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ALTERNATIVE APPROACH TO THE PROBLEM OF PRODUCING CONTROLLED THERMONUCLEAR POWER

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The attempt to achieve controlled thermonuclear power with low-density magnetically confined plasmas is confronted with many difficulties. It is therefore important that we should examine the problem from as many different points of view as possible. In the approach suggested here macroscopic particles are accelerated to 10^8-10^9 cm sec^{-1}; they then collide either with other particles or a target, and their kinetic energy is converted into thermal energy inertially confined to a small region for a short period of time. Only a crude analysis is offered, and the problem of accelerating macroscopic particles to a high energy is ignored.

Provisionally we assume that (i) the energy radiated from the impact region is small compared with the initial kinetic energy, and (ii) all reaction products escape from the impact region. For simplicity it is supposed that the particle is a right cylinder, of diameter D and length L, with its base facing the target.

First, we consider $L=D$ (an approximation for a spherical particle). We assume that a one-dimensional fluid treatment is adequate during the time taken for a shock wave to travel through the particle. Because of (i) and (ii), the temperature T is almost constant during impact and we use the steady-state equations of continuity, motion, and energy. Let a particle have a velocity v and density ρ, and a stationary target have the same density; then behind the shock fronts advancing into the particle and target, the density and pressure are

$$\rho_1 = 4\rho, \quad p_1 = \tfrac{1}{3}\rho v^2 \qquad (1a, b)$$

for $\gamma = \frac{5}{3}$, and neglecting ionization energy. If z is the distance separating the shock fronts, $dz/dt = \frac{1}{3}v$, and $z = \frac{1}{2}D$ at the end of the impact period $t_1 = 3D/2v$.

Optimistically we suppose that the particle and target consist solely of hydrogen isotopes; then $p_1 = 2n_1kT$, $\rho_1 = n_1 m_{\rm H} A$, where n is the ion density and A the mean atomic weight. The power radiated by free-free transitions is[1]

$$P_b = 5.2\times10^{22}\rho_1{}^2 A^{-2} T^{1/2} \text{ erg cm}^{-3}\text{ sec}^{-1}, \qquad (2)$$

and (i) is therefore true if

$$\int_0^{t_1} P_b z\,dt/\tfrac{1}{2}\rho D v^2 \ll 1.$$

Using (1a), (1b), and (2), it follows that

$$\rho D \ll 1.4\times10^{-8} A^{1/2} T \text{ g cm}^{-2}. \qquad (3)$$

The reaction power per unit volume is

$$P_r = \alpha\rho_1{}^2 A^{-2}\langle\sigma v\rangle,$$

and the ratio R of energy released to the energy supplied is

$$R=\int_0^{t_1} P_\gamma z\,dt/\tfrac{1}{2}\rho D v^2.$$

We require at the very least $R>1$, or

$$12\rho D\alpha A^{-2}v^{-3/2}\langle\sigma v\rangle>1. \qquad (4)$$

Clearly, ρD is a minimum[2] where the $\log\langle\sigma v\rangle$ - $\log I$ curves[3] have a slope of $\frac{3}{2}$: $I_{\rm DD}=4.4\times10^8\,^\circ$K, $\langle\sigma v\rangle_{\rm DD}=1.4\times10^{-17}$; and $I_{\rm DT}=2.0\times10^8\,^\circ$K, $\langle\sigma v\rangle_{\rm DT}=3.3\times10^{-16}$. For the combined D-D reaction (assuming H^3 and He^3 are recycled) $\alpha=Q/2m_{\rm H}^2$, $Q=30$ MeV; and for the D-T reaction (assuming equal concentrations) $\alpha=Q/4m_{\rm H}^2$, $Q=26$ MeV; in both cases 8 MeV is added for each neutron. Therefore, from (4), the energy released exceeds the energy supplied when

$$(\rho D)_{\rm DD}>0.8,\quad (\rho D)_{\rm DT}>2.7\times10^{-2} \qquad (5a, b)$$

in g cm^{-2}, at the velocities

$$v_{\rm DD}=7\times10^8,\quad v_{\rm DT}=4\times10^8 \qquad (6a, b)$$

in cm sec^{-1}. Preheating by radiation and the reaction products will presumably reduce these values of ρD and v, but to what extent is not known.

