the resonance multiplet. Thus it appears that experimental resonances do not necessarily arise directly from the poles of a symmetry multiplet but may arise instead from the shadows of these poles.

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†NATO Fellow.

[1]R. J. Oakes and C. N. Yang, Phys. Rev. Letters 11, 174 (1963).

[2]P. V. Landshoff, Nuovo Cimento 28, 123 (1963).

[3]A detailed account of these arguments will be given in a future paper.

[4]R. J. Eden, Proc. Roy. Soc. (London) A210, 388 (1952); 217, 390 (1953).

[5]We are indebted to P. V. Landshoff for discussions on the possibility of false resonances in connection with production thresholds.

[6]Since this Letter was first submitted, we have learned of related work by R. H. Dalitz and G. Rajasekharan, reported at the Siena Conference on Elementary Particles, Siena, Italy, 1963 (unpublished). We are indebted to Professor Dalitz for information about their approach to the Oakes-Yang problem.

QUANTUM ELECTRODYNAMICS*

Kenneth Johnson
Physics Department and Laboratory for Nuclear Science,
Massachusetts Institute of Technology, Cambridge, Massachusetts

and

Marshall Baker† and Raymond S. Willey
Physics Department, University of Washington, Seattle, Washington

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The unrenormalized Schwinger-Dyson equations[1,2] for the Green's functions of ordinary quantum electrodynamics have been examined in a systematic nonperturbative approximation scheme. We have obtained the following results. We can obtain finite solutions of the equations without infinite renormalizations if and only if the mechanical mass of the particle is taken to be zero. This still allows a finite physical mass for the particle. The asymptotic behavior of the finite, unrenormalized electron and photon propagators is found to be

$$1/S(p) \to \gamma p + O(p\beta), \tag{1}$$

as $p \to \infty$, where $\beta = -1 + (1 - 3\alpha_0/\pi)^{1/2}$, and

$$1/D(k^2) \to k^2\{1 + O[(k^2)\beta']\}, \tag{2}$$

as $k^2 \to \infty$, where $\beta' = -\alpha_0/3\pi$ and where α_0 is the bare electron coupling constant. The second form holds only for small α_0. This behavior implies that all renormalization constants are finite. In outline, these results are obtained as follows.

If the unrenormalized Green's functions actually exist, then the following statements can be made. The electron Green's function $S(p)$ has the spectral representation

$$1/S(p) = \gamma p + m_0 - \int dK\, r(K)/(\gamma p + K), \tag{3}$$

where the integral over the mass spectrum converges; and therefore, as $p \to \infty$,

$$1/S(p) \to \gamma p + m_0 \tag{4}$$

for any reasonable $r(K)$. Secondly, because of Ward's identity,

$$k^\mu \Gamma_\mu(p+k, p) = S^{-1}(p+k) - S^{-1}(p), \tag{5}$$

we have for fixed k by differentiating (5) with respect to k as $p \to \infty$

$$\Gamma_\mu(p+k, p) \to \gamma_\mu \tag{6}$$

[actually faster than S approaches its asymptotic value because of the difference which appears in (5)]. Finally the photon Green's function has the spectral form

$$1/D(k^2) = k^2 + k^2 \int d\lambda^2\, s(\lambda^2)/(\lambda^2 + k^2), \tag{7}$$

so

$$D(k^2) \to 1/k^2, \tag{8}$$

as $k^2 \to \infty$ for any reasonable $s(\lambda^2)$. We again emphasize that all of these statements refer to unrenormalized Green's functions, none of which exists in perturbation theory. In ordinary language, the first statement implies that δm and Z_2 $(= Z_1)$ are finite. The final statement implies that Z_3 is finite.

The systematic approximation scheme used by

us yields results at every stage consistent with these properties of the unrenormalized Green's functions if and only if m_0, the mechanical mass, is zero. Thus, from our point of view, quantum electrodynamics makes sense only if the electron mass is totally dynamical.

