mate fit to all the data. It is therefore impossible to tell whether or not the U^{235} sample was superconducting. However, any reasonable choice for $f(T)$ leads to approximately the same value for the hyperfine contribution.

It is not possible at present to give an unambiguous interpretation of the hyperfine specific heat of U^{235}. Both nuclear magnetic and nuclear electric quadrupole interactions give rise to heat capacities proportional to T^{-2} in the high-temperature approximation. The hyperfine term in the Hamiltonian appropriate to a metal in a magnetically ordered state may be written[3]

$$H = a'I_z + P\{I_z^2 - \tfrac{1}{3}I(I+1)\}, \quad (4)$$

where the first term is the magnetic interaction and the second the electric quadrupole interaction, and $I = 7/2$ for U^{235}. If we assume the hyperfine interaction to be entirely magnetic, our value of β gives $a' \approx 12\times10^{-3}$ deg, a value close to that which one would expect[2] if the ions in the metal had three $5f$ electrons, and if the metal were perfectly ordered. However, while there are some indications that uranium metal may be ordered at low temperatures,[9] present experimental evidence indicates it is unlikely that uranium ions in the metal have three $5f$ electrons.[10] Therefore it is possible that the major, and perhaps the total, contribution to the specific heat of U^{235} metal arises from the electric quadrupole interaction. If we assume that the interaction is entirely electric, we obtain $P \approx 8\times10^{-3}$ °K. Such a value of P is comparable to those obtained for U^{235} in various salts.[11,12]

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HELICON-PHONON INTERACTION IN SOLIDS

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The existence of the type of electromagnetic wave propagation in a conducting medium known as the helicon wave is now well established both in theory and by experiments on a number of metals and semiconductors.[1] The phase velocity of a helicon wave can be very much less than the velocity of light (velocities of the order of 100 cm sec^{-1} have been observed) and, in particular, may be equal to an acoustic velocity in a solid. If a mechanism for interaction exists, the two types of wave will be coupled and their behavior may be expected to deviate significantly from the usual. The purpose of this Letter is to present the results of a calculation of the interaction of acoustic waves and helicon waves in a solid. Several effects are suggested which should be experimentally observable.

The dispersion relation for a helicon wave propagating in an infinite isotropic lossless conductor with applied static magnetic field H is

$$\omega = (\omega_c/\omega_p^2)c^2q^2, \tag{1}$$

where $\omega_c = eH/m^*c$ is the cyclotron frequency and ω_p is the plasma frequency defined by $\omega_p^2 = 4\pi Ne^2/m^*$. This quadratic dispersion curve crosses the linear acoustic dispersion curve, $\omega = v_0 q$, at a crossover frequency

$$\omega_x = (\omega_p^2/\omega_c)(v_0/c)^2, \tag{2}$$

where v_0 is a sound velocity. For typical sound velocities $v_x = \omega_x/2\pi \cong 10^{-9}NH^{-1}$; for a metal with $N = 10^{23}$ cm^{-3} in a field of 10^5 Oe, the crossover frequency is about 1 Gc/sec; if $N = 10^{18}$ cm^{-3}, a field of 10^3 Oe gives a crossover at about 1 Mc/sec. Figure 1 shows schematically the dispersion relations of the coupled helicon and acoustic waves.

