Single Photon Delayed Feedback: A Way to Stabilize Intrinsic Quantum Cavity Electrodynamics

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We propose a scheme to control cavity quantum electrodynamics in the single photon limit by delayed feedback. In our approach a single emitter-cavity system, operating in the weak coupling limit, can be driven into the strong coupling-type regime by an external mirror: The external loop produces Rabi oscillations directly connected to the electron-photon coupling strength. As an expansion of typical cavity quantum electrodynamics, we treat the quantum correlation of external and internal light modes dynamically and demonstrate a possible way to implement a fully quantum mechanical time-delayed feedback. Our theoretical approach proposes a way to experimentally feedback control quantum correlations in the single photon limit.

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Introduction.—Single emitters, such as atoms, molecules, or solid-state emitters embedded in photonic nanocavities exhibit a variety of interesting features, arising from the nonlinear and strongly correlated interactions between photons and electrons. Typical examples include Mollow triplets [1,2], single photon emission [3–5], vacuum Rabi oscillations [6–8], and lasing [9–11]. To exploit quantum optical features for quantum information science, the stabilization of nonclassical photon states against decoherence processes is of great importance. Proposed control and protection schemes include protocols for quantum error correction [12], quantum gate purifying [13], or quantum feedback [14].

Particularly promising for nonlinear systems is quantum feedback based on the repeated action of a sensor-controller-actuator loop. Hereby, the quantum system is driven to a target state via the extrinsic control [14,15], and vacuum Rabi oscillations have been stabilized, recently [16]. In addition to these extrinsic control setups, experiments start to explore the variety of intrinsic, delayed feedback control, e.g., by using an external mirror in front of a nanocavity [17]. In such cavities, supporting one high-Q mode, the transition between the quantum to the classical limit can be studied. Classically, this type of feedback is used to stabilize the operation point of semiconductor lasers with mW output power and can be theoretically described with the Lang-Kobayashi model [18–20] for classical fields. However, restricted to coherent fields, this model breaks down in the limit of few photons and emitters, i.e., in the quantum limit. Therefore, a fully quantized model of delayed feedback is useful for the full understanding of the experimental findings and to pave the way for the experimental studies of delayed feedback control of few photon states.

In this Letter, we report on a theoretical description of an optical delayed feedback setup in the quantum limit of few photons. Hereby, we predict the stabilization of vacuum Rabi oscillations via intrinsic quantum control in contrast to the already experimentally demonstrated extrinsic control techniques [14,16]. To achieve self-feedback, our model includes the quantum mechanical coupling of the cavity-QED system to external modes provided by an external mirror. Since we treat this quantum limit, where fluctuations dominate the dynamics, a nonperturbative theory in the electron-photon and photon-photon coupling is developed. We illustrate the approach for the single-photon case and show that the mirror provides a structured external mode continuum, enforcing strong coupling features due to delayed feedback. Our results demonstrate that this feedback can overcome the optical cavity loss and that the internal electron-photon coupling strength can be extracted even if the system remains in the weak coupling regime. This intrinsic quantum mechanical control setup includes quantum mechanical as well as classical effects: For instance, the Lang-Kobayashi case constitutes the classical limit of our approach, if classical initial conditions are chosen and the feedback contributions are restricted to a single-round trip [18].

Model.—The system consists of a microcavity system with a two-level emitter coupled to a single cavity mode. The cavity exhibits a photon loss due to its coupling to an external mode continuum shaped eventually by an external mirror, Fig. 1. The external mirror, placed in a distance of L [21], introduces a boundary condition to the external mode structure and causes a feedback of lost cavity photons into the cavity. This quantum self-feedback depends on the distance L and the speed of light c_0 outside the cavity, corresponding to the delay time: $\tau = 2L/c_0$ [18,22]. To maximize the efficiency of this feedback mechanism, a lens is placed in front of the cavity, which focuses the transmitted photons onto the mirror and back to

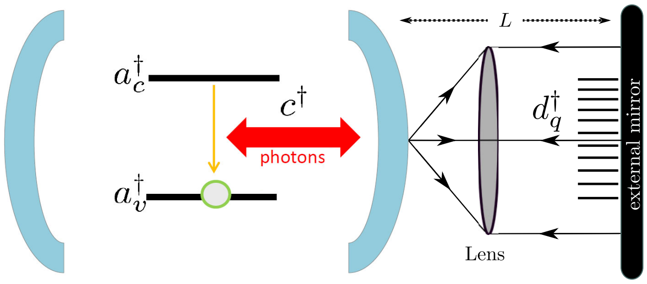

FIG. 1 (color online). Interaction scheme of a strongly coupled electron-photon dynamics in a microcavity with input-output coupling to a photon continuum in front of a mirror.

