CALCULATION OF AUTOIONIZATION LEVELS IN He

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(Received 28 October 1963)

In this paper the results of a calculation are reported in which quantitative agreement is obtained with experimental results on the positions and widths of certain autoionization states in He. Several more detailed predictions of resonances so far unobserved are also made. It is pointed out that verification of these further levels could probably most easily be made by electron-scattering experiments in He^+.

The last few years have seen a considerable increase in the understanding of narrow resonances occurring in electron-atom and electron-ion scattering. These resonances can be simply understood in terms of the formation of long-lived intermediate compound states of the electron-atom or electron-ion system. Such resonant states decay with a characteristic lifetime into the continuum state and produce a characteristic resonant structure in the associated cross section. If the state has a sufficiently long life against autoionization, then radiative transitions to lower levels may become appreciable. Several emission lines in the vacuum ultraviolet in He have been attributed to this source.[1] More recently, experiments on the ionization of He by 500-eV electrons[2] and the absorption of ultraviolet light in He, Ne, and Ar[3] have revealed the detailed positions and widths of certain excited states of these atoms. An analysis of the former has been made by Fano,[4] and a detailed analysis of the latter in terms of the independent-particle approximation has been made by Cooper, Fano, and Prats.[5] Calculations have previously been made[6] of the positions and widths of some of the excited states of He; however, it has been difficult to relate these predictions with experiment.[4] In particular, the widths of the calculated lines have been more than an order of magnitude different from experiment and very sensitive to the details of the theory.

The method adopted for calculating the resonant positions and widths follows closely similar calculations for the e^--H system.[7] It has been shown that the resonances occurring in the elastic-scattering cross section just below the excitation threshold for the $2s$ and $2p$ states depend critically on a correct treatment of the virtual excitation of these levels.[7,8] In the e^--He^+ system the situation is very similar, with the additional complication of the Coulomb force at large distances. In fact, the presence of the Coulomb force makes our method more reliable than in hydrogen. This is because this force being central can be treated exactly and it dominates the scattering at large distances. Our method is to expand the total wave function in the following way:

$$\psi(1,2) = (1 \pm P_{12}) \sum_{n = 1s, 2s, 2p} \psi_n(1) F_n(2), \qquad (1)$$

where 1 and 2 are the coordinates of the two electrons, ψ_n are the helium-ion eigenstates, and F_n describes the motion of the scattered electron. The whole expression is explicitly symmetrized or antisymmetrized by $1 \pm P_{12}$ according to the spin symmetry of the two electrons. If the above expression for $\psi(1,2)$ is substituted into the Kohn variational principal satisfied by the exact wave function, a system of three ($L = 0$) or four ($L > 1$) coupled integrodifferential equations are derived for the functions F_n.[9] Here L is the total orbital angular momentum quantum number and is obtained when a partial-wave analysis of Eq. (1) is carried out. For energies below the excitation energy of the $2s$ and $2p$ states, boundary conditions corresponding to decaying waves in all channels except the $1s$ channel must be imposed. The advantages of the expansion (1) for this calculation are now immediately obvious. Firstly, virtual transitions to the $2s$ and $2p$ states are treated correctly and, to the extent that transitions to higher states can be neglected, expansion (1) is exact. We can thus expect rather good results for those levels which converge to the $n = 2$ level of He^+.[10] Secondly, the mixing of the discrete levels with the continuum is put in from the beginning and is not treated by perturbation theory. We can thus expect better level widths than these latter methods tend to give.

The numerical problem of solution of the cou-

Table I. The positions and widths in electron volts and the probability for autoionization in sec^{-1} for the lowest energy states in He. The level energy is given relative to the ground-state energy of He. No resonances of appreciable width have been found in the ^3S state. (Higher angular momentum states have not been considered.) Errors are one in the last figure quoted except where otherwise stated.

