Professor R. E. Marshak and Professor E. C. G. Sudarshan for many useful conversations.

---

*This work has been supported in part by the U. S. Atomic Energy Commission.

[1]Y. Yamaguchi (unpublished); Y. Ne'eman, Nucl. Phys. 26, 222 (1961); M. Gell-Mann, Phys. Rev. 125, 1067 (1962). Our η is the particle that Gell-Mann and some others denote by χ.

[2]L. Rosenson, Bull. Am. Phys. Soc. 8, 46 (1963); Earle C. Fowler, Frank S. Crawford, Jr., L. J. Lloyd, Ronald A. Grossman, and LeRoy Price, Phys. Rev. Letters 10, 110 (1963).

[3]N. Cabibbo and R. Gatto, Nuovo Cimento 21, 872 (1961).

[4]S. Okubo, Phys. Letters 4, 14 (1963)

[5]We have taken $(1.05\pm0.18)\times10^{-16}$ sec for the decay lifetime of π^0: G. Von Dardel et al., Phys. Letters 4, 51 (1963). Earlier experiments gave values about twice larger: R. G. Glasser, N. Seeman, and B. Stiller, Phys. Rev. 123, 1324 (1962); R. F. Blackie, A. Engler, and J. H. Mulney, Phys. Rev. Letters 5, 384 (1960).

[6]B. Barrett and G. Barton, Phys. Letters 4, 16 (1963).

[7]D. Berley, D. Colley, and J. Schultz, Phys. Rev. Letters 10, 114 (1963); Riazuddin and Fayyazuddin, Phys. Rev. 129, 2337 (1963). There is a difference of a factor 64 between our calculation and that of Riazuddin and Fayyazuddin. We believe that the latter authors have used a wrong definition of λ differing by a factor of 8 from the conventional one, so that we have to multiply their expression by 64.

[8]S. Coleman and S. L. Glashow, Phys. Rev. Letters 6, 423 (1961). Equation (13) has been also noted by E. C. G. Sudarshan (private communication). We may remark that we can estimate γ by another formula, $\gamma=(1/2\sqrt{3})[m^2(\pi_0)+3m^2(\eta)-4m^2(K_0)]$, but this gives an unreasonably large value for γ (see reference 4). It might also be noted that it would not necessarily vanish in the charge-independence limit, whereas the formula used, Eq. (13), does.

[9]J. Hamilton, P. Menotti, G. C. Oades, and L. J. Vick, Phys. Rev. 128, 1881 (1962); B. R. Desai, Phys. Rev. Letters 6, 497 (1961).

[10]K. C. Wali, Phys. Rev. Letters 9, 120 (1962).

[11]If the interaction is not assumed local and minimal, at least one more parameter is needed for its description.

[12]S. Okubo, Progr. Theoret. Phys. (Kyoto) 27, 949 (1962); 28, 24 (1962).

[13]C. J. Goebel, Bull. Am. Phys. Soc. 3, 12 (1958); B. Bosco and V. de Alfaro, Phys. Rev. 115, 215 (1959); K. Itabashi, M. Kato, K. Nakagawa, and G. Takeda, Progr. Theoret. Phys. (Kyoto) 24, 529 (1960); M. Kato, Progr. Theoret. Phys. (Kyoto) 25, 493 (1961).

[14]M. I. Adamovich et al., International Conference on High-Energy Nuclear Physics, Geneva, 1962 (CERN Scientific Information Service, Geneva, Switzerland, 1962); C. S. Robinson, P. M. Baum, L. Criegee, and J. M. McKinley, Phys. Rev. Letters 9, 349 (1962); J. S. Ball, Phys. Rev. Letters 5, 73 (1960); Phys. Rev. 124, 2014 (1961); B. de Tollis et al., Nuovo Cimento 18, 198 (1960).

[15]K. Iatabashi and T. Ebata, Progr. Theoret. Phys. (Kyoto) 28, 915 (1962); M. Monda, Progr. Theoret. Phys. (Kyoto) 28, 904 (1962).

[16]Y. Fujii and M. Kawaguchi, Progr. Theoret. Phys. (Kyoto) 26, 519 (1961).

