

conservation, the recently observed  $\pi$ - $K$  resonance<sup>3</sup> at 723 MeV might be expected to appear at  $Q=85$  MeV. The data are consistent with no formation of this resonance. The value  $Q_0$ , corresponding to  $m=m_0$ , is indicated by the arrow on the  $(\pi^+, p)$   $Q$  distribution, Fig. 3(a). The shift of the theoretical peak from this value represents the combined effects of phase space and the factor  $(\vec{q}')^2$  in expression (10). For fixed barycentric energy, high isobar mass corresponds to low center-of-mass momentum of the  $K^0$ .

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### STRUCTURE OF THE WEAK INTERACTION $\Lambda + N \rightarrow N + N$

M. M. Block\*

Physics Department, Northwestern University, Evanston, Illinois

and

R. H. Dalitz†

The Enrico Fermi Institute for Nuclear Studies and Department of Physics,  
The University of Chicago, Chicago, Illinois

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In this Letter, the empirical evidence available on the nonmesonic decay of  $\Lambda$  hypernuclei will be analyzed in terms of the spin and isospin dependence of the weak interactions

$$\Lambda + p \rightarrow n + p, \quad (1)$$

$$\Lambda + n \rightarrow n + n. \quad (2)$$

Recently, measurements on the nonmesonic decay processes of  ${}_{\Lambda}^4\text{H}$  and  ${}_{\Lambda}^4\text{He}$  hypernuclei produced in  $K^-$ - $\text{He}^4$  reactions have been reported by Block et al.<sup>1</sup> For  ${}_{\Lambda}^4\text{He}$ , they have obtained  $Q({}_{\Lambda}^4\text{He}^5) = 0.52 \pm 0.10$  and  $C({}_{\Lambda}^4\text{He}^4) = 2.2 \pm 0.8$ , where  $Q$  denotes the ratio of nonmesonic to  $\pi^-$  decay modes and  $C$  denotes the ratio of Reactions (1) to (2), determined from the energy spectrum of the final-state protons. For  ${}_{\Lambda}^4\text{H}$ , they have obtained  $Q({}_{\Lambda}^4\text{H}^4) = 0.26 \pm 0.13$ . Using theoretical estimates<sup>2</sup> for the  $\pi^-$ -mesonic decay rates, we may deduce the nonmesonic decay rates for these hypernuclei,

$$\Gamma_{\text{nm}}({}_{\Lambda}^4\text{He}^4) = (0.14 \pm 0.03)\Gamma_{\Lambda}, \quad (3a)$$

$$\Gamma_{\text{nm}}({}_{\Lambda}^4\text{H}^4) = (0.29 \pm 0.14)\Gamma_{\Lambda}, \quad (3b)$$

where  $\Gamma_{\Lambda} = \tau_{\Lambda}^{-1} = (4.25 \pm 0.1) \times 10^9 \text{ sec}^{-1}$  denotes the free  $\Lambda$  decay rate.<sup>3</sup> We note first that these rates are just compatible with the inequality

$$\Gamma_{\text{nm}}({}_{\Lambda}^4\text{H}^4) \leq 2\Gamma_{\text{nm}}({}_{\Lambda}^4\text{He}^4) \quad (4)$$

required by the  $\Delta I = \frac{1}{2}$  rule.<sup>4</sup>

We shall base our detailed analysis on a simplified calculation<sup>2</sup> for the nonmesonic decay rates, which treats the  $\Lambda$  de-excitation by different nucleons as incoherent. This procedure neglects final-state interactions for the two fast outgoing nucleons, and neglects the interference effects which usually arise from antisymmetrization of the final state, corrections which are not expected to be important here, because of the large energy release. In this model, these rates are expressed in terms of the elementary rates  $R_{NS}$  for nonmesonic de-excitation of a  $\Lambda N$  system with total spin  $S$ , for unit density of nucleon  $N$  at the  $\Lambda$  position. In light hypernuclei, the initial  $\Lambda N$  states are  $S$ -wave,  ${}^1S_0$  and  ${}^3S_1$ . The  $\Lambda N - NN$  transitions then possible are listed in Table I, together with the spin dependence of their corresponding matrix