the cavity [23]. This model can be experimentally realized for many different two-level-like emitters, such as atoms, molecules, or semiconductor nanostructures. Our theoretical model is based on the following Hamiltonian in second quantization within the rotating-wave and dipole approximation [24]:

$$H = \hbar\omega_c a_c^\dagger a_c + \hbar\omega_v a_v^\dagger a_v - \hbar M(a_v^\dagger a_c c^\dagger + a_c^\dagger a_v c) + \hbar\omega_0 c^\dagger c - \hbar\sum_q (G_q c^\dagger d_q + G_q^* d_q^\dagger c + \omega_q d_q^\dagger d_q). \quad (1)$$

The electronic two-level system is described via the fermionic annihilation (creation) operator $a_{v/c}^{(\dagger)}$ of the ground (single valence) v and excited (conduction band) state c with energies ω_c and ω_v, respectively. In the following, the band gap energy $\omega_c - \omega_v = \omega_{cv}$ is assumed to be in resonance with the single cavity mode ω_0. A photon annihilation (creation) in the cavity is described with the bosonic operator $c^{(\dagger)}$ and M is the coupling between the two-level system and the cavity mode. We assume a coupling strength in order of magnitude of $M = 50\ \mu$eV [9,25]. The cavity photons interact with the external modes $d_q^{(\dagger)}$ in front of the mirror via the tunnel Hamiltonian coupling element G_q. In the case of the mirror, $G_q = G\sin(qL)$ [24,26], and G is chosen in the order of a cavity loss of $\kappa = \pi G^2/c_0 = 300\ \mu$eV. The coupling element of the tunnel Hamiltonian is derived via the boundary condition of a half-cavity in free space and the energy dispersion of the continuum modes is chosen as the free dispersion of light: $\omega_q = c_0 q$. Note, this widely used tunnel Hamiltonian is an alternative description in comparison to the composite-mode description of two coupled cavities, treating rigorously both cavities as a continuum [27].

The dynamics of the observables of interest is calculated within the framework of the equation of motion approach [28–30], using Heisenberg's equation of motion: $-i\hbar\partial_t\hat{O} = [H, \hat{O}]$ for an observable $\hat{O}$. Given the one-electron assumption: $a_v^\dagger a_v + a_c^\dagger a_c = 1$, the set of equations of motions reads in the rotating frame:

$$\partial_t\langle E\rangle = iM\langle T^\dagger c\rangle - iM\langle Tc^\dagger\rangle, \quad (2)$$

$$\partial_t\langle Td_q^\dagger\rangle = iM\langle d_q^\dagger c\rangle - i\bar{G}_q\langle Tc^\dagger\rangle, \quad (3)$$

$$\partial_t\langle Tc^\dagger\rangle = -iM\langle E\rangle + iM\langle c^\dagger c\rangle - i\sum_q \bar{G}_q^*\langle Td_q^\dagger\rangle, \quad (4)$$

$$\partial_t\langle c^\dagger d_q\rangle = i\bar{G}_q^*\langle c^\dagger c\rangle - iM\langle T^\dagger d_q\rangle - i\sum_{q'}\bar{G}_{q'}^*\langle d_{q'}^\dagger d_q\rangle, \quad (5)$$

$$\partial_t\langle d_{q'}^\dagger d_q\rangle = +i\bar{G}_q^*\langle d_{q'}^\dagger c\rangle - i\bar{G}_{q'}\langle c^\dagger d_q\rangle, \quad (6)$$

with $T := a_v^\dagger a_c$, $E := a_c^\dagger a_c$, and within the corresponding rotating frame $\bar{G}_q := G_q \exp[i(\omega_0 - \omega_q)t]$. The number of excitations in the system N, described with the Hamiltonian in Eq. (2), is the sum of the excitation inside the cavity $N_c = \langle E\rangle + \langle c^\dagger c\rangle$ and outside the cavity $N_{\rm ex} = \sum_q\langle d_q^\dagger d_q\rangle$ and is conserved: $N = N_c + N_{\rm ex}$, therefore: $\partial_t\langle c^\dagger c\rangle = -\partial_t(\langle E\rangle + N_{\rm ex})$.