[17]K. Kawarabayshi and A. Sato, Nuovo Cimento 26, 1015 (1962). We have corrected the value of Λ by using the new experimental value of the π^0 lifetime (see reference 5).

[18]M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Letters 8, 261 (1962)

[19]L. M. Brown and P. Singer, Phys. Rev. Letters 8, 460 (1962).

[20]S. Okubo and R. E. Marshak (to be published).

DETERMINATION OF SPIN AND DECAY PARAMETERS OF FERMION STATES*

N. Byers and S. Fenster
Department of Physics, University of California, Los Angeles, California
(Received 3 June 1963)

In this Letter we discuss the decay process

$$Y^* \to Y + \pi, \tag{1}$$

where Y^* has spin J and decays into a particle with spin 1/2 (Y) and one with spin zero (π). We show how J and the amplitudes for the parity states $l=J\pm1/2$ may be measured if the transverse and longitudinal polarization of Y are appreciable.[1] We denote these amplitudes by

$$a=\text{amplitude for } l=J-1/2,$$
$$b=\text{amplitude for } l=J+1/2, \tag{2}$$

defined so that the lifetime of Y^* is given by

$$\tau^{-1}=2\pi\rho_E(|a|^2+|b|^2);$$

and in accordance with the notation[2] for $J=1/2$, we define the parameters

$$\gamma=(|a|^2-|b|^2)/(|a|^2+|b|^2),$$
$$\alpha=2\,\mathrm{Re}\,ab^*/(|a|^2+|b|^2),$$
$$\beta=2\,\mathrm{Im}\,ab^*/(|a|^2+|b|^2). \tag{3}$$

If parity is conserved in (1), $\alpha=\beta=0$ and $\gamma=\pm1$.

Note that

$$\alpha^2+\beta^2+\gamma^2=1. \tag{4}$$

To see how J and the parameters α, β, and γ may be measured, first consider a collection of Y^* at rest. The state of the system is completely specified by $(2J+1)^2$ real numbers which are conveniently chosen to be the multipole parameters[3] t_L^M (L and M are integers; $0\leq L\leq 2J$, $-L\leq M\leq L$). The t_L^M have the reality property that

$$t_L^{-M}=(-)^M t_L^{M*}. \tag{5}$$

For given L, the $2L+1$ parameters t_L^M are components of an irreducible tensor[4] of rank L. This means that under a spatial rotation the t_L^M obey the same transformation law as the spherical harmonics Y_L^M.[4] The scalar t_0^0 is a normalization constant which we shall take equal to unity. For $L=1$, the t_1^M measure the polarization of the initial state $\vec{P}_i$. They are related to the expectation values of the spin operators $\vec{S}$ by

$$\vec{P}_i=\langle\vec{S}\rangle/J, \tag{6}$$

$$t_1^0=\langle S_z\rangle/[J(J+1)]^{1/2},$$

$$t_1^{\pm1}=\mp\langle S_x\pm iS_y\rangle/[2J(J+1)]^{1/2}. \tag{7}$$

For $L>1$, they are expectation values of irreducible tensors formed from the components of $\vec{S}$. For example,

$$t_2^0\propto\langle 3S_z^2-\vec{S}^2\rangle,$$

$$\mathrm{Re}\,t_2^2\propto\langle S_x^2-S_y^2\rangle,$$

$$\mathrm{Im}\,t_2^2\propto\langle S_xS_y+S_yS_x\rangle.$$

The normalization of the t_L^M is given below [see Eqs. (18) and (19)]. For the present, we shall assume that the t_L^M are unknown parameters (determined by the process which formed the collection of Y^*).

Let $I(\theta,\phi)\Delta\Omega$ be the fraction of Y particles emitted into the element of solid angle $\Delta\Omega$ and let $\vec{P}$ be the polarization of these particles. Since $I(\theta,\phi)$ is a scalar function of the angles θ and φ, the moments of I,

$$\int d\Omega I(\theta,\phi)Y_L^M(\theta,\phi)\equiv\langle Y_L^M\rangle, \tag{8}$$

are proportional to t_L^M; the relation is[5]

$$\langle Y_L^M\rangle=n_{L0}t_L^M \text{ if } L \text{ is even}, \tag{9}$$

$$\langle Y_L^M\rangle=\alpha n_{L0}t_L^M \text{ if } L \text{ is odd}, \tag{9$'$}$$

where

$$n_{L0}=(-)^{J-1/2}[(2J+1)/4\pi]^{1/2}C(JJL;1/2,-1/2), \tag{10}$$

and $C(JJL;mm')=(JmJm'|JJLM)$ is a standard Clebsch-Gordan coefficient.[4] Note that $n_{L0}=0$ for $L>2J$.