Table I. Summary of the properties of the transitions  $\Lambda p \rightarrow np$  from initial  $^1S_0$  and  $^3S_1$  states. The spin  $\vec{\sigma}_Y$  denotes that for the  $\Lambda$  particle or the final neutron, and  $\vec{q}$  denotes the final neutron momentum [ $(q/M)^2 \approx 0.20$ ]. The rates  $R_{NS}$  are given by  $R_{p0} = |a|^2 + |b|^2(q/M)^2$ ,  $R_{p1} = |c|^2 + |d|^2(q/M)^4 + |e|^2(q/M)^2 + |f|^2(q/M)^2$ , and by  $R_{n0} = |a_n|^2 + |b_n|^2 \times (q/M)^2$ ,  $R_{n1} = |f_n|^2(q/M)^2$ . The coefficients  $a, \dots, f$  are given for  $(\Lambda p)$  de-excitation through the  $(V, A)$  interaction of (13) and (14), and through the Karplus-Ruderman processes of Figs. 1(c) and 1(d). The last entry is given for  $(s_0, p_0) = -(s, p)/\sqrt{2}$ , corresponding to the  $\Delta I = \frac{1}{2}$  rule; the value of  $(pq/sq_\Lambda)$  is about 1.45.

Allowed Transitions	Matrix element	Transition rate	$(V, A)$ Interaction	K-R terms (Eq. 15)	
$^1S_0 \rightarrow ^1S_0$ $\rightarrow ^3P_0$	$I = 1$ $\frac{a}{4}(1 - \vec{\sigma}_Y \cdot \vec{\sigma}_N)$ $\frac{b}{8M}(\vec{\sigma}_Y - \vec{\sigma}_N) \cdot \vec{q}(1 - \vec{\sigma}_Y \cdot \vec{\sigma}_N)$	$ a ^2$ $ b ^2 \left(\frac{q}{M}\right)^2$	$a = (f_\Lambda + g_\Lambda)(1 + 3\lambda\eta)$ $b = -2(f_\Lambda + g_\Lambda)(\lambda + \eta)$	$a = \frac{(p_0 + p\sqrt{2})q^2}{Mq_\Lambda} = \frac{pq^2}{\sqrt{2}Mq_\Lambda}$ $b = s_0 + s\sqrt{2} = \frac{s}{\sqrt{2}}$	
$^3S_1 \rightarrow ^3S_1$ $\rightarrow ^3D_1$ $\rightarrow ^1P_1$ $\rightarrow ^3P_1$	$I = 0$ $I = 1$ $I = 1$ $I = 1$	$ c ^2$ $ d ^2 \left(\frac{q}{M}\right)^4$ $ e ^2 \left(\frac{q}{M}\right)^2$ $ f ^2 \left(\frac{q}{M}\right)^2$	$c = (f_\Lambda - 3g_\Lambda)(1 - \lambda\eta)$ $d = \frac{(f_\Lambda - 3g_\Lambda)(1 - \lambda\eta)}{6\sqrt{2}}$ $e = 0$ $f = \frac{\sqrt{2}}{\sqrt{3}}(f_\Lambda + g_\Lambda)(\eta - \lambda)$	$c = \frac{(p\sqrt{2} - p_0)q^2}{3Mq_\Lambda} = \frac{pq^2}{\sqrt{2}Mq_\Lambda}$ $d = \frac{(p\sqrt{2} - p_0)M\sqrt{2}}{3q_\Lambda} = \frac{pM}{q_\Lambda}$ $e = \frac{s_0 - \sqrt{2}s}{\sqrt{3}} = -s\frac{\sqrt{3}}{\sqrt{2}}$ $f = \frac{-2(\sqrt{2}s + s_0)}{\sqrt{6}} = \frac{-s}{\sqrt{3}}$	

element. Thus, for  $^1S_0$  capture, the failure of parity conservation for weak interactions allows both the transitions  $^1S_0 \rightarrow ^1S_0$  and  $^1S_0 \rightarrow ^3P_0$ . We note that  $^1S_0$  capture leads only to  $I = 1$  final states, whereas both  $I = 0$  and  $I = 1$  final states are available for  $^3S_1$  capture. From this, it follows that the validity of the  $\Delta I = \frac{1}{2}$  rule for the nonmesonic decay interactions would require<sup>5</sup>

