# SHRINKAGE OF THE DIFFRACTION PATTERN AND SHORT-RANGE FORCES*

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Recent high-energy experiments have established that in the energy range of about 10-20 BeV/$c$ there is shrinkage of the diffraction pattern in $pp$,[1,2] but not in $\pi^-p$ scattering.[2] This evidence, however, is based on values of momentum transfer, $-t > 0.2$ (BeV/$c$)$^2$, much larger than the expected width of the diffraction pattern of about 0.1 (BeV/$c$)$^2$ ($\approx 4m_\pi^2$). Therefore, it may very well be that the shrinkage in $pp$ is due to strong short-range effects (which would be noticeable at large momentum transfers) and may have no connection to the conjectured Pomeranchuk-Regge pole.[3] In view of the fact that the $\pi^-p$ diffraction pattern does not shrink, it seems that important effects should arise from $\omega$ exchange which is present in $pp$ but not in $\pi p$ scattering.

If Regge poles dominate the high-energy behavior of cross sections, then, in order to fit the $\pi^-p$ data, there seems to be no alternative to assuming that $\alpha_P'(0)$ of the Pomeranchuk trajectory ($P$) be much smaller (<1/5) than originally estimated by Chew and Frautschi.[3,4] There is nothing known in potential theory that contradicts such an assumption of small slopes.[5] With the assumption that $\alpha_P(t) \approx 1$ up to moderately large values of $-t$, we can represent the existing $\pi^-p$ differential cross-section data by $P$ alone.[6,7] We then have

$$d\sigma_{\pi p}/dt = \gamma_P^{\ 2}(t) = \gamma_P^{\ 2}(0)e^{t/C}. \qquad (1)$$

An exponential fall-off for the residue $\gamma_P(t)$, such as the one given above, seems necessary to fit the $\pi^-p$ data.[2] This expression, of course, will not be true for very large values of $-t$.[8,9] Here $\gamma_P^{\ 2}(0) = (0.22\,\sigma_{\pi p}^{\ T})^2$ mb/(BeV/$c$)$^2$, where $\sigma_{\pi^-p}^{\ T}$ is the total $\pi^-p$ cross section.[10] With $C = 0.13$ (BeV/$c$)$^2$ and the known experimental values of $\sigma^T$, very good agreement is obtained with the diffraction data of reference 2. A similar situation will hold in $\pi^+p$ case also where the scattering again can be represented by $P$ alone. In other words, there will not be any shrinkage in $\pi^+p$ scattering.[11]

In $pp$ scattering the observed shrinkage can be understood, as mentioned earlier, in terms of the $\omega$ trajectory. Because of its odd $G$-parity, $\omega$ exchange is present in $pp$ but not in $\pi p$.[12] Even though $\rho$ contributes to both, from $pp$-$np$ and $\pi^{\pm}p$ total cross-section data, it appears that its contribution is very weak and can be ignored.[13] Along with $P$ and $\omega$ we shall consider $P'$ also,[14] since among other things its presence enables $\sigma_{pp}^{\ T}$ to remain a constant. We then have

$$d\sigma_{pp}/dt = |A_{pp}|^2 \qquad (2)$$

$$A_{pp} = i\beta_P(t) + \frac{1+\exp[-i\alpha_{P'}(t)]}{\sin\pi\alpha_{P'}(t)}\beta_{P'}(t) \times \exp\{[\alpha_{P'}(t)-1]\ln E\} + \frac{1-\exp[-i\pi\alpha_\omega(t)]}{\sin\pi\alpha_\omega(t)} \times \beta_\omega(t)\exp\{[\alpha_\omega(t)-1]\ln E\}, \qquad (3)$$

where $E$ is the lab energy and the negative sign in the $\omega$ term comes from its odd signature. In order to give a constant value to the total $pp$ cross section, it is necessary that at $t=0$ the $P'$ contribution in the imaginary part of $A_{pp}$ exactly cancel the $\omega$ contribution.[15] We shall assume that this happens up to moderately large negative $t$ values so that in the imaginary part only $P$ survives. The parameters of $P'$ are then determined by those of $\omega$. In the real part of $A_{pp}$, $P'$ and $\omega$ add. We shall take the slope $\alpha_\omega'(0)$ [$=\alpha_{P'}'(0)$] to be of the same order as $\alpha_P'(0)$ (<1/5) and ignore it in the present calculation.[16] This is a plausible assumption and has the additional advantage of reducing the total number of parameters to just two. For $\alpha_\omega(0)$ [$=\alpha_{P'}(0)$] we shall take the value 0.4 estimated on the basis of forward $\pi N$ dispersion relations and total cross-section data.[13]