The angular distribution of longitudinal polarization $(I\vec{P}\cdot\hat{k})$ is given by the multipole parameters in a similar manner[6]; the moments of this function ($\hat{k}$ is the unit vector with spherical angles θ and ϕ),

$$\int d\Omega I\vec{P}\cdot\hat{k}Y_L^M\equiv\langle\vec{P}\cdot\hat{k}Y_L^M\rangle, \tag{11}$$

are

$$\langle\vec{P}\cdot\hat{k}Y_L^M\rangle=n_{L0}t_L^M \text{ if } L \text{ is odd}, \tag{12}$$

$$\langle\vec{P}\cdot\hat{k}Y_L^M\rangle=\alpha n_{L0}t_L^M \text{ if } L \text{ is even}. \tag{12$'$}$$

The angular distribution of transverse polarization also yields measurable quantities proportional to t_L^M (with odd L). If both γ and β are different from zero, the components of $I\vec{P}_{\mathrm{Tr}}=I\vec{P}-I\vec{P}\cdot\hat{k}\hat{k}$ have even and odd moments. Let[7] $P_0=P_z$ and $\sqrt{2}P_{\pm1}=\mp(P_x\pm iP_y)$; then we find[8]

$$\gamma n_{L1}t_L^M$$
$$=[(L+1)/(2L+1)]^{1/2}\sum_m\langle P_mY_{L-1}^{M-m}\rangle$$
$$\times C(1,L-1,L;m,M-m)+[L/(2L+1)]^{1/2}$$
$$\times\sum_m\langle P_mY_{L+1}^{M-m}\rangle C(1,L+1,L;m,M-m), \tag{13}$$

where

$$n_{L1}=(-)^{J-1/2}[(2J+1)/4\pi]^{1/2}C(JJL;1/2,1/2). \tag{14}$$

Note that $n_{L1}=0$ for L even, and that only components of $\vec{P}_{\mathrm{Tr}}$ contribute in (13). For the odd moments of $\vec{P}_{\mathrm{Tr}}$, we find[8]

$$\beta n_{L1}t_L^M=-i\sum_m\langle P_mY_L^{M-m}\rangle C(1LL;mM-m). \tag{15}$$

The right-hand side of (15) is a sum of averages of the components of the vector $\vec{P}\times\hat{k}$ weighted with even functions of $\vec{k}$. (Therefore, in the absence of final-state interactions, it vanishes if time-reversal invariance holds.) Since for L odd,

$$[L(L+1)]^{1/2} n_{L1} = (2J+1) n_{L0}, \tag{16}$$

every $t_L{}^M$ (with L odd) appreciably different from zero yields a possible measurement of the quantities $\gamma(2J+1)$ and $\beta(2J+1)$. If parity is conserved in (1), $\beta=0$, $\gamma=\pm 1$, and these are measurements of J and l. If (9′) and (15) are different from zero, their measured values combined with (4) yield J (and α, β, γ).

Owing to symmetries in the production process, some of the $t_L{}^M$ may vanish identically. For example, if the Y^* are produced in a parity-conserving reaction

$$A+B=Y^*+D, \tag{17}$$

where the incident and target particles are unpolarized and one sums over the spin states of D particles,[9] the state of the Y^* (in their rest frame) is invariant against rotations of 180° about the normal to the production plane. Consequently if the normal to the production plane is the z axis, $t_L{}^M=0$ for M odd [since $t_L{}^M \to (-)^M t_L{}^M$ under a rotation of 180° about the z axis]. Similarly one may show that, for Y^* produced along the beam direction, the $t_L{}^M$ with L odd vanish identically.