$$R_{n0} = 2R_{p0}, \quad (5a)$$

$$R_{n1} \leq 2R_{p1}. \quad (5b)$$

With this model,<sup>2</sup> the nonmesonic decay rate for hypernucleus  ${}_\Lambda Z^A$  is given by  $\rho_A \bar{R}({}_\Lambda Z^A)$ , where  $\bar{R}$  denotes the spin and charge average of the  $R_{NS}$  appropriate to this hypernucleus, and  $\rho_A$  denotes the mean nucleon density at the  $\Lambda$  position, given by

$$\rho_A = (A - 1) \int \rho_N(\vec{r}) \psi_\Lambda^2(\vec{r}) d^3r,$$

$\rho_N$  being the nucleon density (normalized to unity) and  $\psi_\Lambda$  the wave function for the relative motion between the  $\Lambda$  particle and the nuclear core. For the  $J = 0$  hypernuclei  ${}_\Lambda H^4$  and  ${}_\Lambda He^4$ , we then have

the nonmesonic rates

$$\Gamma_{nm}({}_\Lambda He^4) = \frac{1}{8}\rho_4(3R_{p1} + R_{p0} + 2R_{n0}), \quad (6a)$$

$$\Gamma_{nm}({}_\Lambda H^4) = \frac{1}{8}\rho_4(3R_{n1} + R_{n0} + 2R_{p0}), \quad (6b)$$

where  $\rho_4$  has the value<sup>2</sup>  $0.019 \text{ fm}^{-3}$ , with

$$C({}_\Lambda He^4) = (3R_{p1} + R_{p0})/2R_{n0}, \quad (7a)$$

$$C({}_\Lambda H^4) = 2R_{p0}/(3R_{n1} + R_{n0}). \quad (7b)$$

From (6a), (7a), and the data on  ${}_\Lambda He^4$  alone, we deduce directly  $R_{n0} = (6.6 \pm 2.1)\Gamma_\Lambda \text{ fm}^3$ , and  $\frac{1}{4}(3R_{p1} + R_{p0}) = (7.3 \pm 1.8)\Gamma_\Lambda \text{ fm}^3$ .

In order to carry this analysis further, we shall assume from this point on that the  $\Delta I = \frac{1}{2}$  rule is valid for these interactions. With the equality (5a), this allows the determination of all the  $R_{NS}$  from these data. As remarked above, the nonmesonic rates (3) are compatible with the inequality (4); however, with Eqs. (6) and (7) (which introduce the additional information that  $J = 0$ ), the relations (5) lead to the stronger inequality,<sup>6</sup>

$$\Gamma_{nm}({}_\Lambda H^4) \leq 2\Gamma_{nm}({}_\Lambda He^4) \left\{ 1 - \frac{3}{4[1 + C({}_\Lambda He^4)]} \right\}, \quad (8)$$

with which the data are compatible, within experimental error. In order to obtain consistent equations, we now adopt the value

$$\Gamma_{nm}(\Lambda H^4) = (0.21^{+0.05}_{-0.08})\Gamma_{\Lambda}, \quad (9)$$

where the median value and upper limit are fixed by the upper limit of (8) and the lower limit corresponds to one standard deviation below the measured rate. From the  $\Lambda$ He<sup>4</sup> data, the equality (5a) leads to

$$R_{p1} = (8.6 \pm 3.0)\Gamma_{\Lambda} \text{ fm}^3, \quad (10a)$$

$$R_{p0} = \frac{1}{2}R_{n0} = (3.3 \pm 1.1)\Gamma_{\Lambda} \text{ fm}^3. \quad (10b)$$

Finally, the  $\Lambda H^4$  rate (9) then leads to

$$R_{n1} = (17.2 \pm 6.0)\Gamma_{\Lambda} \text{ fm}^3. \quad (10c)$$

We note that the equality  $R_{n1} = 2R_{p1}$  is required (within experimental errors) by the data; this reflects the fact that the value (9) corresponds to the equality in Eq. (8). Consequently, the  $\Lambda N - NN$  transitions take place dominantly to  $I=1$  final states; further,  $R_{p1} = 2.6R_{p0}$ , so that the triplet interaction rate is the stronger.<sup>7</sup> Indeed, from Table I, we can say that the dominant transition is  ${}^3S_1 - {}^3P_1$ , together with some  ${}^1S_0 - {}^1S_0$  and  ${}^3P_0$  transitions, corresponding to a matrix element of the form<sup>8</sup>

$$M(\Lambda p \rightarrow np) = \frac{f\sqrt{6}}{4M}(\vec{\sigma}_Y + \vec{\sigma}_N) \cdot \vec{q} + \left[ a + \frac{b}{2M}(\vec{\sigma}_Y - \vec{\sigma}_N) \cdot \vec{q} \right] \times \left[ \frac{1}{4}(1 - \vec{\sigma}_Y \cdot \vec{\sigma}_N) \right]. \quad (11)$$

