ELECTROMAGNETIC FORM FACTOR OF THE NEUTRINOS*

J. Bernstein
Physics Department, New York University, New York, New York

and

T. D. Lee†
Physics Department, Columbia University, New York, New York
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The neutrino ν_l ($l=e$ or μ) is known[1] to be a particle of zero charge. However, through its weak interactions with other charged particles, the neutrino can acquire an electromagnetic current distribution. In this note, we assume a zero-mass two-component theory for the neutrino and the existence of an intermediate boson $W^\pm$. The weak-interaction Lagrangean between $W^\pm$ and the leptons is given by

$$-ig\varphi_\lambda^*[\psi_l^\dagger\gamma_4\gamma_\lambda(1+\gamma_5)\psi_{\nu l}]+\text{conjugate term}, \quad (1)$$

where φ_λ, ψ_l, and $\psi_{\nu l}$ are, respectively, the field operators for W^+, l^-, and ν_l. In a two-component theory, the matrix element of the electromagnetic current operator J_μ evaluated between states of a single neutrino depends only on a single scalar function:

$$\langle\nu'|J_\lambda|\nu\rangle=i[u'^\dagger\gamma_4\gamma_\lambda(1+\gamma_5)u]F(q^2), \quad (2)$$

where ν and ν' refer to the initial and final four-momenta of the neutrino, u and u' are the corresponding spinors that satisfy the free neutrino equation, and $q^2=(\nu-\nu')^2$.

We shall calculate $F(q^2)$ by considering the sum of all Feynman graphs which are of arbitrary order in the fine structure constant α but only first order in g^2. Some examples are shown in Fig. 1. In order to render these graphs finite, it is necessary to introduce a cutoff Λ in momentum space. The limit $\Lambda\to\infty$ is to be taken only after the summation of all these graphs. A convenient gauge-invariant way to introduce such a cutoff is the so-called ξ-limiting formalism[2,3] where $\Lambda=\xi^{-1/2}m_W$, and m_W is the mass of $W^\pm$. In this formalism, the Ward identities are satisfied[2] for a finite cutoff. It follows that to every order in α the sum J_λ of all such Feynman graphs satisfies the operator equation

$$q_\lambda J_\lambda=0, \quad (3)$$

which is valid[4] whether or not the initial and final spinors u and u' satisfy the Dirac equation with the correct mass. Differentiating (3) with respect to q_μ and then setting $q=0$, we obtain the well-known condition

$$F(q^2)=0, \text{ at } q^2=0. \quad (4)$$

When $\Lambda\to\infty$, each of the Feynman graphs in Fig. 1 diverges. The degree of divergence depends sensitively on the gyromagnetic ratio $(1+\kappa)$ of the W. It has been pointed out[2] that the case $\kappa\neq0$ is far more singular[5] than $\kappa=0$. As we shall see in the remark (2) below, this singular behavior for $\kappa\neq0$ seems to lead to a physically unacceptable result for the charge radius of the neutrino.

We consider, therefore, the less singular case $\kappa=0$, so that apart from radiative corrections, the magnetic moment of the W is (spin)$\times(\frac{1}{2}e/m_W)$. To discuss the behavior of Feynman graphs as $\Lambda\to\infty$, we expand $F(q^2)$ as a power series in q^2:

$$F(q^2)=\sum_1^\infty(n!)^{-1}(q^2)^nF_n(0), \quad (5)$$

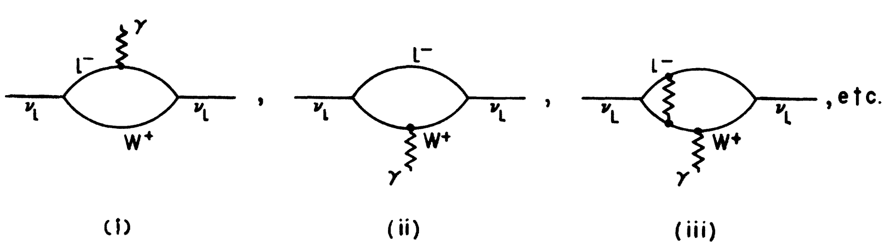

FIG. 1. Examples of Feynman graphs for the charge distribution of ν_l ($l=e$ or μ).

