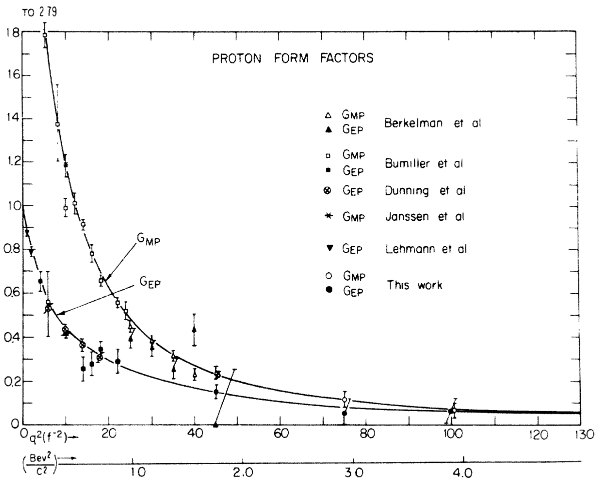

FIG. 3. The solid line gives the best fit for the electromagnetic form factors of the proton, G_{Ep} and G_{Mp}, as obtained in Table II. G_{Ep} and G_{Mp} are equal within statistics. The error limits do not include the 4% scale uncertainty which is common to all points. No separation is made at $q^2 = 125$ F^{-2} since the cross section is measured at one angle only.

core in the electric form factor of 0.2.

If the form factors are due to a multipion resonance, we expect terms of the form $(1/q^2 + m^2)$, where m is the mass of the resonant state. For $q^2 \gg m^2$ we therefore expect $G \sim 1/q^2$. The form factors in this region fall off consistently with this behavior. Our data are therefore consistent with no core in either the electric or magnetic form factor.

We intend to repeat and extend these measurements on hydrogen and deuterium in the near future.

We would like to thank the entire Cambridge Electron Accelerator and the Harvard Cyclotron Laboratory staff who made this experiment possible. Dr. F. Brasse, Dr. P. F. Cooper, Dr. L. N. Hand, Dr. J. R. Rees, and Mr. W. J. Shlaer contributed in the first stage of the experiment. Mr. L. Lanzerotti provided the oscilloscope camera system. Mr. D. Kotz, Mr. E. J. Moore, Mr. P. Palmer, Mr. D. Rosner, Mr. T. Von Foerster, and Mr. W. Wanning helped in the reduction of the data. Dr. E. Engels and Mr. M. Goitein assisted in the runs and in many other ways. We are indebted to Mr. L. H. Chan for his computations on the quadrupole spectrometer.

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†National Science Foundation predoctoral Fellow.

‡On leave from the Department of Physics, University of Liverpool, Liverpool, England.

[1]For a review of the data up to $q^2 = 45$ F^{-2}, see L. N. Hand, D. G. Miller, and Richard Wilson, Rev. Mod. Phys. 35, 335 (1963).

[2]J. R. Dunning, Jr., K. W. Chen, N. F. Ramsey, J. R. Rees, W. Shlaer, J. K. Walker, and Richard Wilson, Phys. Rev. Letters 10, 500 (1963).

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[4]K. Berkelman, M. Feldman, R. M. Littauer, G. Rouse, and R. R. Wilson, Phys. Rev. 130, 2061 (1963).

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FINAL-STATE INTERACTIONS IN THE DECAY $\eta \to 3\pi$†

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In this Letter we present the results of an analysis of the Dalitz-Fabri plot of 97 eta decays, $\eta \to \pi^+ + \pi^- + \pi^0$. The etas were produced in the reaction $\pi^+ + p \to \pi^+ + p + \eta$, by using π^+ of 1170 MeV/c (76 events) and 1050 MeV/c (21 events) incident on the Alvarez 72-in. hydrogen chamber. Our sample differs from previously published samples in two important respects.[1] First, our background is negligible.[2] Second, the contaminating decay mode $\eta \to \pi^+ + \pi^- + \gamma$,