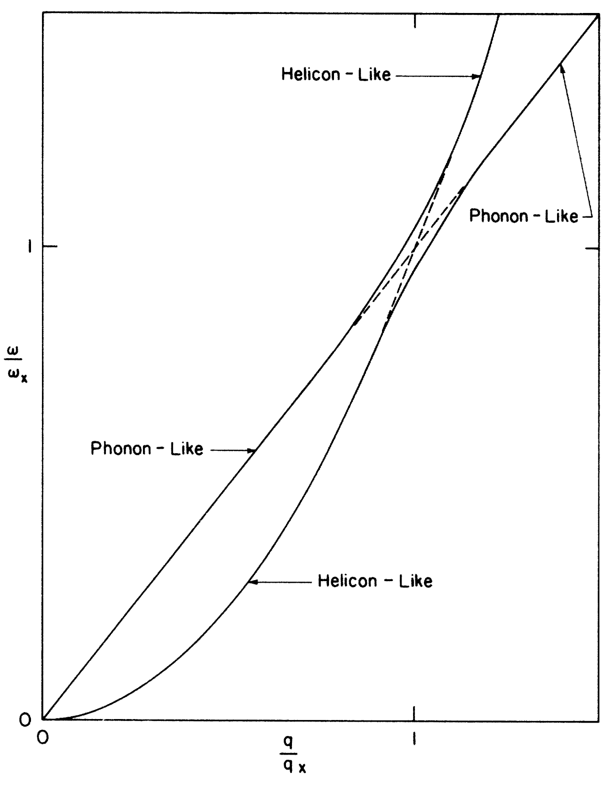

FIG. 1. A schematic plot of the dispersion relations of the two propagating modes with positive circular polarization for the case of negligible attenuation. The dashed curves are for noninteracting acoustic and helicon waves, the solid curves for interacting waves. The character of the modes far from the crossover point is also indicated.

In order to investigate the detailed nature of the effect we have used a model used by Kjeldaas[2] and Cohen, Harrison, and Harrison[3] in developing a theory of the attenuation of acoustic waves in a conductor in the presence of a magnetic field.[4] We consider an infinite medium composed of N electrons per unit volume of charge $-e$ and effective mass m^* in a uniform positively charged background of mass density ρ. A static magnetic field $\vec{H}$ is applied. We assume propagation of a transverse wave with time and space dependence $\exp[i(\vec{q}\cdot\vec{r} - \omega t)]$ and with $\vec{q} \parallel \vec{H}$. The basic equations of the system are Maxwell's equations, the constitutive equation for the electronic part of the total current,

$$\vec{J}e = \sigma_0\sigma' \cdot \left(\vec{E} - \frac{m_0\vec{u}}{e\tau}\right), \tag{3}$$

and the equation of motion of the lattice,[5]

$$\partial^2\vec{r}/\partial t^2 = v_0^2\nabla^2\vec{r} + (Ne/\rho)\vec{E} + (Nm_0/\rho\tau)[\langle\vec{v}\rangle - \vec{u}]. \tag{4}$$

The notation used is that of Cohen, Harrison, and Harrison.[3] σ' is a normalized (to $\sigma_0 = Ne^2\tau/m^*$) magnetoconductivity tensor. The last term in (3) describes a contribution to the electronic current arising from the fact that the electrons tend via relaxation processes to equilibrium in a lattice which is moving with velocity $\vec{u}$.[6] The last term in (4) represents the reaction force on the lattice from the same source; $\langle\vec{v}\rangle$ is the electronic drift velocity. These two terms plus the term in (4) expressing the electrostatic force on the lattice describe the coupling between the helicon and acoustic waves.

Combining (3) and (4) with Maxwell's equations and using circularly polarized field components we find equations for $E_\pm$ and $u_\pm$:

$$\left(q^2 - \frac{i\omega_p^2\omega\tau}{c^2}G^\pm\right)E_\pm = \frac{4\pi iNe\omega}{c^2}\left(1 - \frac{m^0}{m^*}G^\pm\right)u_\pm, \tag{5}$$

$$\frac{i\omega Ne}{\rho}\left(1 + \frac{im_0}{m^*}\frac{c^2q^2}{\omega_p^2\omega\tau}\right)E_\pm = (\omega^2 - v_0^2q^2)u_\pm, \tag{6}$$

where $G^\pm \equiv \sigma_\pm'$ is a function of $\omega\tau$, $\omega_c\tau$, and $qv_t\tau$ given by Kjeldaas[2] and Cohen, Harrison, and Harrison.[3] For a nondegenerate electron gas v_t is an average thermal velocity; for a degenerate gas it is the Fermi velocity. We use here only the limiting value of $G^\pm$ for $\omega_c \gg \omega$, qv_t,[7] which is $G^\pm = (1 \pm i\omega_c\tau)^{-1}$. In doing so we render our equations beyond (7) invalid near the absorption edge.[4]

Setting the determinant of the coefficients of $E_\pm$ and $u_\pm$ in (5) and (6) equal to zero yields an equation for the dispersion relations of the propagating waves,

$$\left(q^2-\frac{i\omega_p{}^2\omega\tau}{c^2}G^\pm\right)(v_0{}^2q^2-\omega^2)-\frac{4\pi N^2e^2\omega^2}{\rho c^2}\left(1-\frac{m_0}{m^*}G^\pm\right)\times\left(1+\frac{im_0c^2q^2}{m^*\omega_p{}^2\omega\tau}\right)=0. \quad (7)$$