where each of the derivatives

$$F_n(0)=[\partial^n F/\partial(q^2)^n]_{q^2=0}$$

can, in turn, be expanded as a power series in α, provided Λ is finite. The contributions to $F_n(0)$ given by the two lowest order graphs, (i) and (ii) in Fig. 1, remain finite as $\Lambda\to\infty$ so long as $n\neq 1$. To obtain $F_1(0)$, it is necessary to include all the higher order Feynman graphs. The most singular part $[F_1(0)]_S$ is found to be

$$[F_1(0)]_S=-(16\pi^2 m_W^2)^{-1}g^2e[\tfrac{5}{3}\ln(\Lambda/m_W)^2+\textstyle\sum_1^\infty a_n(\alpha\Lambda^2/m_W^2)^n], \quad (6)$$

where we keep, to each power of α, only the coefficient of the most divergent term in the power series expansion of $F_1(0)$. The a_n are numbers independent of either α or Λ. By the same type of argument as used in reference 2, we shall assume that the entire sum $[F_1(0)]_S$ exists for $\Lambda=\infty$, although each term in the power series expansion diverges. Equation (6) can be rewritten as

$$[F_1(0)]_S=-(16\pi^2 m_W^2)^{-1}g^2e[-\tfrac{5}{3}\ln\alpha+G(x)],$$

where $x=(\alpha\Lambda^2/m_W^2)$, and for small x the function $G(x)$ can be represented by its series expansion

$$G(x)=\tfrac{5}{3}\ln x+\textstyle\sum_1^\infty a_n x^n.$$

Carrying out the limit $\Lambda\to\infty$ and keeping α fixed, we find[6]

$$[F_1(0)]_S=-(16\pi^2 m_W^2)^{-1}g^2e[-\tfrac{5}{3}\ln\alpha+G(\infty)], \quad (7)$$

where $G(\infty)$ is independent of α. Mathematically, this means that as $\alpha\to 0$, $F_1(0)\to\frac{5}{3}(16\pi^2 m_W^2)^{-1}g^2e\times\ln\alpha$.

Similar considerations can also be applied to the sum of the less singular terms in the power series expansion of $F_1(0)$. For example, the two lowest order graphs (i) and (ii) in Fig. 1 both give finite contributions to $F_1(0)$, in addition to the $g^2e\ln(\Lambda^2/m_W^2)$ term in (6). Furthermore, the finite contribution due to graph (i) contains a term proportional to $g^2e\ln(m_W/m_l)^2$, which is a result of the degeneracy[7] among intermediate states as the mass of the charged lepton $m_l\to 0$. Actually, the magnitude of $\ln(m_W/m_l)^2$ is quite comparable to that of $\ln\alpha$. The constant remainder $G(\infty)$ in (7) is basically determined by divergences connected with virtual W propagators and W-photon interactions. Consequently, $G(\infty)$ does not contain a term proportional to $g^2e\ln(m_W/m_l)^2$. In the following, we evaluate the two lowest order graphs (i) and (ii) exactly. The effects of higher order graphs are included only so far as they convert the $\frac{5}{3}\ln(\Lambda^2/m_W^2)$ term in (6) to the $-\frac{5}{3}\ln\alpha$ term in (7). The result for the complete function $F(q^2)$ is given by

$$F(q^2)=-g^2e(16\pi^2 m_W^2)^{-1}q^2f(q^2)\approx -e(10^{-18}\text{ cm})^2q^2f(q^2),$$

where $(e^2/4\pi)=\alpha$,

$$f(q^2)=\tfrac{5}{3}[\ln(137)+\tfrac{19}{15}]-\left(\frac{m_W^2}{b^2}\right)\left[\frac{5}{3}+\left(\frac{m_l}{m_W}\right)^2-\left(\frac{m_l^2}{m_W^2-m_l^2}\right)\ln\left(\frac{m_W}{m_l}\right)^2\right]-\frac{1}{3}\left[\frac{2m_W^2}{b^2}+5\right]\left(\frac{c}{b}\right)\ln\left(\frac{c+b}{c-b}\right)$$
$$+\int_0^1 dx\left\{-\left(\frac{2m_W^2}{ab}\right)I\ln\left(\frac{a+bx}{a-bx}\right)+\left(\frac{m_W^2a'}{b^3}\right)I'\ln\left(\frac{a'+bx}{a'-bx}\right)\right\}, \quad (8)$$

where

$$b=(\tfrac{1}{4}q^2)^{1/2},\quad c=(m_W^2+b^2)^{1/2},$$

$$a=[b^2x^2+(m_l^2-m_W^2)x+m_W^2]^{1/2},$$

$$a'=[b^2x^2+(m_W^2-m_l^2)x+m_l^2]^{1/2},$$

$$I=1-\tfrac{1}{4}x\left[2-\left(\frac{m_l}{m_W}\right)^2\right]-\left(\frac{a^2}{4b^2}\right)\left[2+\left(\frac{m_l}{m_W}\right)^2\right],$$

and

$$I'=1+\tfrac{1}{2}\left(\frac{m_l}{m_W}\right)^2+\left(\frac{1}{a'}\right)^2\left[\tfrac{1}{2}(m_W^2-m_l^2)-\left(\frac{m_l^2b^2}{m_W^2}\right)\right].$$