For given J, the $t_L{}^M$ satisfy certain inequalities. The density matrix ρ for the Y^* state may be written as

$$\rho = (2J+1)^{-1} \sum_{L,M} (2L+1) t_L{}^{M*} T_L{}^M, \tag{18}$$

where the matrix $T_L{}^M$ is formed from the components of the spin operator $\vec{S}$ as the spherical harmonic $Y_L{}^M$ is formed from the components of a unit vector. The normalization is given by the relation

$$\mathrm{Tr}\left(T_{L'}{}^{M'} T_L{}^{M\dagger}\right) = [(2J+1)/(2L+1)]\delta_{LL'}\delta_{MM'}. \tag{19}$$

In a representation in which $T_1{}^0$ is diagonal, the matrix elements are Clebsch-Gordan coefficients; viz.,

$$(T_L{}^M)_{mm'} = C(JLJ; m'M). \tag{20}$$

Using (19), one sees that[10]

$$t_L{}^M = \mathrm{Tr}\rho T_L{}^M. \tag{21}$$

To obtain the range of these parameters, one may form the Hermitian matrices

$$R_L{}^M = (T_L{}^M + T_L{}^{M\dagger})/2 \text{ and } I_L{}^M = (T_L{}^M - T_L{}^{M\dagger})/2i$$

and obtain the relations

$$|\mathrm{Re}\, t_L{}^M| \le \text{largest eigenvalue of } R_L{}^M,$$

$$|\mathrm{Im}\, t_L{}^M| \le \text{largest eigenvalue of } I_L{}^M. \tag{22}$$

That all $t_L{}^M$ cannot simultaneously reach their upper bounds (22) may be seen from the restriction $\mathrm{Tr}(\rho^2) \le \mathrm{Tr}\rho$ which yields

$$\sum_{L,M} (2L+1)|t_L{}^M|^2 \le 2J+1. \tag{23}$$

A lower bound on the sum in (23) for Y^* states which are incoherent mixtures of Q pure states is given by a generalization of the Eberhard-Good theorem (see Capps[1]) $Q\,\mathrm{Tr}(\rho^2) \ge \mathrm{Tr}\rho$; this gives

$$Q \sum_{L,M} (2L+1)|t_L{}^M|^2 \ge 2J+1. \tag{24}$$

We are grateful to Professor H. K. Ticho and Professor P. Schlein for bringing this problem to our attention, and to H. H. Joos for enlightening and encouraging discussions.

---

*Work supported in part by the National Science Foundation.

[1]Assuming $J \le 3/2$, R. H. Capps, Phys. Rev. 122, 929 (1961), and R. Gatto and H. P. Stapp, Phys. Rev. 121, 1553 (1961), showed that J and l could be measured in parity-conserving decays. Our work reformulates and generalizes their results.

[2]James W. Cronin and Oliver E. Overseth, Phys. Rev. 129, 1795 (1963).

[3]See, for example, U. Fano, Rev. Mod. Phys. 29, 74 (1957); M. I. Shirokov, Zh. Eksperim. i Teor. Fiz. 32, 1022 (1957) [translation: Soviet Phys.—JETP 5, 835 (1957)]. These parameters are often called statistical tensors.

[4]See, e.g., A. R. Edmonds, Angular Momentum in Quantum Mechanics (Princeton University Press, Princeton, New Jersey, 1957).

[5]Equation (A1) of T. D. Lee and C. N. Yang, Phys. Rev. 109, 1755 (1958), combined with our (18), yields (9).

[6]Loyal Durand, III, Leon F. Landovitz, and Jack

Leitner, Phys. Rev. 112, 273 (1958).

[7]Except when specified otherwise, components of vectors and tensors refer to any right-handed set of Cartesian coordinate axes chosen without reference to the Y^* decay.

[8]Details of the derivation are available in a University of California, Los Angeles, California, preprint (unpublished). Using helicity states for the decay products, these results are obtained using standard methods [see, e.g., reference 3 and M. Jacob and G. C. Wick, Ann. Phys. (N.Y.) 7, 404 (1959)]. We thank S. Berman for pointing this out.

[9]If the Y^* state is formed from all Y^* produced with momentum $\vec{u}'$ and $\vec{u}$ is the incident momentum, the t_L^M are polynomials in the components of $\hat{u}$ and $\hat{u}'$: see, e.g., H. H. Joos, Forstchr. Physik 10, 65 (1962).