In the limit of very large relaxation times this becomes

$$\left(q^2\mp\frac{\omega_p{}^2\omega}{c^2\omega_c}\right)(v_0{}^2q^2-\omega^2)-\frac{4\pi N^2e^2\omega^2}{\rho c^2}=0. \quad (8)$$

In the absence of the last term we would have the dispersion relations for uncoupled helicon (upper sign choice) and acoustic waves. This term introduces a coupling, however, and at the crossover frequency the wave vectors of the two modes are given by

$$q_x{}^2=\frac{\omega_x{}^2}{v_0{}^2}\left[1\pm\frac{v_0}{\omega_x c}\left(\frac{4\pi N^2e^2}{\rho}\right)^{1/2}\right]. \quad (9)$$

The fractional difference in wave vector for the two modes is, for typical acoustic velocities and material densities,

$$\frac{\Delta q_x}{q_x}\cong\frac{H}{v_0(4\pi\rho)^{1/2}}\cong 5\times10^{-7}H, \quad (10)$$

which is small but should be experimentally detectable in available magnetic fields. The relation between the magnetic induction associated with the wave and the lattice particle velocity is, at the crossover,

$$\left(\frac{B_+}{u_+}\right)_x=-\frac{icq_x}{\omega_x}\left(\frac{E_+}{u_+}\right)_x\cong(4\pi\rho)^{1/2}. \quad (11)$$

Particle velocities of the order of 0.1 to 1 cm sec^{-1} are obtainable using conventional ultrasonic techniques. For sufficiently long relaxation times these particle velocities will be associated with magnetic inductions of the order of 1 to 10 gauss.

Calculation of the dispersion relations and relative field amplitudes for finite relaxation times can be carried out in the following manner: We distinguish two cases. First, we assume propagation of the mode which can be identified as a weakly perturbed acoustic wave with dispersion relation $v_0{}^2q^2=\omega^2(1+\delta_P)$ where δ_P describes the perturbation due to the interaction and it is assumed that $|\delta_P|\ll1$. Substitution of this dispersion relation into (5) and (6) permits calculation of δ_P and E_+/u_+, hence B_+/u_+, at the crossover. The result is

$$\left(\frac{B_+}{u_+}\right)_x\cong\frac{iH\omega_c\tau}{v_0}=\frac{10^{-8}i\mu H^2}{v_0}, \quad (12)$$

$$\delta_{Px}\cong\frac{iH^2\omega_c\tau}{4\pi\rho v_0{}^2}, \quad (13)$$

where μ is the carrier mobility measured in $\text{cm}^2\ \text{V}^{-1}\ \text{sec}^{-1}$. These equations are approximate relations subject to the condition $1\ll\omega_c\tau\ll v_0(4\pi\rho)^{1/2}\times H^{-1}$. The condition on the relaxation time for realizing the full effect described by (11) is $\omega_c\tau\gg v_0(4\pi\rho)^{1/2}H^{-1}$, a much more stringent condition than $\omega_c\tau\gg1$. Nevertheless, the magnitudes are such that the effect should be observable with attainable relaxation times and fields. Figure 2 shows schematically the dependence of $(B_+/u_+)_x$ on the relaxation time for a fixed magnetic field.

A factor $(i\omega_c\tau+1-m_0/m^*)$ appears in the equations leading to (12) and (13). In order to simplify (12) and (13) we have assumed that if m_0/m^* differs from one, $\omega_c\tau\gg|1-m_0/m^*|$. This fac-