A measurement of  $C(\Lambda H^4)$  would provide a sensitive test of the  $\Delta I = \frac{1}{2}$  assumption. Since  $C(\Lambda H^4) = 2R_{p0}/(3R_{n1} + R_{n0})$ , this would determine  $R_{p0}$  directly and test the significant equality (5a). The value expected from the  $R_{NS}$  given above is  $C(\Lambda H^4) = 1/(8.8 \pm 3.2)$ . No empirical estimate of this ratio is available at present.

Finally, we discuss briefly the possibilities for a simple interpretation of these values  $R_{NS}$ . With the  $\Delta I = \frac{1}{2}$  rule, these four-fermion interactions have form limited to

$$\sum_i \{ f_{\Lambda} (\bar{N}K_i \Lambda_S) (\bar{N}K_i' N) + g_{\Lambda} (\bar{N}L_i \vec{\tau} \Lambda_S) (\bar{N}L_i' \vec{\tau} N) \}, \quad (12)$$

where  $\Lambda_S$  denotes the spurion ( $I = \frac{1}{2}$ ,  $I_3 = -\frac{1}{2}$ ) wave function for the  $\Lambda$  particle. Rewriting this inter-

action, we have the general form

$$\sum_i \{ f_{\Lambda} (\bar{N}K_i \Lambda) (\bar{p}K_i' p + \bar{n}K_i' n) + g_{\Lambda} [ -(\bar{N}L_i \Lambda) (\bar{p}L_i' p - \bar{n}L_i' n) + 2(\bar{p}L_i \Lambda) (\bar{n}L_i' p) ] \}. \quad (13)$$

The primary four-fermion interaction must also be of this general form; its contributions to the process  $\Lambda + p \rightarrow n + p$  are illustrated in Figs. 1(a) and 1(b). However, with four strongly interacting particles, the primary form may be distorted by mesonic corrections in quite a complicated way. For example, Karplus and Ruderman<sup>9</sup> have discussed a class of mesonic corrections directly related with the  $\Lambda - N + \pi$  interaction, illustrated in Figs. 1(c) and 1(d), although there is no reason at present to believe that these necessarily represent the dominant corrections.

As the simplest possibility, we consider a general  $(V,A)$  four-fermion interaction (12), with

$$K_i = L_i = \gamma_{\mu} + \lambda \gamma_{\mu} \gamma_5, \quad K_i' = L_i' = \gamma_{\mu} + \eta \gamma_{\mu} \gamma_5, \quad (14)$$

for which the individual transition amplitudes have been listed in Table I. In order to suppress both the  $I=0$  transitions and the  ${}^1S_0 - {}^1S_0$  transition, it is necessary to choose  $(A - 3B) \approx 0$  and  $(1 + 3\lambda\eta) \approx 0$  quite closely. The parameters  $\lambda, \eta$  can then be obtained to fit the ratio of the  ${}^3S_1 - {}^3P_1$  and  ${}^1S_0 - {}^3P_0$  transition rates; only an upper limit is known for the latter rate, since we know only the sum  $R_{N0}$  of the  ${}^1S_0 - {}^1S_0$  and  ${}^1S_0 - {}^3P_0$  rates. The data allow four possible regions for  $(\lambda, \eta)$ :

- (i)  $0.6 \leq \eta \leq 0.75$  with  $\lambda$  from  $-0.6$  to  $-0.45$ ,
- (ii)  $-0.6 \geq \eta \geq -0.75$  with  $\lambda$  from  $0.6$  to  $0.45$ ,
- (iii)  $0.6 \geq \eta \geq 0.45$  with  $\lambda$  from  $-0.6$  to  $-0.75$ , and
- (iv)  $-0.6 \leq \eta \leq -0.45$  with  $\lambda$  from  $0.6$  to  $0.75$ ;

in each case, the right-hand limit corresponds to the zero rate for the  ${}^1S_0 - {}^1S_0$  transition, the left-hand limit to zero rate for the  ${}^1S_0 - {}^3P_0$  transition. We note explicitly that these interactions are