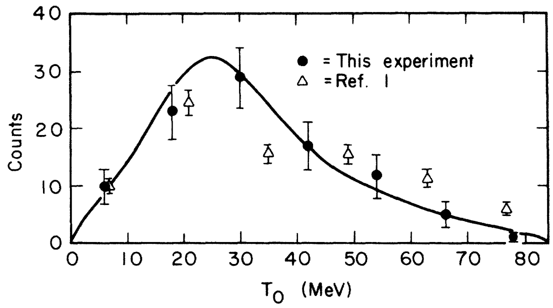

FIG. 1. Spectrum of kinetic energy of π^0 from $\eta \rightarrow \pi^+ + \pi^- + \pi^0$. Solid circles represent the present experiment; open triangles represent the compilation of reference 1, renormalized so as to give the same area. The solid curve is a best fit of our data to the theory of Brown and Singer, and is included in this figure only to aid comparison of the present experiment with the previous compilation. (The same solid curve appears in Fig. 2.)

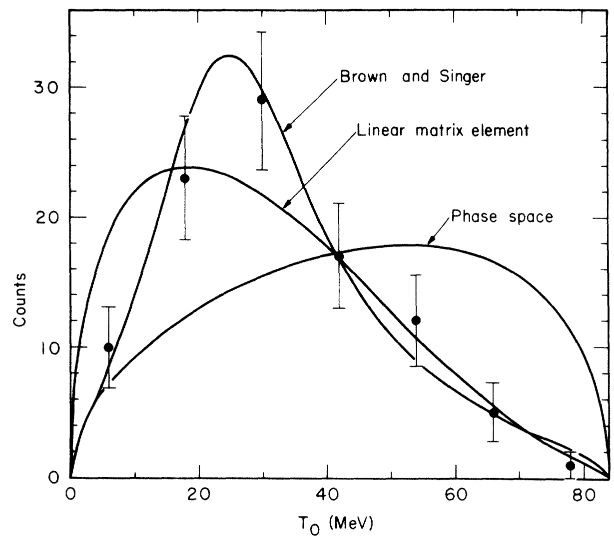

FIG. 2. Spectrum of the π^0 kinetic energy from $\eta \rightarrow \pi^+ + \pi^- + \pi^0$. The experimental points are from the present experiment. The three solid curves correspond to phase space, the linear-matrix-element theories,[6,7] and the $I=0$, $J=0$ di-pion-resonance theory of Brown and Singer.[9,10]

which is $26 \pm 8\%$ as probable as the $\pi^+\pi^-\pi^0$ mode,[3] has been clearly separated out and removed.[4]

We do not present here the complete Dalitz-Fabri plot,[5] but only its projection on the T_0 axis, where T_0 is the kinetic energy of the π^0. We first compare our spectrum with that given in the compilation of Berley et al.[1] The comparison is shown in Fig. 1. Agreement is only fair. In particular, our data show a more rapid decrease in intensity for T_0 greater than about 30 MeV than does the compilation. Our belief is that the disagreement is due to the unsubtracted background and the unseparated $\pi^+\pi^-\gamma$ decays contained in the compilation.[1]

We now compare our spectrum with two theories. The first theory we call the linear-matrix-element theory.[6,7] We fit our spectrum to the formula

$$dN/dT_0 = C\,|1 + \alpha y \exp(i\beta)|^2 \varphi(y)$$
$$= C(1 + 2\alpha y \cos\beta + \alpha^2 y^2)\varphi(y),$$

where $y = 2(T_0/T_0^{\max}) - 1$, so that $-1 \leq y \leq +1$; and where $\varphi(y)$ is the Lorentz-invariant phase space. The constant C is chosen to normalize the area to 97 counts. We find a minimum $\chi^2 = 6.1$ for $\cos\beta = -1$ and $\alpha = 0.71 \pm 0.09$. The expected χ^2 is 4.0 and the χ^2 probability for a fit as bad or worse is about 20%. The best fit to this theory is the "linear-matrix-element" curve shown in Fig. 2. From these parameters one can predict[6] the branching ratio

$$R \equiv \Gamma_\eta(000)/\Gamma_\eta(+-0) = (3/2)P/[1 + (\alpha^2/4)],$$

where $P \sim 1.1$ corrects for $m_{\pi^0} \neq m_{\pi^+}$. Inserting our best-fit value $\alpha = 0.71$, we obtain the prediction $R = 1.50 \pm 0.04$. This can be compared with the directly measured value,[8]

$$R = 0.83 \pm 0.32. \qquad (1)$$

The χ^2 probability for agreement between the predicted and measured values of R is 3.8%. Thus the agreement is only fair.[9]