At $I\sim10^8$, $\rho\sim1$, the bound-free transitions in hydrogen are negligible, and the radiative opacity is determined by electron scattering.[4] The particle is therefore optically thin for $\rho D<2.5A$, thus justifying the use of (2). The minimum values of ρD given by (5) satisfy condition (3), and the radiated energy is consequently small. The mean free path λ in the high-temperature gas is given by $\rho\lambda\sim10^{-3}$, and therefore a fluid treatment seems not unreasonable. For $\rho\simeq1$, the D-D reaction requires the large value of $D>1$ cm and is ruled out on practical grounds; the D-T reaction, however, requires the small value of $D>0.3$ mm.

If the collision occurs between particles moving with equal speed in opposite directions, the values of ρD and v in (5) and (6) are each halved. The values of ρD may also be reduced by allowing for reactions occurring at $t>t_1$. Let us suppose that after a collision between two particles the gas forms a sphere of uniform density which expands adiabatically at the velocity of sound. Using an analytic expression for the reaction probability

$$\langle\sigma v\rangle\propto T^{-2/3}\exp(-bT^{-1/3})$$

(where $bT^{-1/3}=\frac{13}{2}$ at $t\leq t_1$), one finds the additional reactions reduce ρD to $\frac{5}{7}$ of its previous value. More sophisticated arguments are needed when a particle strikes a large target. Since the local region is heated by the reaction products, and at t_1 the impact region still possesses half the kinetic energy of the original particle, it is likely that the total energy released is greater than in the two-particle model.

We consider now a needlelike particle of $L\gg D$ plunging into a target. A lower limit for R is estimated as follows. Applying a one-dimensional treatment to the nose of the shock wave, over an area of $\frac{1}{4}\pi D^2$, the steady-state equations show that for a radial flow of uniform density the shock layer at the nose has a depth of $\frac{1}{2}D$. For this region $\rho_1=4\rho$, $p_1=3\rho v^2/16$, $t_1=2L/v$, and $R=2P_\gamma D/\rho v^3$. Thus, taking into account only the reactions in the nose of the shock wave, we find ρD is reduced to $\frac{8}{9}$ and v is increased to $\frac{4}{3}$ of the values in (5) and (6). An upper limit for R is found by supposing that the shock layer is contoured to the particle as in the Newtonian flow theory[5] and expands negligibly in the nose-confinement time $2L/v$. It follows that $R=P_\gamma L/\rho v^3$, and therefore the values of ρD in (5) are reduced by $16D/9L$, or

$$(\rho L)_{\rm DD}>1.4,\quad (\rho L)_{\rm DT}>5\times10^{-2} \qquad (7a, b)$$

and the velocities of (6) are increased by $\frac{4}{3}$. More precise criteria would presumably lie somewhere between (5) and (7).

We have not considered the form in which the hydrogen isotopes exist in the solid state. The target might be a droplet of liquid deuterium and tritium ($\rho\simeq0.2$). If the particle is a hydride, such as[6] $LiBD_4+LiBT_4$, containing 37% hydrogen isotopes by weight, the previous $\rho D, L$ values are not greatly affected. Even if the particle contains no hydrogen isotopes, but is of a material best suited for acceleration, the $\rho D, L$ values are not altered by any large amount provided the shock layers of the two materials do not intermix during the impact period.

It is possible that on closer examination the above approach will pose problems at least as difficult as those which beset the electromagnetic confinement of low density plasmas. The acceleration of "macrons" (macroscopic particles) to velocities of 10^8-10^9 cm sec^{-1} requires the development of new techniques. With the advent of macron accelerators a new field of physics will undoubtedly open up, and one of the achievements in this field might well be controlled thermonuclear power.