The unrenormalized equation for the electron Green's function is

$$1/S(p) = \gamma p + m_0 + ie_0^2 \int [(dp')/(2\pi)^4] \times D_{\alpha\beta}(p-p')\gamma^\alpha S(p')\Gamma^\beta(p',p), \quad (9)$$

where $D_{\alpha\beta}(k) = \pi_{\alpha\beta}D(k)$ and $\pi_{\alpha\beta} = g_{\alpha\beta} + k_\alpha\lambda_\beta + k_\beta\lambda_\alpha \cdot \lambda_\alpha(k)$ is an arbitrary vector function of k which fixes the gauge. If one could solve the full theory exactly, it would make no difference for the physical content of the theory which gauge was used. Also, if one could solve the unrenormalized Green's function equations with perturbation theory, it would make no difference which gauge was used. Any nonperturbative approximation will, in general, depend upon the choice of gauge. For a particular approximation scheme, there may be a gauge in which that approximation is best. We have found an approximation method and a gauge $\lambda_\alpha(k) = -\frac{1}{2}k_\alpha/k^2$ (the "Landau" gauge) which yield the high-energy behavior of the exact theory at the first stage. The approximation method described below does not yield results for physical quantities which are independent of this choice. However, this lack of strict gauge invariance affects only the low-energy contributions which can be treated with the usual methods. The approximation method is, nevertheless, gauge invariant in the restricted sense that Ward's identity (5) is maintained at each stage. Our procedure is as follows: Assume that in (9) we can replace Γ_β by its asymptotic value γ_β because in virtue of the difference in (5), Γ_β should approach its asymptotic value more rapidly than S. Furthermore, we use the high-k limit (8) for $D(k^2)$. Thus, we obtain the equation[3]

$$1/S(p) \sim \gamma p + m_0 + ie_0^2 \int [(dp')/(2\pi)^4] \times D_{\alpha\beta}{}^0(p-p')\gamma^\alpha S(p')\gamma^\beta, \quad (10)$$

where $D_{\alpha\beta}{}^0(k) = (g_{\alpha\beta} - k_\alpha k_\beta/k^2)/k^2$. It can easily be shown analytically that there are no finite solutions to (10) in any covariant gauge except the Landau gauge. The unique feature of this gauge is the ultraviolet finiteness of Z_2 in the lowest order of perturbation theory. However, in the Landau gauge we find finite solutions of (10) only if $m_0 = 0$. The asymptotic behavior of the solution depends only on the asymptotic behavior of the integrand, and it is given by (1). Since we obtain this result only when $m_0 = 0$, it must be possible to obtain the total mass from the self-energy by the equation

$$m = -\int dK\, r(K)/(K-m), \quad (11)$$

where since (1) is equivalent to

$$r(K) \to K^\beta, \quad (12)$$

the integral (11) converges. In perturbation theory r remains constant for large K and the integral (11) is logarithmically divergent. Since the physical mass appears only as a scale, the self-energy must be proportional to m so that (11) reduces to an equation for α_0 of the form

$$m = mF(\alpha_0). \quad (13)$$

It can be shown analytically that (13) is satisfied in the limit $\alpha_0 \to 0$.

Although for finite values of p we do not expect (10) to be quantitatively reliable, it is interesting to regard it as an approximate equation for $S(p)$ valid for all values of p. In order to avoid an infrared divergence, it is necessary to give the photon a small mass λ. Equation (10) was solved using an electronic computer for $\lambda = 0.1m$. Using the solution to (10) obtained by the computer, it appears as if (13) is approximately satisfied within a few percent for $0 < \alpha_0 \lesssim 0.3$.

For $\alpha_0 = 0.3$ the solution of the equation (10) for $r(k)$ remains close to its perturbation value for $K \lesssim 10m$, at which point it joins smoothly to its damped asymptotic form (12).