We next compare our spectrum with the theory of Brown and Singer.[10,11] In order to explain the unexpectedly large competition of the isospin-violating decay $\eta \rightarrow \pi^+ + \pi^- + \pi^0$ with the electromagnetic decay $\eta \rightarrow \pi^+ + \pi^- + \gamma$, they postulate that $\eta \rightarrow 3\pi$ proceeds via $\eta \rightarrow \sigma + \pi^0$, followed by $\sigma \rightarrow \pi^+ + \pi^-$ or $\sigma \rightarrow \pi^0 + \pi^0$. Here σ represents an $I=0$ di-pion resonance with 0^{++} quantum numbers. Angular momentum conservation forbids $\eta \rightarrow \sigma + \gamma$, so that the 3π mode is enhanced but the $\pi^+\pi^-\gamma$ mode is not. Following Brown and Singer,[11] we fit our spectrum to the expression

$$dN/dT_0 = C\varphi(T_0)[(T_0 - A)^2 + B^2]^{-1},$$

where $\varphi(T_0)$ is phase space, C normalizes the area to 97 counts, $A = [(m_\eta - m_0)^2 - m_\sigma^2]/2m_\eta$, and $B = m_\sigma \Gamma_\sigma / 2m_\eta$. We find $\chi_{\min}^2 = 2.7$, where 4.0 is expected. The best-fit parameters are

$$m_\sigma = 381 \pm 5 \text{ MeV, and}^{12,13}\ \Gamma_\sigma = 48 \pm 8 \text{ MeV}. \qquad (2)$$

The best-fit curve is shown in Fig. 2, labeled "Brown and Singer." From the parameters m_σ and Γ_σ, Brown and Singer can predict the branching ratio R.[10] We shall not write down their formula.[13] Using our results (2) and their formula, we obtain the prediction

$$R=1.02\pm0.07.$$

The χ^2 probability for agreement with the measured value (1) is 57%.

In summary, our data are in only fair agreement with the linear-matrix-element theories, and in excellent agreement with the $I=0$, $J=0$ di-pion resonance hypothesis of Brown and Singer. However, we cannot rule out the possibility that other hypotheses involving final-state interactions might also fit the data. In particular we emphasize that it is only after we assume the existence of the σ resonance that we can determine the parameters of Eq. (2). Thus it is not possible in our experiment to determine whether the resonance actually does or does not exist.[12]

It is a pleasure to acknowledge the advice and support of Luis W. Alvarez. One of us (E.C.F.) wishes to express his gratitude for the hospitality shown to him by the Alvarez bubble chamber group, and to acknowledge financial support during part of the experiment from the Lawrence Radiation Laboratory and from Yale University.

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†Work done under the auspices of the U. S. Atomic Energy Commission.

[1]D. Berley, D. Colley, and J. Schultz, Phys. Rev. Letters 10, 114 (1963), have compiled 511 charged eta decays from eight different experiments. (For references see their Table I.) They estimate that not more than ~100 of the events are background. The $\pi^+\pi^-\gamma$ decays were not separated out, and should constitute ~100 of the nonbackground events. Thus perhaps as many as 200 of the 500 events are spurious. The 69 events labeled "Berkeley-c" in Table I were our preliminary results, obtained before we had eliminated a small background and separated out the $\pi^+\pi^-\gamma$ decays.