For negative $t$ values we shall represent $\beta$'s by

$$\beta_P^{\ 2}(t) = \beta_P^{\ 2}(0)e^{t/a}; \qquad \beta_\omega^{\ 2}(t) = \beta_\omega^{\ 2}(0)e^{t/b}. \qquad (4)$$

As in the $\pi p$ case, the exponential fall-off for $P$ is indicated by the sharply falling diffraction pattern.[17] The exponential $t$ dependence for $\omega$ is taken in analogy with $P$ and is a convenient way of parametrization. The above form of $\beta$'s will certainly be incorrect for very large $-t$ values, as they are inconsistent with analyticity requirements.[8,9] We then have

$$d\sigma_{pp}/dt=\beta_P{}^2(0)e^{t/a}+[\cot\tfrac{1}{2}\pi\alpha_\omega(0)+\tan\tfrac{1}{2}\pi\alpha_\omega(0)]^2$$

$$\times\beta_\omega{}^2(0)\exp\{2[\alpha_\omega(0)-1]\ln E\}e^{t/b}, \qquad (5)$$

where

$$\beta_P{}^2(0)=(0.22\,\sigma_{pp}{}^T)^2\ \mathrm{mb/(BeV}/c)^2,$$

$$\beta_\omega{}^2(0)\exp\{2[\alpha_\omega(0)-1]\ln E\}=[0.11(\sigma_{\bar{p}p}{}^T-\sigma_{pp}{}^T)]^2,$$

and $\alpha_\omega(0)=0.4$.[11,12] We therefore have a formula in terms of two parameters $a$ and $b$, where $a$ determines the width of the diffraction and $b$ should be roughly proportional to the $(\mathrm{range})^{-2}$ of the hard core generated by $\omega$. The values for $\sigma_{pp}{}^T$ and $\sigma_{\bar{p}p}{}^T$ are taken from known experimental data. For $a=0.10$ and $b=0.24$ we have plotted $d\sigma^R/dt=(d\sigma/dt)/[\sigma_T(E)/\sigma_T(20\ \mathrm{BeV}/c)]^2$ against $-t$ in Fig. 1, and in Fig. 2 the same quantity is plotted against $\log_{10}s$ $(=\log_{10}2E)$. The experimental points up to $-t\approx0.6$ and $E\simeq20$ BeV/$c$ are indicated. We observe that good agreement with experiment is obtained for values up to $-t\simeq0.5$. For higher $t$ values, disagreements are expected because there the assumption of the exponential $t$ dependence and the neglect of slopes of $\alpha$ will not be correct. Disagreement at low values of $E$ is also expected (see Fig. 2) because $\sigma_{pp}{}^T$ there is not a constant as is assumed in (5). It is also interesting to note that $a$ $(\approx4m_\pi{}^2)$ and $b$ $(\approx9m_\pi{}^2)$ have the expected order of magnitude.

Qualitatively we can understand the above situation as follows: The first term in (5), the Pomeranchuk term, controls the low momentum transfer behavior $(-t\lesssim a)$ of $d\sigma^R/dt$. The effect of the second term is such as to add a small amount to the $t=0$ value of $d\sigma^R/dt$, but the slope of $d\sigma^R/dt$ at $t=0$ is essentially determined by $a^{-1}$ of the $P$ term and is energy independent. As $E$ is increased the second term decreases, and,