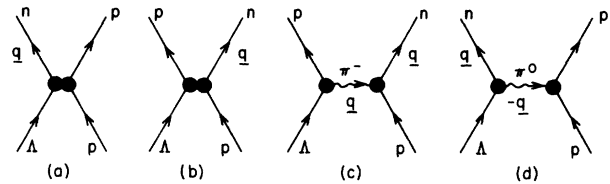


FIG. 1. Graphs (a) and (b) show the primary processes contributing to the  $\Lambda + p \rightarrow n + p$  interaction. Graphs (c) and (d) show the mesonic correction terms discussed by Karplus and Ruderman.

quite different in character from the ( $V-A$ ) interaction, for which  $\eta = \lambda = +1$  holds and which allows only the transitions  $^1S_0 \rightarrow ^1S_0$  and  $^3P_0$ , contrary to observation. We have no interpretation to offer for the  $(3 + \vec{\tau}_Y \cdot \vec{\tau}_N)$  form required by the data; all simple models of weak interactions considered at present lead naturally to a form  $\vec{\tau}_Y \cdot \vec{\tau}_N$ , since the strangeness-conserving weak current necessarily has an isovector component and is usually assumed to be pure isovector. With these sets  $(\lambda, \eta)$ , it is of interest to note that the value required for  $g_\Lambda$  to fit the total transition rate is  $g_\Lambda = (0.35 \pm 0.05) \times 10^{-5}/M^2$ , which may be compared<sup>10</sup> with the beta-decay coupling parameter  $g_\beta = 1.02 \times 10^{-5}/M^2$ .

The Karplus-Ruderman terms provide a second possibility of particular interest. For the process  $\Lambda + p \rightarrow n + p$ , these have the form

$$D \{ (s_0 + p_0 \vec{\sigma}_Y \cdot \vec{q}/q_\Lambda) (-\vec{\sigma}_N \cdot \vec{q}/M) - P_{YN} \sigma \sqrt{2} (s - p \vec{\sigma}_Y \cdot \vec{q}/q_\Lambda) (\sigma_N \cdot \vec{q}/M) \}, \quad (15)$$

where  $(s_0, p_0)$  and  $(s, p)$  denote the  $\Lambda \rightarrow n + \pi^0$  and  $\Lambda \rightarrow p + \pi^-$  decay amplitudes,  $\vec{\sigma}_Y$  denotes the spin of the  $\Lambda$  particle or the final neutron,  $P_{YN}^\sigma$  is the spin-exchange operator, and the coefficient  $D = G/2[M(M_\Lambda - M) + m_\pi^2]$ , where  $G^2/4\pi \approx 14.7$  is the pion-nucleon coupling constant.<sup>11</sup> The transition amplitudes corresponding to (15) are given in Table I. With the  $\Delta I = \frac{1}{2}$  rule,  $s_0/s = p_0/p = -1/\sqrt{2}$ , and the dominant transitions are those from the  $^3S_1$  state to final  $I=0$  states, quite contrary to the observations<sup>12</sup>; further, the  $^1S_0 \rightarrow ^3P_0$  and  $^3S_1 \rightarrow ^3P_1$  and  $^1P_1$  transitions contribute comparably to the rates  $R_{NS}$ , in the ratio 3:2:9. We note that a linear combination of a ( $V-A$ ) interaction with the Karplus-Ruderman terms cannot fit the data, since this necessarily gives strong  $I=0$  transitions; in particular, any linear combination of a ( $V, A$ ) interaction (14) with the Karplus-Ruderman terms (15) necessarily leaves a strong  $^3S_1 \rightarrow ^1P_1$  amplitude. The only conclusion which can be drawn at this stage is that, if the  $\Delta I = \frac{1}{2}$  rule holds for the  $\Lambda N - NN$  weak interaction and our present notions about the current-current nature of weak interactions are valid, then it must be that the higher order mesonic corrections to this primary four-fermion interaction are sufficiently large to mask the simplicity of their primary form.

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<sup>4</sup>See R. H. Dalitz and L. Liu, Phys. Rev. 116, 1312 (1959), Appendix.