Resonance level (resonance number in brackets)	Position (eV)	Width (eV)	Probability of autoionization (sec^{-1})
^1S (1)	57.87	0.158	2.40×10^{14}
^1S (2)	62.86	0.049	7.4×10^{13}
^1S (3)	64.18	0.019 ± 0.002	2.9×10^{13}
^1P (1)	60.23	0.041	6.2×10^{13}
^1P (2)	63.68	0.0092	1.40×10^{13}
^1P (3)	64.49	0.0037	5.6×10^{12}
^1P (4)	64.82	0.0018	2.7×10^{12}
^3P (1)	58.36	0.0107	1.63×10^{13}
^3P (2)	...	...	...
^3P (3?)	64.26	0.0010	1.52×10^{12}

pled integrodifferential equations was achieved by an iteration method for each energy, angular momentum, and spin state. The phase shift was thus obtained as a function of energy, and a Breit-Wigner resonance fit was carried out for each level. Table I represents the collected results obtained from several hundred phase-shift calculations for the four states ^1S, ^3S, ^1P, and ^3P. In certain regions of energy, convergence in the neighborhood of resonance was difficult, and results have so far not been obtained for ^3P resonances in the energy range 60.5 eV to 63.9 eV. We are currently working on a more powerful iteration technique[11] which it is hoped will fill in this gap. Finally, no evidence of any resonant level wider than about 0.001 eV has been found in the ^3S state. The calculations were carried out on the United Kingdom Atomic Energy Agency IBM-7090 and STRETCH computers.

Our results for the ^1S, ^1P, and ^3P states are presented in Table I. The experimental confirmation discussed earlier is limited to the ^1P levels. Fano's[4] analysis of experiment for the first resonance gives a position of 60.1 eV with a width of 0.04 eV in good agreement with our result. The other ^1P resonances found by Madden and Codling[3] appear to be also in good agreement with our work. Holøien[6] has discussed the positions of the first two ^1S resonances, although not their widths. He finds energies of 57.27 eV and 62.16 eV in fair agreement with us. His wave function is not, however, orthogonal to the infinity of lower levels of He, so his result does not represent a bound. One important feature is apparent from this work, and that is that the $2s$ and $2p$ mixing is so strong that two separate He series are not found. If we use the effective-range theory which has recently been developed,[12] it is easy using our results to predict the positions and widths of the infinitely many higher resonances in each series. In conclusion, it is worth pointing out that in the proposed elastic-scattering experiment e^--He^+,[13] all resonances given here should be observable, unlike experiments involving the ground state of He where the ^1P resonances are preferred in the dipole approximation.

Note added in proof:—Recent experimental work by Simpson, Mielczarek, and Cooper[14] confirms to a large extent our predictions on the lowest ^1S and ^3P resonances, while confirming our agreement with previous experimental work[3] on the ^1P resonances. We would also like to note that recent work by Cooper[6] shows that, contrary to previous indications, the widths of the resonances are not very sensitive to the details of the theory

to within a factor of 2 or less. We would like to thank Professor U. Fano for bringing these results to our attention.

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ELECTRON-PROTON SCATTERING AT HIGH MOMENTUM TRANSFERS*

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This Letter reports some recent results on the elastic electron-proton scattering cross-section measurements carried out at the 6-BeV Cambridge Electron Accelerator. Data have been obtained for electron-scattering angles at 35° and 70° in the laboratory. Incident electron energies are varied to attain q^2 (4-momentum transfer squared in F^{-2}) = 45, 75, 100, and 125 F^{-2} for 35°, and $q^2 = 75$ and 100 F^{-2} for 70°, respectively. This work represents the first attempt to measure the electromagnetic form factors of the proton beyond $q^2 = 45$ F^{-2}.[1] Our data are consistent with a suggestion that G_E and G_M are equal and fall off as $1/q^2$ at large momentum transfers.

The internal electron beam of the Cambridge Electron Accelerator is allowed to strike a liquid hydrogen target. The bremsstrahlung from the electrons traversing the target is monitored in the forward direction by a total integral ionization chamber, the quantameter, as well as a thin-walled helium-filled ionization chamber. The target container is made out of 0.0005-inch Mylar. The target illumination is governed by horizontal and vertical clippers located at an integral number of betatron wavelengths upstream from the target. During each run the distribution of bremsstrahlung production in the full target as a function of horizontal position in the median plane of synchrotron is measured.