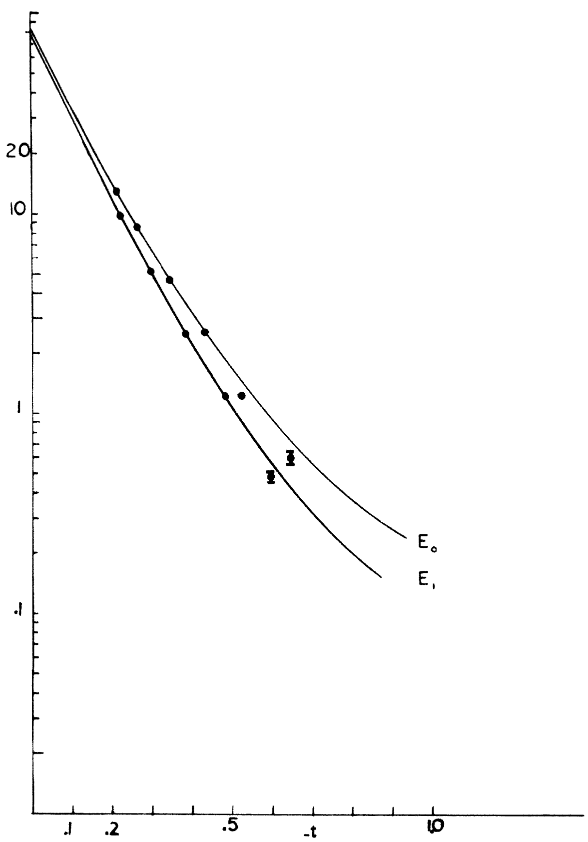


FIG. 1. Differential cross section $(d\sigma/dt)/[\sigma^T(E)/\sigma^T(20\ \mathrm{BeV}/c)]^2$ mb/(BeV/$c$)$^2$ for $pp$ scattering vs momentum transfer $-t$ (BeV/$c$)$^2$ up to $-t$ about 0.6. $E_0$ and $E_1$ indicate lab energies 10.79 and 19.59 BeV/$c$, respectively. Circles indicate experimental points from reference 2. The experiments were not done for $-t$ less than 0.2.

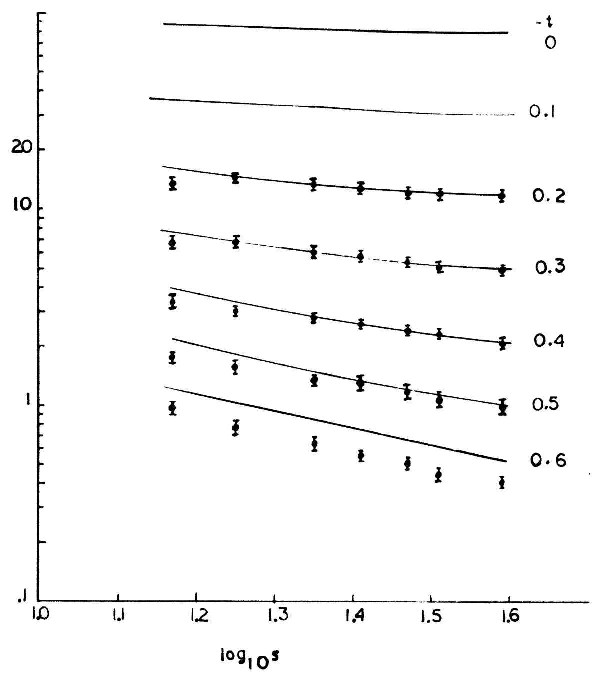


FIG. 2. Differential cross section $(d\sigma/dt)/[\sigma^T(E)/\sigma^T(20\ \mathrm{BeV}/c)]^2$ mb/(BeV/$c$)$^2$ for $pp$ scattering vs $\log_{10}s$ $(=\log_{10}2E)$ for different values of $-t$ up to 0.6, where $E$ is the lab energy in BeV/$c$. The experimental points from reference 2 are indicated. The experiments were not done for $-t$ less than 0.2.

therefore, the diffraction pattern, up to $-t \approx a$, will move downward parallel to itself. At moderately large values of $-t$ the $\omega$ term is important. At 10 BeV/$c$ it is rather large since the difference $(\sigma_{\bar{p}p}{}^T - \sigma_{pp}{}^T)$ there is large, but it decreases as $E$ is increased. Therefore, at moderately large values of $-t$ there will be a substantial shrinkage as $E$ is increased. This is what one observes in $pp$ scattering for $-t \gtrsim 0.2$ (BeV/$c$).[1,2] The essential difference between this mechanism and the shrinkage caused by the Pomeranchuk pole with $\alpha_P{}'(0) \approx 1$ would be apparent at low $-t$. With a single Pomeranchuk pole, the slopes of $d\sigma/dt$ near $t=0$ (i.e., $-t \lesssim a$) would be different for different energies (the value of $d\sigma/dt$ at $t=0$ being energy independent) and the shrinkage larger.[18]