<sup>5</sup>The  $\Delta I = \frac{1}{2}$  rule requires the following values for the  $(\Lambda n)$  amplitudes:  $a_n = 2a$ ,  $b_n = 2b$ ,  $c_n = d_n = e_n = 0$ ,  $f_n = 2f$ . The  $(\Lambda n)$  reaction rates are only twice those for the  $(\Lambda p)$  reactions to the same  $I=1$  final states, because the phase space available for the  $n$  system is only half that for the  $p$  system.

<sup>6</sup>These relations also lead to the inequality  $1 \geq C(\Lambda \text{He}^4) \geq [4C(\Lambda \text{He}^4)]^{-1}$ .

<sup>7</sup>These values for  $R_{NS}$  lead to the following predictions for  $\Lambda \text{He}^5$  decay:

$$Q(\Lambda \text{He}^5) = \left\{ \frac{1}{5} \rho_5 (3R_{p1} + R_{p0} + 3R_{n1} + R_{n0}) \right\} / \{0.25 \Gamma_\Lambda\} \\ = 1.68 \pm 0.36,$$

$$C(\Lambda \text{He}^5) = (3R_{p1} + R_{p0}) / (3R_{n1} + R_{n0}) = 0.5,$$

where the value  $\rho_5 = 0.038 \text{ fm}^{-3}$  is given in reference 2. The first prediction leads to good agreement with the estimates available in the literature [see P. Schlein, Phys. Rev. Letters 2, 220 (1959), and earlier references cited there] for  $Q(\Lambda \text{He})$  in emulsion, where the events correspond to a mixture of  $\Lambda \text{He}^4$  and  $\Lambda \text{He}^5$  in a ratio about 1:4. This agreement is not to be regarded as strong support for the  $\Delta I = \frac{1}{2}$  rule, however, since the input data and positive definiteness for the  $R_{NS}$  already constrain  $Q(\Lambda \text{He}^5)$  to lie between  $1.2 \pm 1.0$  and  $2.3 \pm 1.0$ . No measurement of  $C(\Lambda \text{He}^5)$  is yet available; an equivalent estimate may possibly be made by comparing the proton spectrum observed from nonmesonic decay of heavy hyperfragments with the proton spectra computed for elementary  $(\Lambda p)$  and  $(\Lambda n)$  de-excitation processes.

<sup>8</sup>If time-reversal invariance holds for these interactions, the phases of  $a, b$  and  $f$  are given by the nucleon-nucleon scattering phases in the corresponding final states.

<sup>9</sup>R. Karplus and M. Ruderman, Phys. Rev. 76, 1458 (1949). See also F. Cerulus, Nuovo Cimento 5, 1685 (1957); S. B. Treiman, Proceedings of the 1958 Annual International Conference on High-Energy Physics at CERN (CERN Scientific Information Service, Geneva, Switzerland, 1958), p. 276.

<sup>10</sup>It is amusing to note that the beta-decay interaction  $\sqrt{2} g_\Lambda [\bar{p} \gamma_\mu (1 + \lambda \gamma_5) \Lambda] [\bar{e} + \gamma_\mu \frac{1}{2} (1 + \gamma_5) \nu]$  which corresponds to the  $(\bar{N} \vec{\tau} \Lambda_S)$  current of (12) leads to a branching ratio

for  $\Lambda$  beta decay given by  $0.016(g_\Lambda/g_\beta)^2(1+3\lambda^2)/[1+3(1.14)^2]$ . With  $\eta=+0.75$ , the value appropriate to  $\lambda$  is  $-0.45$ , and this branching ratio becomes  $0.016/27=0.6\times 10^{-3}$ , quite compatible with the value  $(0.82\pm 0.13)\times 10^{-3}$  reported recently by R. Ely, G. Gidal, G. Kalamus, L. Oswald, W. Powell, W. Singleton, F. Bullock, C. Henderson, D. Miller, and F. Stannard, Phys. Rev. **131**, 868 (1963).

<sup>11</sup>In general, the factor  $D$  will include form factors

corresponding to the two vertices shown in Figs. 1(c) and 1(d). Since  $q=400$  MeV/c, there could be quite appreciable uncertainty in the magnitude of the matrix element given for the Karplus-Ruderman terms by expression (15).