A systematic approximation scheme has been developed in which Eq. (10) is the first step. The first nontrivial approximation for Γ is

$$\Gamma_\mu(p+k,p) = \gamma_\mu - ie_0^2 \int [(dp')/(2\pi)^4] D_{\alpha\beta}(p-p') \times \gamma^\alpha S(p'+k)\Gamma_\mu(p'+k,p')S(p')\gamma^\beta, \quad (14)$$

where S is the solution to (10). It is easy to show that (5) is satisfied. Therefore the asymptotic behavior of Γ_β obtained from (14) is consistent with the original assumption of replacing Γ_β by γ_β in (9). It must still be proved that the corrections to Γ_β given by (14) are such that they

do not influence our original approximation (9) for S in the asymptotic region. We can compute the asymptotic corrections to D using the so-determined S and Γ. We find that they are finite, and hence the original assumption that $D(k^2) \to 1/k^2$ is self-consistent. It has been shown further that these corrections do not influence our original approximation for S asymptotically.

The expression used to calculate the first correction to $D(k^2)$ is

$$1/D(k^2) = k^2[1 + \rho(k^2)], \tag{15}$$

where

$$\rho(k^2)(k^2 g^{\mu\nu} - k^\mu k^\nu) = -ie_0^2 \int [(dp)/(2\pi)^4] \mathrm{tr}\gamma^\mu [S(p + \tfrac{1}{2}k)\Gamma^\nu(p + \tfrac{1}{2}k, p - \tfrac{1}{2}k) \times S(p - \tfrac{1}{2}k) - S(p)\Gamma^\nu(p,p)S(p)], \tag{16}$$

and where the subtraction of the second term results from using the properly gauge-invariant form of the current. In perturbation theory, (16) diverges logarithmically. We find, using the integral equation (14) for Γ, that the leading asymptotic terms in $S(p + \frac{1}{2}k)\Gamma^\nu S(p - \frac{1}{2}k)$ have the same behavior as in perturbation theory with the exception of the terms which give a contribution to ρ according to (16). These are vanishingly small in comparison. To correctly compute them, it is necessary to also include in a self-consistent way the corrections to D given by (15) and (16) where they multiply the dominant terms in $S\Gamma S$. We then find that the asymptotic behavior of the relevant amplitudes is such that

$$s(\lambda^2) \to \mathrm{const}(1/\lambda^2)^{\alpha_0/3\pi}, \tag{17}$$

which contrasts with the perturbation result $s(\lambda^2) \to$ const. The form of the exponent is valid for small α_0. Such an asymptotic behavior for s implies a finite Z_3 and an asymptotic behavior for D which is consistent with the original assumption used to derive (9).

Quantum electrodynamics with no mechanical mass but finite physical mass represents a spontaneous breakdown of γ_5 invariance. Hence one might be concerned that there may be zero-mass pseudoscalar particles in such a theory. However, we believe that this does not necessarily occur.[4] The muon-electron mass difference, for example, may also arise from a spontaneous breakdown of the original symmetry in the Lagrangian which includes both fields coupled to the electromagnetic field.[3]

The fact that we have shown that quantum electrodynamics is a finite theory according to our method makes it extremely attractive to try to make use of a similar method of coupling of all Fermi fields to neutral vector-meson fields. The property which distinguishes these theories from all other field theories is that they are the only ones which make sense as local relativistic theories. This is at least the case when they are treated according to the above scheme.

In conclusion, we find that quantum electrodynamics may be regarded as a perfectly consistent theory. The usual divergences from our point of view arise from an unjustified use of perturbation theory. On the other hand, renormalized perturbation theory for finite quantities is modified only in that integrals which are convergent in that theory are made slightly more convergent. Only the asymptotic dependence on energy is modified. Detailed calculations must be done to determine quantitatively these effects.

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†Alfred P. Sloan Research Fellow. Present address: Departamento de Fisica, Universidad de Buenos Aires, Buenos Aires, Argentina.

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