takes place when $f(w) = \omega_H/sq$, an equation that can be solved graphically to obtain the frequency ω_d where the acoustic and electromagnetic branches are degenerate. Ordinarily this transcendental equation possesses only one real root. However, there is a range of the magnetic field for which it can have up to three real roots. In this Letter we shall limit ourselves to the former situation. By treating the right-hand side of Eq. (5) as a perturbation, the solution for the eigenfrequencies at the crossover to first order in Ω_0 is

$$\omega = \omega_d[1 \pm (\Omega_0/2f_d\omega_d)^{1/2}]. \quad (9)$$

For a metal such as sodium in a magnetic field of the order of 5×10^4 G, the crossover frequency ω_d is approximately equal to 3.6×10^9 sec^{-1}. The splitting at this point is about 12% of ω_d. The considerable admixture of acoustic and helicon modes in this region suggests the possibility of exciting transverse phonons in metals by means of electromagnetic radiation of the appropriate frequency. In Fig. 1 we give a graphical representation of the zeros of Eq. (5) in terms of the parameter $w = qv_0/\omega_0$. For the purpose of this calculation we have considered sodium at a magnetic field for which w at the crossover is $w_d = 2$. Under this assumption $B_0 = 4.8\times10^4$ G and $f(w_d) = 0.684$. The foregoing results are, of course, only valid if $\omega_0\tau \gg 1$. Further details of this work will be the subject of a subsequent publication.

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de HAAS–van ALPHEN EFFECT AND FERMI SURFACE IN NICKEL*

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The Fermi surfaces of nonferromagnetic metals have been extensively investigated by de Haas–van Alphen (dHvA) and related techniques. Corresponding data on the magneto-oscillatory properties of ferromagnetic metals are much less complete, although preliminary dHvA effect results on Fe have been reported by Anderson and Gold,[1] and Shubnikov–de Haas oscillations have been detected in Co by Fawcett and Reed.[2] In this Letter we describe dHvA oscillations observed in nickel single crystals by means of a null deflection torsion balance in steady magnetic fields up to 40 kG. The dHvA oscillations were studied as a function of magnetic field orientation in the (001) and $(1\bar{1}0)$ crystallographic planes. The variation of the period of the oscillations in these planes suggests that at least one sheet of the Fermi surface of nickel is similar to that of the noble metals, in accord with a model of the Fermi surface proposed by Fawcett and Reed on the basis of magnetoresistivity studies.

The torsion balance used for this study was modified so that the large steady torques exerted on the sample due to the ferromagnetic anisotropy of nickel could be nullified. Samples were cut from a single crystal nickel rod [$\rho(300°\text{K})/\rho(4.2°\text{K}) \approx 990$] in the form of disks (0.20-in. diam $\times$ 0.030 in. thick) and each was mounted with its axis of rotation vertical. In the first sample the $[1\bar{1}0]$ axis coincided with the disk axis, so that the magnetic field was located in the $(1\bar{1}0)$ plane. The [001], [110], and [111] symmetry directions in this plane were accurately located by the vanishing of the ferromagnetic torques when the field was along any one of these directions. The dHvA oscillations (a typical recording is shown in Fig. 1) were found to be periodic in $(H_0 + H_m)^{-1}$ where H_0 is the applied external magnetic field and H_m

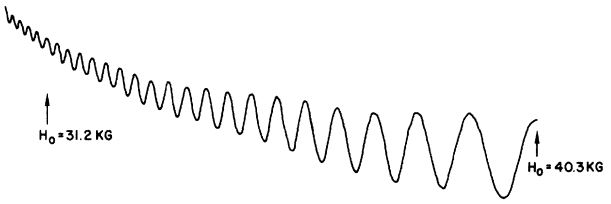

FIG. 1. de Haas–van Alphen oscillations in the torque versus magnetic field for H approximately parallel to [110].

