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GIANT OSCILLATORY ATTENUATION OF HELICON AND ALFVÉN WAVES

P. B. Miller
IBM Watson Research Center, Yorktown Heights, New York
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The propagation and attenuation of helicon[1-4] and Alfvén waves[5-7] in solids have been discussed using classical models of transport. By the use of a quantum model to calculate the current density, we have found that giant oscillations are present in the attenuation and in the real part of the conductivity. The meaning of giant is that the attenuation coefficient in cm^{-1} (Q_I) varies with magnetic field H so that $(Q_I^{\text{Max}} - Q_I^{\text{Min}}) \gg Q_I^{\text{Min}}$. Such giant oscillations have been found in the magnetoacoustic attenuation by Gurevich et al.[8] We find similar giant oscillations in the attenuation of helicon and Alfvén waves although different selection rules on the Landau quantum number n lead to a period which is approximately periodic in H^{-1} but which differs from the usual de Haas–van Alphen period and is a slowly varying function of H.

Consider an electromagnetic wave (helicon or Alfvén wave) propagating in a solid state plasma with a static magnetic field (H) along the z axis. If the charge carriers have an isotropic effective mass, the normal modes in both cases will be circularly polarized with an electric field

$$\vec{E} = E_t(\hat{x} - i\hat{y})\exp[i(\omega t + Qz)] \tag{1}$$

where from Maxwell's equations Q is given by the solution of

$$Q^2 = (-4\pi i\omega/C^2)\sigma(Q) \tag{2}$$

where $\sigma(Q)$ is the wave-number-dependent conductivity. When $\text{Re}\sigma(Q)$ vanishes, undamped propagation occurs for those modes where $\text{Im}\sigma(Q)$ is positive. Equation (2) is strictly valid only when the real part of $\sigma(Q)$ is much smaller than the imaginary part since $\sigma(Q)$ is only defined for real Q and hence may be used to describe quantitatively both undamped and weakly damped propagation. In a qualitative sense, it also describes the strong-damping region.

The current response to an electric field given by (1) is calculated by a perturbation method using the Landau levels in a magnetic field as basis states. With an appropriate choice of gauge, the levels are labeled by quantum numbers $|n, k_y, k_z\rangle$ and energy $E_n(k_z) = (n+\frac{1}{2})\hbar\omega_c + h^2k_z^2/2m$ where ω_c is the cyclotron frequency. For isotropic carriers, the use of the selection rules $\Delta n = 1$ and $\Delta k_z = Q$ in a transition leads to a conductivity given by

$$\sigma(Q) = \frac{Ne^2}{im\omega} - \frac{e^2}{\pi i}\frac{\omega c^2}{\hbar\omega} \times \sum_{n,k_z}(n+1)\frac{f[E_n(k_z)] - f[E_{n+1}(k_z+Q)]}{\omega - \omega_c - \hbar k_z Q/m - is} \tag{3}$$

where f is the Fermi function and $s \to 0$ in the limit of no scattering. When $\hbar\omega_c \ll kT$ we may replace the sum over n by an integration and we obtain the same result as from a Boltzmann equation treatment,[9,2] with no oscillatory terms. When $\hbar\omega_c \gg kT$ the summation over n cannot be replaced by integration. To evaluate $\text{Re}\sigma$ one replaces the energy denominator by a delta function to obtain

$$\text{Re}\sigma(Q) = \frac{1}{2\pi}\frac{me^2\omega_c^2}{\hbar^2\omega Q} \times \sum_n\{f[E_n(k_{z0})] - f[E_{n+1}(k_{z0}+Q)]\}(n+1), \tag{4}$$

where

$$k_{z0} = \frac{m}{\hbar Q}(\omega - \omega_c). \tag{5}$$

Energy-momentum conservation implies that only a single k_z value (k_{z0}) contributes to absorption and leads to large oscillations in $\mathrm{Re}\sigma$.