In $\bar{p}p$ scattering the residue $\beta_\omega$ in $A$ [see expressions (2) and (3)] will acquire a negative sign, and, therefore, $P'$ and $\omega$ will add in the imaginary part and subtract in the real part of $A$.[12,13] Compared to the $pp$ scattering, the imaginary part here is large (as $\sigma_{\bar{p}p}$ is large), and, because of the choice of $\alpha_\omega(0)$, the real part is negligible. In Fig. 3 we have plotted $d\sigma^R/dt = (d\sigma/dt)/[\sigma^T(E)/\sigma^T(20\text{ BeV}/c)]^2$ for $\bar{p}p$ with the same values of $a$ and $b$ as obtained in $pp$ scattering and for the same energies. There are certain differences from the $pp$ case which should be noticed. Because the real part is negligible, the $t=0$ value of $d\sigma^R/dt$ is the same for both energies ($\approx$10 and $\approx$20 BeV/$c$). The slopes at $t=0$ of $d\sigma^R/dt$, however, are different for different energies, being proportional to $a^{-1}$ times $(\sigma_{\bar{p}p}{}^T)^{-1}$. The shrinkage for $-t \lesssim a$ is slightly more than for $pp$ because of the interference effects of $\omega$ with $P$. For $-t \gtrsim b$ the shrinkage of $d\sigma^R/dt$ in $\bar{p}p$ is less than in $pp$ scattering by a factor proportional to $[\sigma_{\bar{p}p}{}^T(E_1)/\sigma_{\bar{p}p}{}^T(E_0)]^2$, where $E_1 > E_0$.

The $\omega$ exchange is present in $K^\pm p$ scattering also. The expressions for $d\sigma^R/dt$ and the corresponding diffraction pattern for $K^+p$ and $K^-p$ will, therefore, be similar to $pp$ and $\bar{p}p$, respectively, with only minor differences in the parameters $a$ and $b$ of $pp$ scattering. Because $(\sigma_{K^-p}{}^T - \sigma_{K^+p}{}^T)$ and its rate of decrease with energy are small, however, the shrinkage will be less strong than in $pp$ and $\bar{p}p$ scattering. A rough estimate using the same $a$ and $b$ values as before shows that the shrinkage in $d\sigma^R/dt$ for $-t = 0.2$, 0.3 would be ~75% of the corresponding shrinkage in $pp$.

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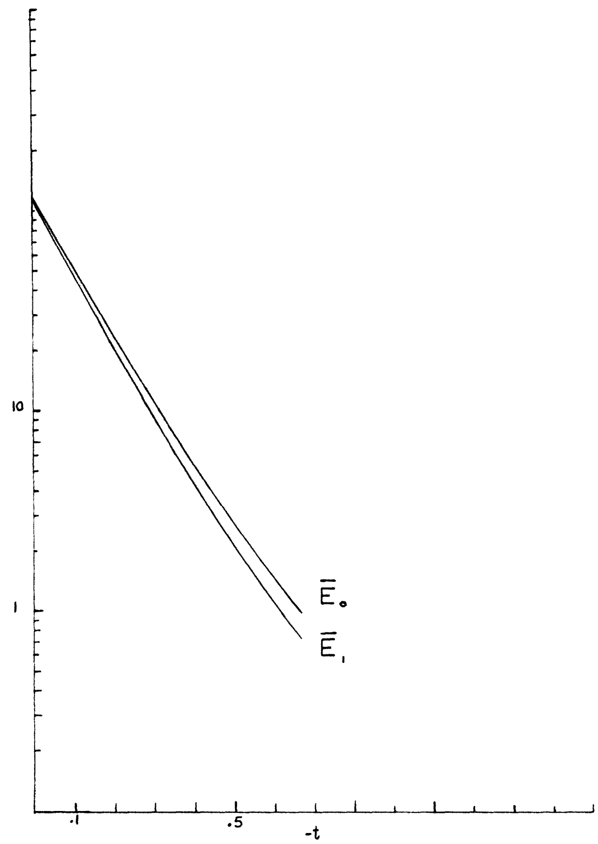


FIG. 3. Differential cross section $(d\sigma/dt)/[\sigma^T(E)/\sigma^T(20\text{ BeV}/c)]^2$ mb/(BeV/$c$)$^2$ predicted for $\bar{p}p$ scattering vs momentum transfer $-t$ (BeV/$c$)$^2$. $\bar{E}_0$ and $\bar{E}_1$ indicate lab energies 10.79 and 19.59 BeV/$c$, respectively.

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[2]K. J. Foley, S. J. Lindenbaum, W. A. Love, S. Ozaki, J. J. Russell, and L. C. L. Yuan, Phys. Rev. Letters 10, 376 (1963).

[3]G. F. Chew and S. C. Frautschi, Phys. Rev. Letters 7, 394 (1961).