The values for $f(q^2)$ can be obtained by numerical integration. The results for ν_e and ν_μ are given in Figs. 2 and 3 where the mass m_W is chosen

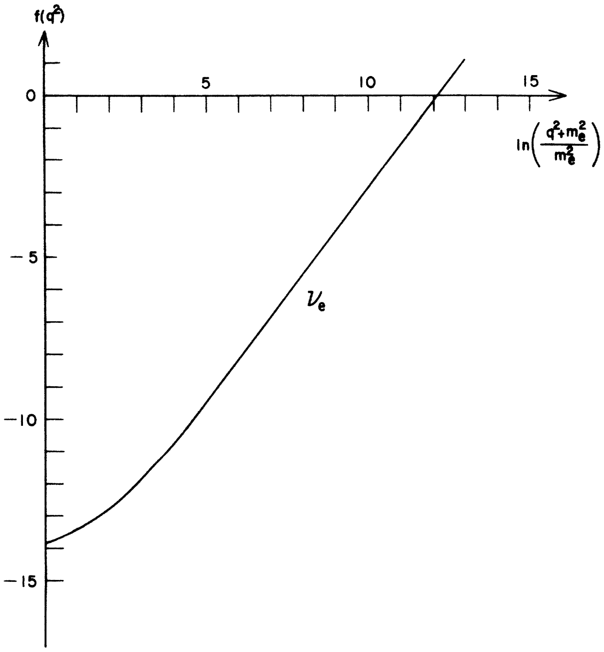

FIG. 2. Values of $f(q^2)$ for ν_e at low q^2. The form factor $F(q^2)=-g^2e(16\pi^2 m_W^{\;2})^{-1}q^2 f(q^2)$. Note that $[f(q^2)]_{\nu_e} \cong -13.9$ at $q^2=0$ and $[f(q^2)]=0$ at $q^2 \cong 4m_\mu^{\;2}$. The mass of W is arbitrarily chosen to be $9m_\mu^{\;2}$.

arbitrarily to be $9m_\mu$.

If we neglect both $(m_l/m_W)^2$ and $(\frac{1}{4}q^2/m_W^{\;2})$ as compared to 1, but keep all orders in $(q^2/m_l^{\;2})$, Eq. (8) becomes simply

$$f(q^2) \cong \tfrac{5}{3}[\ln(137) - \tfrac{38}{15}] - \tfrac{4}{3}\ln\left(\frac{m_W}{m_l}\right)^2 + \tfrac{16}{3}\left(\frac{m_l^{\;2}}{q^2}\right) + \tfrac{4}{3}\left(1-\frac{2m_l^{\;2}}{q^2}\right)\left[1+\left(\frac{4m_l^{\;2}}{q^2}\right)\right]^{1/2} \times \ln\left\{\frac{1+[1+(4m_l^{\;2}/q^2)]^{1/2}}{-1+[1+(4m_l^{\;2}/q^2)]^{1/2}}\right\}. \qquad (9)$$

As $q^2 \to 0$,

$$f(0) \cong \tfrac{5}{3}\ln(137) - \tfrac{4}{3}\ln(m_W/m_l)^2 - 2, \qquad (10)$$

where the first term, $\frac{5}{3}\ln(137)$, can be regarded as that due to the positive charge distribution (W^+) and the second term, $-\frac{4}{3}\ln(m_W/m_l)^2$, as that due to the negative charge distribution (l^-). Because of the difference in the masses between e^- and μ^-, the form factors $f(q^2)$ belonging to ν_e and ν_μ for $q^2 \lesssim 4m_\mu^{\;2}$ are very different. For very large values of q^2, we can neglect m_l in (8); the two form factors then become the same.

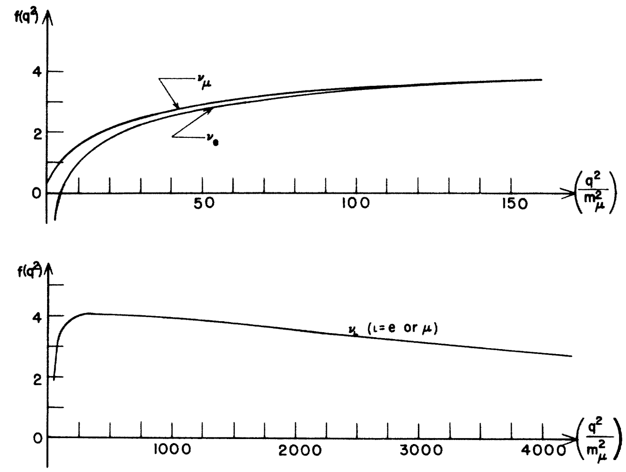

FIG. 3. Values of $f(q^2)$ for ν_e and ν_μ at intermediate-q^2 and at high-q^2 regions with $m_W=9m_\mu$. Note that $[f(q^2)]_{\nu_\mu} \cong 0.30$, at $q^2=0$.