To reveal single photon events, we choose in the following $N = 1$ as the true quantum limit, where factorizations of the correlations in Eqs. (2)–(6) are not applicable: For only few quantum excitations, fluctuations dominate the combined electron and photon dynamics. In this limit, fundamental quantum processes become experimentally accessible and cQED experiments are conducted successfully in recent years [31,32].

For $N = 1$, higher-order photon-assisted excited state density, e.g., $\langle Ec^\dagger c\rangle$ or $\langle Ec^\dagger d_q\rangle$, are zero for all times and do not need to be included [33]. The set of Eqs. (2)–(6) is closed and can be solved exactly. To make the discussion explicit, we use parameters of a self-organized InAs quantum dot microcavity systems, widely used in experiments [2,10,17,25,34,35]. Note, our approach is not limited to a quantum dot as a two-level emitter. In contrast, it applies to other system such as atomic or molecular systems, also.

Single photon feedback.—We start our discussion of quantum feedback with Fig. 2, including three limiting cases of the electron-photon dynamics (a)–(c) and our main result in case (d).

Preliminary, the Jaynes-Cummings model is represented by case (a) without outcoupling ($G_q = 0$) and case (b) the weak coupling limit without an external mirror ($G_q =$ const). For case (c) and (d), the external mirror is introduced via $G_q = G\sin(qL)$. Without a microcavity, case (c), no Rabi oscillations are seen in the weak coupling limit. In contrast, by studying the full system, including a microcavity, strong coupling features become clearly visible even in the weak coupling limit; cf. Fig. 2(d).

Case (a): The upper panel in Fig. 2 shows the photon dynamics in case of a vanishing outcoupling element $G_q = 0$ with an initially excited two-level system: For this benchmark calculation, the model contains no cavity loss, since outcoupling is suppressed. The system operates

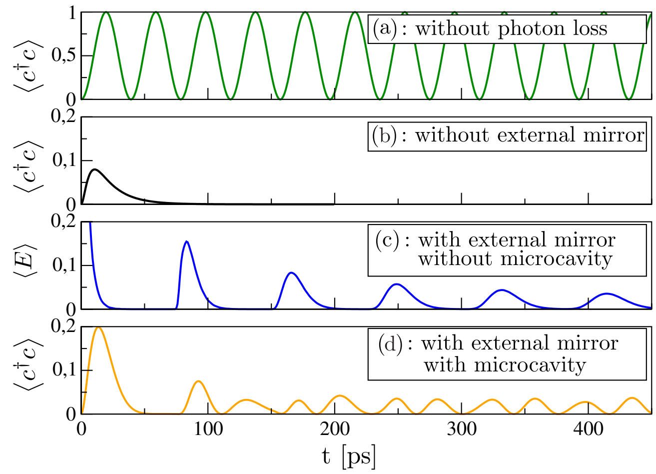

FIG. 2 (color online). Dynamics in the single-excitation limit. Case (a): Without outcoupling, Rabi oscillations occur. In the weak coupling limit without external mirror (b) or with feedback in a bigger cavity (c), no Rabi oscillations occur. But in (d), although in the weak coupling limit, with an external mirror and a microcavity, the delayed feedback effect leads to Rabi oscillations again. Note, that in case (c) the occupation of the upper level $\langle E\rangle$ is plotted, in cases (a),(b),(d) the photon density is $\langle c^\dagger c\rangle$.

in the strong coupling regime, visible at the occurring Rabi oscillations with amplitude 1, Fig. 2(a). The excitation is transferred coherently between the photonic and electronic system at a rate directly proportional to the coupling element, i.e., with $2M$. The solution is given analytically by the Jaynes-Cummings model [24]: $\langle c^\dagger c\rangle(t) = \frac{N_c}{2}[1 - \cos(2Mt)]$, and the number of excitations inside the cavity N_c remains constant for all times at $N_c(t) \equiv 1$.

Case (b): Next, we consider an outcoupling mechanism into the external modes without an external mirror, i.e., $G_q = \text{const} \equiv G \neq 0$. This limit represents the case (b) of an exponential rate for the cavity loss, where excitation is lost to an unstructured continuum: This is the case, if the outcoupling element G does not strongly depend on the wave number q. In the weak coupling regime, i.e., for $G \gg M$ and an initially fully excited two-level system, the solution reads $\langle c^\dagger c\rangle(t) \approx \sin^2(Mt)\exp(-\kappa t)$; cf. Fig. 2(b). It can be recognized that due to the spontaneous emission a photon density is built up at the beginning of the dynamics. Since the cavity loss is large in comparison to the electron-photon coupling, a reversible exchange of excitation between the cavity photon and the electron of the two-level system is inhibited. We observe a decay of the photon density after its initial excitation by the electronic relaxation. In this case, the strength of the coupling element M is clearly not accessible.