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GIANT OSCILLATORY ATTENUATION OF HELICON AND ALFVÉN WAVES

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The propagation and attenuation of helicon[1-4] and Alfvén waves[5-7] in solids have been discussed using classical models of transport. By the use of a quantum model to calculate the current density, we have found that giant oscillations are present in the attenuation and in the real part of the conductivity. The meaning of giant is that the attenuation coefficient in cm^{-1} (Q_I) varies with magnetic field H so that $(Q_I^{\mathrm{Max}} - Q_I^{\mathrm{Min}}) \gg Q_I^{\mathrm{Min}}$. Such giant oscillations have been found in the magnetoacoustic attenuation by Gurevich et al.[8] We find similar giant oscillations in the attenuation of helicon and Alfvén waves although different selection rules on the Landau quantum number n lead to a period which is approximately periodic in H^{-1} but which differs from the usual de Haas–van Alphen period and is a slowly varying function of H.

Consider an electromagnetic wave (helicon or Alfvén wave) propagating in a solid state plasma with a static magnetic field (H) along the z axis. If the charge carriers have an isotropic effective mass, the normal modes in both cases will be circularly polarized with an electric field

$$\vec{E} = E_t(\hat{x} - i\hat{y}) \exp[i(\omega t + Qz)] \quad (1)$$

where from Maxwell's equations Q is given by the solution of

$$Q^2 = (-4\pi i\omega/C^2)\sigma(Q) \quad (2)$$

where $\sigma(Q)$ is the wave-number-dependent conductivity. When $\mathrm{Re}\sigma(Q)$ vanishes, undamped propagation occurs for those modes where $\mathrm{Im}\sigma(Q)$ is positive. Equation (2) is strictly valid only when the real part of $\sigma(Q)$ is much smaller than the imaginary part since $\sigma(Q)$ is only defined for real Q and hence may be used to describe quantitatively both undamped and weakly damped propagation. In a qualitative sense, it also describes the strong-damping region.

The current response to an electric field given by (1) is calculated by a perturbation method using the Landau levels in a magnetic field as basis states. With an appropriate choice of gauge, the levels are labeled by quantum numbers $|n, k_y, k_z\rangle$ and energy $E_n(k_z) = (n+\frac{1}{2})\hbar\omega_c + h^2 k_z^2/2m$ where ω_c is the cyclotron frequency. For isotropic carriers, the use of the selection rules $\Delta n = 1$ and $\Delta k_z = Q$ in a transition leads to a conductivity given by

$$\sigma(Q) = \frac{Ne^2}{im\omega} - \frac{e^2}{\pi i}\frac{\omega c^2}{\hbar\omega} \times \sum_{n, k_z}(n+1)\frac{f[E_n(k_z)] - f[E_{n+1}(k_z+Q)]}{\omega - \omega_c - \hbar k_z Q/m - is} \quad (3)$$

where f is the Fermi function and $s \to 0$ in the limit of no scattering. When $\hbar\omega_c \ll kT$ we may replace the sum over n by an integration and we obtain the same result as from a Boltzmann equation treatment,[9,2] with no oscillatory terms. When $\hbar\omega_c \gg kT$ the summation over n cannot be replaced by integration. To evaluate $\mathrm{Re}\sigma$ one replaces the energy denominator by a delta function to obtain

$$\mathrm{Re}\sigma(Q) = \frac{1}{2\pi}\frac{me^2\omega_c^2}{\hbar^2\omega Q} \times \sum_n\{f[E_n(k_{z0})] - f[E_{n+1}(k_{z0}+Q)]\}(n+1), \quad (4)$$

where

$$k_{z0} = \frac{m}{\hbar Q}(\omega - \omega_c). \quad (5)$$

Energy-momentum conservation implies that only a single k_z value (k_{z0}) contributes to absorption