In the expression (8) above, we include only those higher order effects that give the $-\frac{5}{3}\ln\alpha$ term for $f(0)$. Therefore, we expect $f(0)$ to be accurate to the order of either $|\ln\alpha|^{-1}$ or $|\ln(m_W/m_l)^2|^{-1}$. On the other hand, the shape of the curve $[f(q^2)-f(0)]$ is of a much higher accuracy. Similar considerations lead one to expect Eq. (8) to give the correct values for the slope $(\partial f/\partial q^2)$ at $q^2=0$ to the accuracy of $|\alpha\ln\alpha|$, and for all the other derivatives $\partial^n f/\partial(q^2)^n$ to the accuracy of α.

Throughout our considerations of higher order graphs we have implicitly assumed that $(q^2/m_W^{\;2})$ is not larger than $(1/\alpha)$. Other correction terms may become important if this does not hold.

In considering possible measurements of $f(q^2)$ one may take as an example the reaction $\nu_l + p \to \nu_l + p$. The differential cross section for this reaction through the virtual emission and absorption of photons can be written as

$$d\sigma = [(4\pi)^{-1}(137)^{-1}m_W^{\;-2}(q^2+m_W^{\;2})f]^2 d\sigma_v, \qquad (11)$$

where $d\sigma_v$ is identical in form with the corresponding expression for the reaction $\nu_l + n \to l^- + p$ at the same q^2, provided m_l and the axial-vector form factor are set equal to zero and the vector form factors are replaced by the corresponding electromagnetic form factors of the proton. The numerical values of $d\sigma_v$ have been given in the literature.[8] The rate for $(\nu_l + p \to \nu_l + p)$ is, thus, expected to be $\approx 10^{-4}$ times that for $(\nu_l + n \to l^- + p)$. This small cross section makes the experimental detection of $f(q^2)$ difficult, though not impossible.

We hope the marked difference between $f(q^2)$ for ν_e and ν_μ at small q^2 may stimulate some ingenious use of not only the high-energy ν_μ beam from the multiple BeV range accelerators, but also the large, available low-energy ν_e fluxes from either nuclear piles or other radioactive sources. In the latter case, one might study nuclear excitations that could be induced by the charge distribution of the neutrino. An alternative method is to use such neutrinos to study the reaction $\nu_e + e^- \to \nu_e + e^-$ or $\bar{\nu}_e + e^- \to \bar{\nu}_e + e^-$. The lowest order weak process is through an exchange of $W^\pm$. In addition to the radiative corrections, there is now a further correction of a magnitude $\sim\alpha$ due to the interference term between the W-exchange process and the γ-exchange process.

Remarks. – (1) To give a simple illustration of the limiting method $\Lambda \to \infty$ which yields the dominant $\ln\alpha$ term from a divergent power series expansion such as (6), we may consider a simple integral

$$H(\Lambda) = \int_0^\Lambda (x+1)^{-1} h(\alpha x^2) dx, \tag{12}$$

where $h(z)$ is an arbitrary function of z that has a power series expansion at $z=0$, and as $z \to \infty$, $h \to 0$ sufficiently fast so that $H(\infty)$ exists. Expanding $h(\alpha x^2)$ as a power series in α, the most singular part $[H(\Lambda)]_S$ of the power series expansion for $H(\Lambda)$ is found to be of the form

$$[H(\Lambda)]_S = \tfrac{1}{2} h(0) \ln\Lambda^2 + \sum_1^\infty a_n (\alpha\Lambda^2)^n,$$

where a_n are numbers. Using the same argument as that given above, we find, in the limit $\Lambda \to \infty$, $[H(\infty)]_S = -\tfrac{1}{2} h(0) \ln\alpha + \text{constant}$. In this simple example, the same asymptotic expansion can be directly obtained from (12), which, at $\Lambda \to \infty$, may be written as

$$H(\infty) = \int_0^\infty (y + \alpha^{1/2})^{-1} h(y^2) dy$$

by substituting $y^2 = \alpha x^2$. It is easy to see that as $\alpha \to 0$, $H(\infty) \to -\tfrac{1}{2} h(0) \ln\alpha$, which confirms the result obtained by the limiting process. Furthermore, by considering the sums of the less singular terms in the power series expansion of $H(\Lambda)$, one can obtain systematically the complete asymptotic expansion of $H(\infty)$ for small α.