For helicon waves propagating in metals $\hbar\omega \ll kT$ and Eq. (4) becomes

$$\mathrm{Re}\sigma(Q) = \frac{3\pi Ne^2}{4mQv_F}\left[1 - \left(\frac{\omega_c - \omega}{Qv_F}\right)^2\right]\frac{\hbar\omega_c}{kT}$$

$$\times \cosh^{-2}\left[\frac{E_F - (n_0 + \frac{1}{2})\hbar\omega_c - \hbar^2 k_{z0}^2/2m}{2kT}\right] \qquad (6)$$

where n_0 is the integer for which the argument of the cosh function is a minimum. The relation (6) describes oscillations of giant amplitude $(\sigma_R^{\mathrm{Max}} - \sigma_R^{\mathrm{Min}}) \gg \sigma_R^{\mathrm{Min}}$ in the real part of the conductivity which are approximately periodic in H^{-1} with a period given by

$$\Delta\left(\frac{1}{H}\right) = \frac{e\hbar}{mE_F c}\,\frac{1}{1 - [(\omega_c - \omega)/Qv_F]^2}. \qquad (7)$$

The attenuation coefficient Q_I is found from (2) to be proportional to $[(\sigma_I^2 + \sigma_R^2)^{1/2} - \sigma_I]$ in the qualitative sense previously discussed and hence also shows giant oscillations with the period (7). The denominator in (7) is a slowly varying function of H compared to the very short de Haas–van Alphen period in metals. No giant oscillations occur in $\mathrm{Im}\sigma(Q)$ since the summation over k_z is not restricted. Thus the real part of Q, Q_R, is given by the usual helicon dispersion relation $Q_R \simeq (4\pi Ne\omega/cH)^{1/2}$. The onset of the absorption is at $Qv_F = \omega_c - \omega$.[2-4]

When we consider Alfvén waves in semimetals, we need to evaluate (4) in the limit $\hbar\omega \gg kT$ and sum over both electrons and holes. The initial state taking part in the transition can now be in a band of energies from E_F to $E_F - h\omega$ with a fixed k_z value given by (5) for electrons and for holes. The number of Landau levels contributing to the attenuation will therefore oscillate. It has been shown[6,7] that attenuation occurs below a threshold value defined as the field where the equality $\omega_c = \omega + v_F Q_R$ is satisfied for any carrier. The real part of the wave number is given by $Q_R = \omega H^{-1} \times (4\pi \sum N_j m_j)^{1/2}$ and the sum is over all carriers.[5] Below the onset the attenuation will thus be oscillatory with each carrier contributing with a period given by Eq. (7).

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RELATIVISTIC EFFECTS IN THE BAND STRUCTURE OF PbTe*

L. E. Johnson, J. B. Conklin, and G. W. Pratt, Jr.
Materials Theory Group, Department of Electrical Engineering,
Massachusetts Institute of Technology, Cambridge, Massachusetts
(Received 16 October 1963; revised manuscript received 12 November 1963)

The purpose of this Letter is to point out that relativistic interactions have a drastic effect on the energy band structrue of PbTe and are of major importance in understanding the energy gaps and effective masses. These interactions are commonly derived by converting the 4-component Dirac equation into a second-order equation and then reducing this to a two-component form. When this is done, as in the Pauli approximation, two other terms appear besides the spin-orbit interaction, namely, the mass-velocity energy correction and a term of the form $(i\mu_0/2mc)\vec{\epsilon}\cdot\vec{p}$, where $\vec{\epsilon}$ is the electric field seen by an electron and $\vec{p}$ is its momentum. If the spatial components of the vector potential are assumed to be zero, the one-electron Hamiltonian is[1]

$$\mathcal{H} = -(\hbar^2/2m)\nabla^2 - e\varphi - (1/2mc^2)(E + e\varphi)^2$$
$$- (i\mu_0/2mc)\vec{\epsilon}\cdot\vec{p} + (\mu_0/2mc)\vec{\sigma}\cdot(\vec{\epsilon}\times\vec{p}).$$

We show in this Letter that the mass-velocity