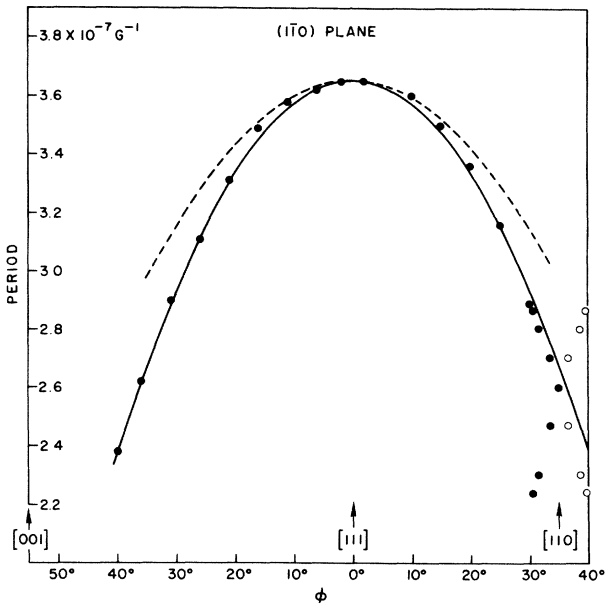

FIG. 2. de Haas–van Alphen period versus φ in the (1$\bar{1}$0) plane. Note deviation of the experimental curves from the result (dashed line) to be expected for a cylindrical Fermi surface segment. The solid line is obtained from Eq. (2) with the parameter m_t/m_l chosen to be 0.40 ± 0.05.

$= 6400 \pm 500$ G is the internal magnetization field. This periodicity in the reciprocal of the magnetic induction is in agreement with the results on Fe mentioned above. In Fig. 2 we have shown the values (P) of the period of the oscillations as a function of φ, the angle between the magnetic field and the [111] axis in the (1$\bar{1}$0) plane. At $\varphi = 0°$ (H along [111]) the amplitude of the dHvA oscillations vanishes, indicating that the [111] axis is a symmetry axis for the Fermi surface segment responsible for the oscillations. The extrapolated maximum value of period along [111] is 3.65×10^{-7} G^{-1} which corresponds to a minimum cross-sectional area of 0.026 Å^{-2}. Approximately 40° away from the [111] axis the oscillations due to the Fermi surface segment centered on [111] disappear. The temperature variation of the amplitude of the oscillation at $\varphi = 2°$ (H approximately 2° from [111]) and 35° (H parallel to [110]) yields values of the effective mass of the pertinent carriers of $(0.26 \pm 0.04)m_0$ and $(0.370 \pm 0.006)m_0$ respectively.

The second sample was cut and suspended so that the magnetic field was located in the (001) plane. We observed very weak oscillations within 6° of the [110] axis, and these disappeared along [110]. This behavior would be expected for a Fermi surface segment centered along [111].

Although the present data are not sufficient for a complete determination of the relevant Fermi surface sheet in nickel, they are in accord with the model proposed by Fawcett and Reed.[3] Their magnetoresistance measurements indicated behavior very similar to that of copper and led them to postulate that one sheet of the Fermi surface in nickel, like that of copper, has necks protruding in the [111] directions. The above dHvA data can be attributed to electron orbits around such a neck, having a cross section approximately ten times smaller than that in copper. If we assume a circular cross section for the necks we find the minimum diameter to be 0.18 Å^{-1}, which subtends an angle of $6.8° \pm 0.2°$ from Γ at the center of the Brillouin zone. This is in excellent agreement with that found by Fawcett and Reed.[4]