[2]This is demonstrated in Fig. 1 of reference 3, which contains 76 of our present 97 events. First, the figure shows that essentially all events of the type $\pi^++p \rightarrow \pi_1^+ + p + \pi_2^+ + \pi^- + \pi^0$ are due to η production, with $\eta \rightarrow \pi^+ + \pi^- + \pi^0$. Second, the figure (and our calculation) shows that about 16% of the events are ambiguous with respect to the π^+, when $\pi_1^+\pi^-\pi^0$ and $\pi_2^+\pi^-\pi^0$ both have the η mass. We always choose that combination yielding a mass closest to $m_\eta=548.0$. Therefore we choose the wrong π^+ in 8% of the cases. We have examined our Dalitz-Fabri plot with the ambiguous events deleted and also with the ambiguous positive pions interchanged, and find no distinguishable change in the shape. Third, we discard events with "$m(e^+e^-)$" <100 MeV to eliminate $\pi^++p\rightarrow\pi^++p+e^++e^-\cdots$ from our sample. This cutoff also eliminates any event $\pi^++p\rightarrow\pi^++p+\pi^++\pi^-+\pi^0$, where the π^- and one π^+ have the same direction in the laboratory. We have examined the cutoff events and find that about 10 of them correspond to η production and decay into $\pi^+\pi^-\pi^0$. Adding these events to the spectrum for T_0 produces no detectable effect on the shape of the spectrum. Finally, we have examined the events discarded because they satisfy $\pi^++p\rightarrow\pi^++p+\pi^++\pi^-$, with a Coulomb scatter on one track (1-C fit; see reference 3). Six of these events are probably $\eta\rightarrow\pi^++\pi^-+\pi^0$. None satisfy the cutoff criterion on the error in m^2(neutral) described in reference 4.

[3]Earle C. Fowler, Frank S. Crawford, Jr., L. J. Lloyd, Ronald A. Grossman, and LeRoy R. Price, Phys. Rev. Letters 10, 110 (1963).

[4]The technique is described in reference 3. See especially Fig. (3b). If we relax our cutoff on the error in $m^2(\text{neutral})=m^2(\pi^0 \text{ or } \gamma)$, our 97 $\pi^+\pi^-\pi^0$ events became 146 $\pi^+\pi^-\pi^0$ events with a small (but not easily measured) contamination from $\pi^+\pi^-\gamma$. The T_0 spectrum of these 146 $\pi^+\pi^-\pi^0$ events is not distinguishable from that of our reduced sample of 97, which has nearly zero contamination. Thus the error cutoff does not distort the T_0 spectrum.

[5]Our complete Dalitz-Fabri plot (not shown) only confirms the already well-established 0^{-+} quantum numbers for the eta. See, for instance, the review by G. Puppi, in Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962 (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 713; see also C. Alff et al., Phys. Rev. Letters 9, 325 (1962); M. Chrétien et al., Phys. Rev. Letters 9, 127 (1962).

[6]M. Gell-Mann and A. H. Rosenfeld, Ann. Rev. Nucl. Sci. 7, 407 (1957) (they discuss $K^+\rightarrow 3\pi$); K. C. Wali, Phys. Rev. Letters 9, 120 (1962).

[7]G. Barton and S. P. Rosen, Phys. Rev. Letters 8, 414 (1962); M. A. B. Bég, Phys. Rev. Letters 9, 67 (1962).

[8]Frank S. Crawford, Jr., L. J. Lloyd, and Earle C. Fowler, Phys. Rev. Letters 10, 546 (1963).

[9]In reference 8, we obtained the η branching ratios $\Gamma(\gamma\gamma)/\Gamma(\text{charged})\equiv x=0.99\pm0.48$ and $\Gamma(000)/\Gamma(\text{charged})\equiv y=0.66\pm0.25$. Other experiments (see footnote 2 of reference 8) yield a combined value $\Gamma(\text{neutral})/\Gamma(\text{charged})=x+y=2.7\pm0.5$. If we average these three results by the method of least squares, we obtain $x=1.42\pm0.36$, $y=0.78\pm0.23$, and $x+y=2.20\pm0.37$, with $\chi^2=1.9$ for one degree of freedom, and an off-diagonal error term $dxdy=-0.027$. Combining this averaged value of y with $\Gamma(\text{charged})/\Gamma(+-0)=1.26\pm0.08$, we obtained $R\equiv\Gamma(000)/\Gamma(+-0)=0.98\pm0.30$. If we compare this value with the prediction $R=1.50$ of the linear-matrix-element theory, we find a χ^2 probability for agreement of 8%.

[10]L. M. Brown and P. Singer, Phys. Rev. Letters 8, 460 (1962).