[10]Using (20) and (21), one may evaluate the t_L^M when ρ is known; for an example, see R. K. Adair, Phys. Rev. 100, 1540 (1955). In the Adair analysis, only the t_L^M with $M=0$ and L even are different from zero. If parity is violated in the decay, a unique function $I\vec{P}\cdot\vec{k}$ is obtained for each J [see Eq. (12')].

UPPER LIMITS FOR COUPLING CONSTANTS IN QUANTUM FIELD THEORY

B. V. Geshkenbein and B. L. Ioffe
Institute of Theoretical and Experimental Physics, Moscow, U.S.S.R.
(Received 11 March 1963)

In quantum field theory it is usually considered that the magnitudes of the coupling constants can assume any value irrespective of the particle mass. In the case of a bound state in the nonrelativistic theory, however, Heisenberg[1] showed that the coupling constant is expressed through the constant in the asymptotics of the wave function of this state. Subsequently in a number of reports[2-8] it was shown that the physical (renormalized) coupling constants are bounded above by definite boundaries which depend on the particle mass. In these reports, however, the considerations either had resort to models or[5] were based on the assumption that the interaction of the elementary particles is characterized by an effective radius, which does not increase with the increase of the coupling constant. We will show here that the restrictions on the magnitudes of the coupling constants at any given mass follow from the general principles of quantum field theory with no additional assumption. (A detailed report appears elsewhere.[9])

Our assumption will be based on the representation of Green's function in the form of the Lehman-Källén[10] expansion; i.e., we will consider that all the conditions under which this expansion takes place are fulfilled.

We will first consider a case of interaction of three boson fields with zero spin a, b, and c, and we will obtain the restriction on the magnitude of the coupling constant g^2 of these three fields. We will consider that particles a, b, and c are stable, $m_a < m_b < m_c$; we will assume also that b and c are the closest particles (with regard to total mass) to particle a. Based on the Lehman-Källén expansion of the Green's function of boson a in our previous work,[7] we obtain the following inequality restricting the possible magnitude of the renormalized coupling constant g^2:

$$\frac{1}{4\pi}\frac{g^2}{(m_b+m_c)^2}\Phi < 1,$$

$$\Phi = 2(m_b+m_c)^2\int_{(m_b+m_c)^2}^{\infty}\frac{|\Gamma(\kappa^2)|^2}{(\kappa^2-m_a^2)^2}\frac{q(\kappa^2)}{\kappa}d\kappa^2. \quad (1)$$

Here $\Gamma(\kappa^2)\equiv\Gamma(\kappa^2, m_b^2, m_c^2)$ is the vertex part for a, b, c particle interaction; $q(\kappa^2)=(2\kappa)^{-1}[\kappa^2-(m_b+m_c)^2]^{1/2}[\kappa^2-(m_b-m_c)^2]^{1/2}$ is the momentum of b and c particles in the center of mass. In order to obtain from inequality (1) definite restriction for g^2, the magnitude of Φ at given masses m_a, m_b, m_c must be bounded below.

Let us find the minimum in the class of functions $\Gamma(\kappa^2)$ having the following characteristics:

(I) $\Gamma(\kappa^2)$ is a holomorphic function of κ^2 in the complex κ^2 plane with a cut along the real axis, extending from point $\kappa^2=(m_b+m_c)^2$ to infinity. To the left of the point $\kappa^2=(m_b+m_c)^2$ on the real axis, $\Gamma(\kappa^2)$ is real.

(II) The rate of increase of $\Gamma(\kappa^2)$ as κ^2 tends to ∞ is not more rapid than an exponential increase.

(III) At the point $\kappa^2=m_a^2$, $\Gamma(m_a^2)=1$.

We assume that $\Gamma(\kappa^2)$ has no poles in the complex plane. In principle, the poles of $\Gamma(\kappa^2)$ could be situated on the real axis in the interval $m_a^2 < \kappa^2 < (m_b+m_c)^2$ at point κ^2 at which the Green's function $D(\kappa^2)$ becomes zero, and $\Gamma(\kappa^2)D(\kappa^2) \to$ const when $\kappa^2 \to \kappa_n^2$. [The latter case follows from, for example, Schwinger's equation for the