Over the range of measurement the variation of the experimental period in the (1$\bar{1}$0) plane approximates that which would be obtained for an energy surface having the form of a hyperboloid of one sheet,

$$E = (\hbar^2/2)[(k_x^2 + k_y^2)/m_t - k_z^2/m_l], \qquad (1)$$

although it should be noted that this expression satisfies the crystal symmetry only for small values of k. In this equation, E is measured from the symmetry point L on the hexagonal face of the Brillouin zone, the z axis is taken along the axis of the neck, and m_t and m_l are transverse and longitudinal effective masses relative to the z axis. The angular variation of the period and effective mass corresponding to Eq. (1) is given by the expression[5]

$$P(\varphi)/P(0) = m(0)/m(\varphi) = \cos\varphi[1 - (m_t/m_l)\tan^2\varphi]^{1/2} \qquad (2)$$

where the angle φ is measured from the [111] axis in the $(1\bar{1}0)$ plane. This expression is plotted in Fig. 2 (solid line), and good agreement with the data is obtained when $m_t/m_l = 0.40 \pm 0.05$. From the measured values of $P[111]$, $P[110]$, and $m[110]$ we calculate $m_t = (0.26 \pm 0.02) \times m_0$ and $m_l = 0.65 \pm 0.10$. The calculated value of m_t is in good agreement with the experimental value of $(0.26 \pm 0.04) m_0$.

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[4]The provisional estimate for the size of the neck in nickel from the galvanomagnetic measurements is that it subtends an angle $6.5° \pm 1.0°$ at the center of the Brillouin zone. The value determined from the angular widths of the peaks in the magnetoresistance associated with open orbits is $6° \pm 1.5°$. The independent estimate obtained from the difference in the Hall coefficient when measured for a general magnetic field direction and with the field along the symmetry axis is $7° \pm 1.5°$. The latter estimate depends upon the assumption that the multiply connected sheet of the Fermi surface contains electrons of only one sign of spin; if the spin degeneracy were not resolved the Hall effect data would correspond to an angle 3.5°. E. Fawcett and W. A. Reed (private communication).

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MANY-ELECTRON EFFECTS AND EXCHANGE SPLITTINGS IN NICKEL

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Magnetoresistance measurements have shown[1] that one sheet of the Fermi surface of Ni has the same topology as that of Cu, Ag, and Au.[2] It consists of one spheroid in each Brillouin zone connected by narrow necks passing through the L centers of the (111) zone faces. The diameter of the necks are about three times smaller than those of Cu. In the preceding Letter, measurement of the effective masses of the neck is reported. We show here that the exchange splittings[3]

$$\Delta E_d = E(L_3\downarrow) - E(L_3\uparrow),$$

$$\Delta E_c = E(L_{2'}\downarrow) - E(L_{2'}\uparrow) \quad (1)$$

can be estimated from this and other data.

If we assume that the $d\downarrow$ bands of Ni are full, the high-field Hall data[4] give

$$n(s\downarrow) + n(s\uparrow) + n(d\uparrow) = 1.0 \quad (2)$$

and from saturation magnetization data and gyromagnetic ratios[5]

$$1.0 + n(s\downarrow) - n(s\uparrow) - n(d\uparrow) = 0.54 \quad (3)$$

where, e.g., $n(s\downarrow)$ is the number of electrons per atom contained in the $s\downarrow$ Fermi surface. Combining (2) and (3), we have

$$n(s\downarrow) = 0.27. \quad (4)$$

Three models of the band structure of Ni have been proposed to explain the multiply connected Fermi surface. These are shown in Fig. 1. Model (a) uses[6] the Cu band structure for spin $\downarrow$, with $E_F \simeq 9$ eV reduced by $\delta E_F = 0.7$ eV relative to Cu[7] to yield the reduced neck diameter. The calculated density of states[8] near $L_{2'}$ is 1.6 times the free electron value. This gives a precise value in (a) for

$$n(s\downarrow) = 0.50(1 - 1.6 \times \tfrac{3}{2} \delta E_F / E_F) \quad (5)$$

and

$$n(s\downarrow) = 0.41 \quad (6)$$

in comparison to the rough value of 0.33 obtained previously.[6]

The discrepancy between (4) and (6) suggested[9] models (b) and (c). In (b), $n(s\downarrow)$ is reduced compared to (a) because of increased s-d repulsion. Calculation shows[9] that (4) is satisfied for

$$E_g = E(L_{2'}\downarrow) - E(L_3\downarrow) \leq 0.8 \text{ eV} \quad (7)$$