[11]L. M. Brown and P. Singer, Phys. Rev. (to be published). Compare their theory with the data on

K and γ decay compiled in reference 1. We are grateful to these authors for several enlightening discussions of their preprint.

[12]The off-diagonal error term is $\delta\Gamma_\sigma\delta m_\sigma = 12$ (MeV)^2.

[13]Our values for m_σ and Γ_σ, Eq. (2), may be compared with the values $m = 395 \pm 10$ MeV and $\Gamma = 50 \pm 20$ MeV for a $\pi^+\pi^-$ resonance observed by N. P. Samios, A. H. Bachman, R. M. Lea, T. E. Kalogeropoulos, and W. D. Shephard, Phys. Rev. Letters 9, 139 (1962), and assigned $I = 0$ or 1 by them. The agreement is striking, but could be accidental. The existence of this resonance has not yet been directly confirmed, either in other experiments [see, for instance, C. Alff et al., Phys. Rev. Letters 9, 322 (1962)] or in the present experiment. It is not possible to prove conclusively the existence of the σ resonance in the present experiment, mainly because the width $\Gamma_\sigma = 48$ MeV is not small compared with $T_0^{\max} = 84$ MeV. Assuming the existence of the resonance, we determine the parameters of Eq. (2). Thus we do not regard our results as sufficient to confirm the observation of Samios et al.

[14]If $\eta \to 3\pi$ went exclusively via $\eta \to \sigma + \pi_d^0$ (d for direct), and if the width Γ_σ were zero, then π_d^0 would not interfere with either of the neutral pions from $\sigma \to 2\pi^0$. The direct pion, π_d^0, would be distinguishable by its energy in the η frame. Then $R = (1/2)P \sim 0.55$ follows from the hypothesis $I_\sigma = 0$. In the limit $\Gamma_\sigma \to \infty$, any one of the three neutral pions could be regarded as direct, so that the $|\text{amplitude}|^2$ for $\eta \to 3\pi^0$ would be enhanced over the case $\Gamma_\sigma = 0$ by a factor $|(1+1+1)/\sqrt{3}|^2 = 3$, because of the three possible assignments for π_d^0. In that limit, one has $R = (3/2)P = 1.7$. The $3\pi^0$ are then in the totally symmetric $I = 1$ state.

ERRATA

NUCLEON AXIAL-VECTOR FORM FACTOR. Ching-Hung Woo [Phys. Rev. Letters 11, 385 (1963)].

The value of $B(0)$ in section (c) is in error; it should be $0.4g_A M^2/\mu$ instead of $0.7g_A M^2/\mu$. This value is for $B(0)$ defined through the relation

$$2(E_\pi E_\rho)^{1/2}\langle 0|A_\mu|\pi\rho\rangle = [\eta_\mu B(s) + \text{other form factors}] \times \epsilon_{ijk}\tfrac{1}{2}(\delta_{i1} + i\delta_{i2}),$$

where j, k refer to π and ρ isotopic spin indices.

This does not change any of the results in that paper. However, it will change the $W \to 3\pi$ rate as computed by Feinberg and Mani [Phys. Rev. Letters 11, 448 (1963)]. Because of this reduction in the value of $B(0)$ and correcting for a difference in convention, the $W \to 3\pi$ rate should be 6.7 times smaller than the value given in their paper. The author wishes to apologize to these two authors for supplying them with the incorrect value of $B(0)$, and to thank Dr. B. Bég for bringing the error to his attention.

POSITION OF RESONANCE POLES NEAR THE THRESHOLD OF A CHANNEL. Marc Ross [Phys. Rev. Letters 11, 450 (1963)].

In the next to last paragraph, replace the sentence "On each of the n sheets ⋯" with the following: "If only one channel, k_1, is considered in addition to that explicitly involved above, there will be one additional pair of sheets and one additional pair of poles (corresponding essentially to $k_1 \to -k_1$) for a total of four each. In the n-channel problem, there will be a doubling of the sheets of the $(n-1)$-channel case. Pairs of poles are roughly reflected on appropriate new sheets so that there are 2^n poles associated with one narrow resonance (or corresponding bound state)." The author would like to thank C. Goebel for calling this to his attention.