After these two benchmarks without a structured external continuum, we now focus on the mirror induced delayed feedback case.

Case (c): Now, since $G_q = G\sin(qL)$, the external mode continuum is structured due to the presence of the external mirror at a distance L in front of the two-level system, which introduces a self-feedback at time $\tau = 2L/c_0$. The dynamics in Fig. 2(c) describes a typical situation, for demonstration, the delay time τ is set to 75 ps [36]. For $t < \tau$, the structured character of the external modes has yet not resolved in time and the situation does not differ from the unstructured case in (b): The excitation decays. For $t > \tau$, the structured character of the continuum is resolved and a pronounced feedback is visible at multiples of τ. Note, in case (c), an approximate analytical solution can be given for an excited state dynamics of $\langle E\rangle(0) = 1$ in the time interval $[m\tau, (m+1)\tau]$ [23]:

$$\langle E\rangle(t) = (m!2^m)^{-1}[\kappa(t-\tau m)]^{2m}\exp[-\kappa(t-m\tau)], \tag{7}$$

with the decay rate κ given above. Besides the initial amplitude of 1 for $m = 0$ and $t = 0$, minimum-maximum analysis of Eq. (7) shows the maximum amplitude is reached at $t_{\text{max}} = \tau + 2/\kappa$ with a value of $\langle E\rangle(t_{\text{max}}) = \exp(-2)$; cf. Fig. 2(c). This value represents an upper limit for delayed feedback effects. Concluding the discussion of case (c), it is important to note that a two-level system in front of a mirror without additional losses does not show any Rabi oscillations. This system corresponds to a bigger cavity, where the lens collects all the emission of the two-level system as well as of the mirror and a continuous energy exchange between the two-level system and the mirror is established [23,37].

Case (d): Next, we study the full system, consisting of a microcavity with a two-level system and an external mirror. We still focus on the case of a weak electron-photon dynamics inside the cavity ($M \ll G$) with an initially excited two-level system and for $M = 55$ μeV, $G = 100$ meV and $G_q = G\sin(qL)$. At the beginning of the dynamics, spontaneous emission leads to a decay of the electronic excitation and to a raise of the photon density under cavity excitation loss to the structured external continuum. Therefore, Rabi oscillations do not occur. In contrast, the total amount of excitation is completely transferred in the first 50 ps to the external continuum. After 75 ps, the delayed feedback drives the cavity system again. In strong contrast to cases (b),(c), irregular Rabi oscillations with an approximate frequency of $2M$ as in the Jaynes-Cummings model occur. These Rabi oscillations become highly regular at 225 ps. From this time on, the oscillations occur clearly with the frequency $2M$ determined by the microscopic electron-photon coupling strength. Surprisingly, this happens in the weak coupling regime, $M \ll G$, when no Rabi oscillations are expected. The observed oscillation frequency depends only on the underlying electron-photon coupling strength and is not affected by other outcoupling parameters. In comparison to pure Rabi oscillations of the isolated cavity-emitter system, the amplitude of the observed feedback oscillations is

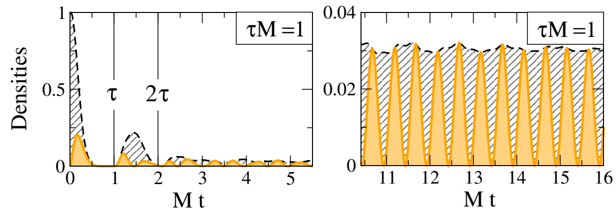

FIG. 3 (color online). Dynamics of the photon density (solid line) and total amount of excitation energy N_c (dashed line) with parameters of Fig. 2(d). The constant, nonvanishing amount of excitation inside the cavity leads to regular Rabi oscillations, revealing the electron-photon coupling M.

