) = \arctan(y/\beta x)$ , which describes a vortex for  $\beta = 1$  and a domain wall for  $\beta \rightarrow 0^+$ . Thus, the domain wall is a continuous deformation of a vortex and the  $\pi$  jump arises from the vortex winding. The interface modes are therefore smoothly connected to the vortex core states.

*Photoinduced current.*—Recall that imposing a phase twist on a superconductor  $\Delta e^{i\alpha(\vec{r})}$  results in a Josephson current  $\vec{j}_S = \rho_s \vec{\nabla} \alpha$  [29]. Motivated by the analogy to  $p$ SC, we consider the effect of a slowly varying phase twist  $\alpha(y) = \alpha_0 + y \partial_y \alpha$ , where  $\partial_y \alpha$  is a small constant. This can be achieved, for example, by shining a coherent light ray incident at an angle to the surface of the sample. Indeed, we find that the system experiences a dc current along  $\hat{y}$ , and compute an analogue of the superfluid stiffness  $\rho_s$  in terms of the vectors  $\vec{d}_k$  and  $\vec{n}_k$ .

By Noether's theorem, the current density operator is

$$\vec{j} = -\vec{\nabla}_k (\vec{d}_k \cdot \vec{\sigma}), \quad (13)$$

where  $\vec{\nabla}_k = (\partial_{k_x}, \partial_{k_y})$ . Our goal is to compute the expectation value of Eq. (13) with respect to the state at half-filling, in which all eigenstates in the Floquet valence band are fully occupied,

$$\langle \vec{j} \rangle = \sum_{\psi_k} \langle \psi_k | R(t) \hat{j} R^\dagger(t) | \psi_k \rangle_r, \quad (14)$$

where the sum is over all the negative quasienergy states. Here, we insert  $R(t) = \exp(-i \hat{d}_k \cdot \vec{\sigma} \frac{\omega t}{2})$  since the current is computed in the lab frame.

In order to derive an expression for the current, we compute  $H_F$  to linear order in  $\partial_y \alpha$  in the RWA. We obtain  $H_F = H_{F, \alpha_0}^{\text{RWA}} + (\partial_y \alpha) H_1^{\text{RWA}}$ , where  $H_{F, \alpha_0}^{\text{RWA}}$  is given by Eq. (10) and

$$H_1^{\text{RWA}} = \frac{y}{4} V_{k, \perp} (\cos \alpha_0 \hat{w}_k - \sin \alpha_0 \hat{v}_k) \cdot \vec{\sigma} + \text{H.c.}$$

We then write  $|\psi_k\rangle_r = |\psi_k^0\rangle_r + \partial_y \alpha |\psi_k^1\rangle_r$ , where  $|\psi_k^0\rangle_r$  are eigenstates of  $H_{F, \alpha_0}^{\text{RWA}}$ , and  $|\psi_k^1\rangle_r$  is obtained from 1st order

perturbation theory. The resulting current, to  $O(\partial_y \alpha)$ , vanishes along  $\hat{x}$ , while along  $\hat{y}$  it is

$$\frac{\langle j_y \rangle}{\partial_y \alpha} = \iint \frac{d^2 k}{(2\pi)^2} \frac{V_0}{n_k} (\vec{d}_k \times \nabla_k \vec{d}_k) \cdot \hat{v}_{kr} \langle \psi_k^0 | i \vec{\partial} | \psi_k^0 \rangle_r,$$

where  $\langle \psi | \vec{\partial} | \psi \rangle = \langle \psi | \partial \psi \rangle - \langle \partial \psi | \psi \rangle$ . In this expression, we have summed over the contributions coming from both sub-Hamiltonians,  $\tilde{H}_k$  and  $\tilde{H}_{-k}^*$ , appearing in Eq. (1). In terms of  $\vec{d}_k$  and  $\vec{n}_k$ , this can be rewritten as

$$\frac{\langle j_y \rangle}{\partial_y \alpha} = \iint \frac{d^2 k}{(2\pi)^2} \frac{V_0}{2} \frac{d_k}{n_k} \frac{\hat{n}_z}{\hat{n}_x^2 + \hat{n}_y^2} [(\vec{d}_k \times \partial_{k_y} \hat{n}_k) \cdot \hat{z}]^2, \quad (15)$$

where  $d_k = |\vec{d}_k|$  and  $n_k = |\vec{n}_k|$ .

Integrating Eq. (15) over the Brillouin zone gives a nonzero dc current. As a check of our analysis, we compared these results with numerical simulations on the lattice model and found good agreement. This result is reminiscent of the photogalvanic effect proposed to exist at the surface of 3D topological insulators radiated with circularly polarized light [30].

*Discussion.*—The analogy to  $p$ SC relies on particle-hole symmetry. Thus, it is natural to ask which of our results rely on this symmetry. For instance, Eq. (15) for the photoinduced current is valid even when PHS is broken. In contrast, PHS plays an important role in preventing the interface modes from opening a gap, as discussed above. For real systems PHS is only approximate, and we must consider the effects of weak PHS breaking. We find that this induces a small mixing of the interface modes. For example, using the same parameters as in Fig. 2, but adding a PHS breaking term to Eq. (3) of the form  $\varepsilon_k = -0.2(\cos k_x + \cos k_y - 2)$ , we find that a small gap  $\approx 10^{-5}$  opens up. Thus, experiments carried out at temperatures above this gap will not be sensitive to PHS breaking. The vortex core state shows higher degree of robustness to breaking of PH symmetry. Although PH symmetry breaking shifts the bound state quasienergy away from zero, it remains a midgap state provided the symmetry is weakly broken. In particular, the fractional nature of the excitation is unaffected.

We now briefly discuss the physical manifestations of our results in realistic systems. Earlier work has shown that resonantly-driven systems can reach a steady state, with occupation given by the Fermi-Dirac distribution based on the Floquet spectrum, and with an effective temperature that is determined by interaction and phonon relaxation rates [31,32]. Then, a standing wave pattern is expected to create one dimensional channels along the domain walls and thus induce anisotropic conductivity. It may be possible to generalize our results to systems in which the light frequency exceeds the bandwidth (that is, systems that are driven off-resonance). The advantage of doing this is that physical properties of systems driven off-resonance are much easier to understand [16,17,33]. Modulated FTIs



driven off-resonance, and in three dimensions, will be discussed in future work.

In conclusion, we provided a set of schemes by which the properties of Floquet topological insulators can be manipulated using modulated light. We proposed explicit setups by which bulk 1D channels, fractionalized excitations, and light induced currents can be generated. Our analysis demonstrates the great potential for tunability and control of light-induced topological phases.

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