[4]Unless otherwise stated, we take BeV as the unit of energy.

[5]The resonance $f_0$ is at $\approx 64\, m_\pi{}^2$, while $\omega$ and $\rho$ are at $\approx 25\, m_\pi{}^2$, and, therefore, there is no reason why they should all be connected to their $t=0$ trajectory end points by straight lines with a slope of 1 (BeV/$c$)$^{-2}$. Since $f_0$ and $\rho$ are presumably generated by $\rho$ exchange, the corresponding slopes $\alpha'(0)$ may be $\sim m_\rho{}^{-2}$, much

smaller than $(2m_\pi)^{-2}$ assumed in reference 3. For a fixed $V_0R^2$, where $V_0$ is the strength and $R$ the range of interaction, smaller $R$ corresponds to larger resonance energies (as is the case for $f_0$ and $\rho$). This means that for shorter ranges the trajectories $\mathrm{Re}\alpha(t)$ vs $t$, for positive $t$, do not turn over quite so quickly as they do for Yukawa potentials with unit range [A. Ahmadzadeh, P. G. Burke, and C. Tate, University of California Radiation Laboratory Report UCRL-10216 (unpublished); C. Lovelace and D. Masson (to be published)] (the total number of bound states and resonances or the shape of the trajectory in the $l$ plane, of course, remain unchanged). In this respect their behavior would be similar to the conjectured trajectories of reference 3. Furthermore, because of the presence of factors like $(t-4m_\pi^2)^{\alpha_0-1/2}$, the slope $\alpha'(t)$ near threshold would increase for positive $t \gtrsim 4m_\pi^2$, if $\alpha_0 > 1/2$. This together with the influence of inelastic channels plus relativistic effects may enable the Regge poles to attain sufficiently large values for positive $t$, even though they start at $t=0$ with rather small slopes.

[6]We are only considering the highest ranking trajectories (such as $P, \omega, P', \rho$) at the present moment. Other trajectories such as $\phi$, $\eta$, $ABC$, etc. will presumably have weaker effects.

[7]The residues for $P'$ and $\rho$ in $\pi p$ scattering are small and are therefore neglected (see reference 13).

[8]If the amplitudes do not have essential singularities and satisfy dispersion relations (as is assumed here), then the residues must go as some power of $t$. From the recent results of R. Serber, Phys. Rev. Letters 10, 357 (1963), it seems that $\beta^2(t)$ should behave as $t^{-5}$ for very large $-t$ (>0.5). For moderately large values $-t$ up to $\approx 0.5$ our exponential form should be a good approximation.

[9]The residues $\beta(t)$ have a cut from $t=4m_\pi^2$ to $\infty$ which may be replaced by a finite sum of poles in order to represent $\beta(t)$ for negative values of $t$. This should be a better approximation than the exponential one but will have more parameters; the smaller the "width" of $\beta(t)$, the larger the number of poles required.

[10]This follows from the optical theorem. The number 0.22 arises from the choice of units.

[11]The parameters in $\pi^+p$ scattering will only be slightly different from the corresponding quantities $C$ and $\gamma$ of $\pi^-p$. This will arise because of the small difference in the $\pi^-p$ and $\pi^+p$ cross sections.

[12]B. M. Udgaonkar, Phys. Rev. Letters 8, 142 (1962).

[13]S. D. Drell, Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962 (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 897.

[14]K. Igi, Phys. Rev. Letters 9, 76 (1962).

[15]F. Hadjioannou, R. J. N. Phillips, and W. Rarita, Phys. Rev. Letters 9, 183 (1962).

[16]On the basis of potential theory results, it appears, however, that $\omega$ probably has a slightly larger slope than $P$, if one believes that it is a less strongly interacting system than $P$.

[17]We would like to add that such an assumption of a sharply falling function was necessary also in the earlier fits for $pp$ scattering with $\alpha_P'(0)=1$ (see references 1 and 2), the reason being that the energy-dependent "width" $[\alpha_P'(0)\ln E]^{-1}$ was $\geqslant 0.3$ (BeV/$c$) (for $E$ in the range 10-20 BeV/$c$) and was inadequate to explain the actual width of about 0.1 (BeV/$c$).

[18]Shrinkage corresponding to $P$ should, of course, exist but only at much higher energies. If $\alpha_P'(0)$ is as small as 1/5 $(\mathrm{BeV}/c)^{-2}$, then a noticeable shrinkage for low $t$ values would be observed only at fourth or fifth power of the presently available energies.