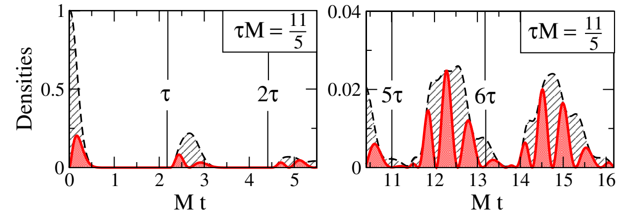

FIG. 4 (color online). Robustness discussion: Dynamics of the photon density (solid line) and total amount of excitation energy N_c (dashed line) for $\tau = \frac{11}{5} M^{-1}$. Even for fluctuating N_c, Rabi oscillations proportional to M occur.

decreased strongly. The amplitude is determined by the outcoupling strength G_q and limited by the upper limit discussed for case (c). Here, the amplitude is in the order of around 5% of the initial excitation. To clarify our results in Fig. 2(d), we now turn to the explanation why the delayed feedback can restore the signatures of strong coupling even in the weak coupling limit.

Feedback-induced oscillations.—The crucial quantity for the explanation of the occurrence of Rabi oscillations in the weak coupling limit is the total amount of excitation inside the cavity: $N_c(t) = \langle c^\dagger c\rangle + \langle E\rangle = 1$. For $N_c =$ constant in a dynamically evolving system of oscillators, both oscillators (two-level system and cavity photon) exchange periodically this excitation in the course of time. In Fig. 3, now for the external mirror case, we plot the dynamics of the total amount of excitation in the cavity N_c (black, dashed line). After dropping completely to zero in the first τ interval, e.g., $Mt = 1/2$, and vanishing again at $Mt = 2$ (left figure), the cavity dynamics and the feedback mechanism establishes a nonvanishing excitation amount inside the cavity; i.e., $N_c(t) > 0$ and const for $Mt > 3$; cf. Fig. 3(left). Obviously, the delayed feedback causes a nearly stationary N_c, due to a constant backcoupling into the cavity. Similar to the case of the ideal Rabi oscillations, for a nonvanishing electron-photon coupling and total excitation $N_c > 0$ inside the cavity, a continuous transfer of excitation between the electronic and photonic system at approximately the rate M occurs. If the fluctuation of the excitation amount N_c is negligible, the oscillation frequency is exactly $2M$ as in the Jaynes-Cummings model. Clearly, the delayed feedback results in a situation comparable to the Jaynes-Cummings model, where also the amount of excitation is fixed due to an ideal, closed cavity. The analytical solution in Fig. 3(right) is approximately $\langle c^\dagger c\rangle(t) = \frac{N_c}{2}[1 - \cos(2Mt)]$ for $Mt > 5$. Fortunately, the presence of the two-level system inside the cavity supports the storage of the cavity excitation, since the outcoupling mechanism acts only on the cavity photons. As the excitation is transferred between the cavity and the external modes, the electronic system stores a necessary amount of excitation to induce the feedback-induced Rabi oscillations. Furthermore, for an increasing number of round trips, the feedback signal smears out and spreads the excitation flow equally in time at the cost of the initially higher amplitude; cf., in Fig. 3(left). In consequence, feedback induced Rabi oscillations are a very robust feature, since two mechanisms guarantee the constant amount of cavity excitation, eventually.

To demonstrate the robustness of our result with respect to different delay times, we calculate the photon density dynamics for another delay time τ without changing other parameters. In Fig. 4, we present the dynamics of the photon density (straight, red line) and total amount of excitation in the cavity (black, dashed line) for $\tau = \frac{11}{5} M^{-1}$. It can be recognized that now with an increased round trip time the total amount of energy N_c drops down to zero longer and more often in comparison with the previous case $M\tau = 1$; cf. Fig. 4(left). For $Mt > 20$ (not shown), the amount of energy inside the cavity becomes approximately constant again and the approximate solution given above with a slightly decreased amplitude is valid again. Regular Rabi oscillations already take place during the transient switch-on period and reveal the electron-photon coupling strength; cf. Fig. 4(right) at, e.g., $Mt > 11$. The constant photonic exchange rate G_q and the presence of the electronic system as an excitation storage on a time scale of M^{-1} leads always to a constant amount of cavity excitation, which leads to regular Rabi oscillations with frequency $2M$.

Conclusion.—We present a fully quantized model to calculate the photon-photon delayed feedback in the single-photon limit beyond typical perturbation approaches. Our model incorporates fundamentally new delay effects into the Jaynes-Cummings model and paves the way to the description of delay-controlled cavity coupled electron-photon dynamics. As a first example, we showed delay-induced Rabi oscillations in the weak coupling regime. We expect exciting new experiments relying on delayed feedback setups in the quantum limit of light, e.g., the photon-photon exchange between wave guide coupled emitter-single-mode cavity systems, and provide a theoretical tool to interpret and predict results yet inaccessible for typical, classical feedback models.

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