<sup>12</sup>With the Karplus-Ruderman terms alone,  $(\Lambda p)$  de-excitation would be the dominant process. With  $x=(pq/sq_\Lambda)^2\approx 2.2$ , the ratio  $C(\Lambda\text{He}^3)$  would be  $(6+14x)/(3+x)\approx 7.1$ .

## ELECTROMAGNETIC AND WEAK INTERACTIONS IN THE UNITARY SYMMETRY SCHEME\*

S. P. Rosen

Purdue University, Lafayette, Indiana

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In the unitary symmetry scheme of Gell-Mann<sup>1</sup> and Ne'eman,<sup>2</sup> particles in a given unitary multiplet are usually classified by means of isotopic spin and hypercharge. It has, however, been observed by Levinson, Lipkin, and Meshkov<sup>3,4</sup> that other classifications can be obtained by considering  $U_2$  subgroups of  $U_3$  that are different from the isotopic-spin subgroup. Here we take advantage of these alternative classifications to derive general formulas for magnetic moments and electromagnetic mass differences of elementary particles, and to make some speculations about the weak interactions. As far as the metastable baryons are concerned, our formulas yield no relations other than those obtained by other authors<sup>5-8</sup>; they are, however, valid for all representations of  $SU_3$ , and, as an illustration, they are applied to the baryon-meson resonances of the "tenfold way."<sup>9</sup>

Following Okubo<sup>7,8</sup> we consider the generators  $A_{\nu}^{\mu}$  ( $\mu, \nu=1, 2, 3$ ) of infinitesimal unitary transformations in  $U_3$ . Their commutation rules

$$[A_{\beta}^{\alpha}, A_{\nu}^{\mu}] = \delta_{\beta}^{\mu} A_{\nu}^{\alpha} - \delta_{\nu}^{\alpha} A_{\beta}^{\mu} \quad (1)$$

and the unitary restriction

$$(A_{\beta}^{\alpha})^{\dagger} = A_{\alpha}^{\beta} \quad (2)$$

enable us to divide the generators into three sets, each containing an angular momentum type operator and a corresponding hypercharge operator. They are

$$T_{+} = -A_1^2, \quad T_{-} = -A_2^1, \quad T_3 = \frac{1}{2}(A_2^2 - A_1^1), \quad Y_T = A_3^3,$$

with

$$T_{\pm} = T_1 \pm iT_2; \quad (3)$$

$$L_{+} = -A_1^3, \quad L_{-} = -A_3^1, \quad L_3 = \frac{1}{2}(A_3^3 - A_1^1), \quad Y_L = A_2^2,$$

with

$$L_{\pm} = L_1 \pm iL_2; \quad (4)$$

and

$$K_{+} = -A_2^3, \quad K_{-} = -A_3^2, \quad K_3 = \frac{1}{2}(A_3^3 - A_2^2), \quad Y_K = A_1^1,$$

with

$$K_{\pm} = K_1 \pm iK_2. \quad (5)$$

From each of these sets we can construct a set of mutually commuting operators

$$\vec{T}^2 = T_1^2 + T_2^2 + T_3^2, \quad T_3, \quad Y_T; \quad (6)$$

$$\vec{L}^2 = L_1^2 + L_2^2 + L_3^2, \quad L_3, \quad Y_L; \quad (7)$$

and

$$\vec{K}^2 = K_1^2 + K_2^2 + K_3^2, \quad K_3, \quad Y_K. \quad (8)$$

Because of the commutation rules in (1),  $\vec{T}^2$ ,  $\vec{L}^2$ , and  $\vec{K}^2$  do not commute with one another; hence, only one of the three sets of operators (6), (7), (8) can be diagonalized in an arbitrary matrix representation of the  $A_{\nu}^{\mu}$ .

We identify  $\vec{T}^2, T_3$  with the usual isotopic-spin operators, and  $Y_T$  with the usual hypercharge

$$Y_T = (B+S), \quad (9)$$

where  $B$  denotes baryon number and  $S$  strangeness. If we restrict ourselves to representations  $U(f_1, f_2, f_3)$  of  $U_3$ , such that<sup>9</sup>

$$f_1 + f_2 + f_3 = 0, \quad (10)$$

then<sup>8</sup>

$$A_1^1 + A_2^2 + A_3^3 = 0. \quad (11)$$