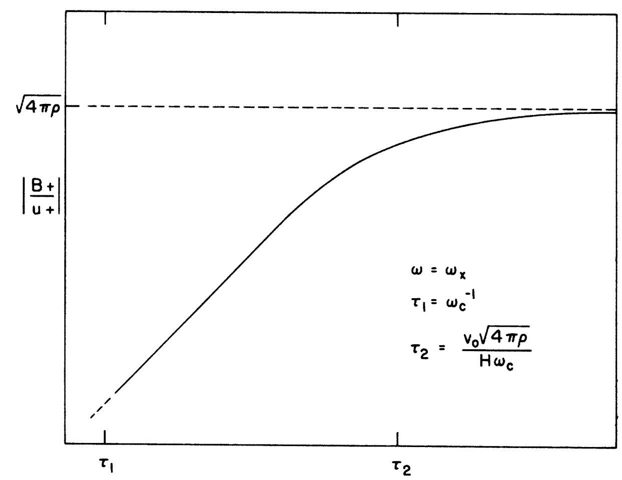

FIG. 2. This shows schematically the dependence on relaxation time of the magnitude of the ratio of the magnetic induction and the acoustic particle velocity at the crossover frequency for the mode which is phononlike for weak coupling. τ_1 is the relaxation time for which $\omega_c\tau$ becomes equal to one. For shorter relaxation times, the helicon wave is severely damped and cannot be said to exist as a propagating wave. τ_2 is the relaxation time for which the condition $\omega_c\tau=v_0(4\pi\rho)^{1/2}H^{-1}$ is satisfied and marks the transition to the region in which the modes are completely mixed at the crossover.

tor leads to phase and amplitude effects which should make it possible to establish experimentally whether m^* or m_0 should appear.[6]

In the second case we assume propagation of a mode identifiable as a slightly perturbed helicon wave with dispersion relation $c^2q^2(1+i\omega_c\tau) = i\omega_p^2\omega\tau(1+\delta_h)$. A calculation similar to the above and subject to the same conditions yields

$$\left(\frac{u_+}{B_+}\right)_x \cong -\frac{iH\omega_c\tau}{4\pi\rho v_0} = -\frac{10^{-8}i\mu H^2}{4\pi\rho v_0}, \tag{14}$$

$$\delta_{hx} \cong -\frac{iH^2\omega_c\tau}{4\pi\rho v_0^2}. \tag{15}$$

These results indicate that in appropriate materials it should be possible to observe a number of effects associated with the helicon-phonon interaction including dispersion or absorption of either type of wave at the crossover or generation of one type by introducing the other type into the system from outside. The latter includes the interesting possibility of generating high frequency acoustic waves in metals.

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[4]In their treatment, Cohen, Harrison, and Harrison describe a peak in the acoustic attenuation [their Eq. (5.5)] which is, in fact, due to the helicon-phonon coupling although it is not identified as such. Kjeldaas has recently calculated the helicon attenuation in the region of the absorption edge predicted by Stern [E. A. Stern, Phys. Rev. Letters 10, 91 (1963)], and, in addition, also notes the existence of enhanced absorption of helicon and acoustic waves at the helicon-phonon crossover (private communication, to be published).

[5]The Lorentz force on the moving lattice has been omitted in Eq. (4). Its inclusion leads to negligible terms in the equations given below for the wave vectors and fields at the crossover frequency.

[6]Harrison has stated that the mass m_0 which appears in this term should be the free electron mass, not the effective mass, because the scattering process involves a transfer of real momentum, not crystal momentum [M. J. Harrison, Phys. Rev. 119, 1260 (1960)]. However, there are scattering processes, e.g., scattering from unscreened ionized impurities, for which the effective mass must certainly appear. In general, m_0 or m^* will probably enter accordingly as the scattering occurs via a short-range or a long-range interaction. We retain m_0 in the equations and leave open the possibility of putting $m_0 = m^*$.

[7]The magnetic fields required to satisfy this condition for metals are large but realizable.

HELICON-PHONON INTERACTION IN METALS*

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The purpose of this Letter is to investigate the interaction between helicon waves and transverse acoustic waves propagating parallel to a dc magnetic field in metals. We assume that a metal behaves as a free electron gas embedded in a lattice of positive ions, the latter being isotropic and capable of sustaining both longitudinal and shear waves. In the absence of the Coulomb interaction between the electrons and the positive ions, the transverse modes of vibration of the system consist of two shear acoustic waves of frequency $\omega = \pm sq$ and an electromagnetic wave (helicon[1]) of frequency

$$\omega_H = c^2q^2\omega_0/\omega_p^2. \tag{1}$$

Here s is the velocity of transverse acoustic waves, q the wave vector, $\omega_0 = +|e|B_0/mc$ (e = charge on the electron) is the electron cyclotron