(2) For $\kappa \neq 0$, the power series in α is much more divergent. For example, instead of (6), we find

$$[F_1(0)]_S = g^2 (\kappa e \Lambda^2) \sum_0^\infty b_n (\kappa^2 \alpha \Lambda^4)^n.$$

Substituting $x = \kappa^2 \alpha \Lambda^4$ and carrying out the limit $\Lambda \to \infty$, keeping κ and α finite, we find the rather strange result $[F_1(0)] = \text{constant} \times g^2$ so that this contribution to the charge radius is independent of e. For a spin-$\frac{1}{2}$ charged particle without strong interactions, it is well known that a theory with an intrinsic anomalous magnetic moment (which corresponds to the $\kappa \neq 0$ case) leads to a much more singular power series expansion. Such a theory does not correspond to any known particle. The above result suggests to us that this may also be true for spin-1 charged particles.

(3) In deriving (8), the ξ-limiting formalism is used in which both W propagator and vertex function depend on ξ explicitly. They are modified, together, because of gauge invariance. In the limit $\xi \to 0$ (i.e., $\Lambda \to \infty$), these modifications in the vertex functions are found to give finite contributions to (8). It may be emphasized that (8) is actually independent of the explicit form in which Λ is introduced, provided gauge invariance is strictly maintained.

(4) The higher order weak interactions are not included in the above derivation. Equation (8) is expected to hold provided $(\Lambda/m_W)^2 \gg \alpha^{-1}$. On the other hand, the higher order weak interactions become important only if $(\Lambda/m_W)^2$ is greater than g^{-2} which is a completely different region. If we envisage the physical result as being reachable by gradually increasing the cutoff, the higher order electromagnetic processes become important at much smaller cutoff than the higher order weak interactions; therefore, they should be summed first. Mathematically, this corresponds to the introduction of a different cutoff Λ' for the weak interactions. The limit $\Lambda \to \infty$ for the electromagnetic interaction should be taken before the limit $\Lambda' \to \infty$. It is possible that the second limit $\Lambda' \to \infty$ may bring some additional changes. However, because of the necessary modifications of the W propagator by its electromagnetic interaction, the magnitude of these higher order weak interactions is expected to be quite different (and perhaps much smaller) than would be indicated by the use of the free W propagators (without electromagnetic interactions).[9]

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[1]See J. Bernstein, M. Ruderman, and G. Feinberg, Phys. Rev. 132, 1227 (1963), for a discussion of the present experimental limits on the charge, mass, etc., of the neutrinos.

[2]T. D. Lee, Phys. Rev. 128, 899 (1962). Throughout the present paper, all unexplained notations are the same as those in this reference.

[3]T. D. Lee and C. N. Yang, Phys. Rev. 128, 885 (1962).

[4]This is to be contrasted with a similar relation for a charged particle, which is valid only if both the initial and the final wave functions satisfy the correct Dirac equation.

[5]This can be seen by considering a three-point W-W-γ vertex $V_\lambda(p,p')$ where both the initial and final W lines are virtual. If $\kappa \neq 0$, then both of these W propagators can assume their singular form $[p\tilde{p}/(p^2+m^2)]$ at large p. However, $\tilde{p}'V_\lambda(p',p)p=0$ if $\kappa=0$. Therefore, for $\kappa=0$ only one of these two propagators can take on the singular form. The relevant expansion parameter for $\kappa=0$ is $(\alpha\Lambda^2)$, and that for $\kappa\neq 0$ is $(\alpha\kappa^2\Lambda^4)$.

[6]The same considerations have been used in reference 2 to calculate the radiative corrections to the electromagnetic properties of $W^\pm$ for $\kappa\neq 0$. Similar calculations for $\kappa=0$ will be given in a separate publication.

[7]For a general discussion of degenerate systems and mass singularities, see T. D. Lee and M. Nauenberg, Phys. Rev. (to be published).

[8]See G. Danby et al., Phys. Rev. Letters 10, 260 (1963).

[9]See G. Feinberg and A. Pais, Phys. Rev. 131, 2724 (1963).

RESONANCE MULTIPLETS AND BROKEN SYMMETRY*

R. J. Eden and J. R. Taylor†
Clare College, Cambridge, England

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In this Letter we describe some experimental and theoretical consequences of the fact that resonance poles may appear on more than one Riemann sheet of the S matrix. We have in mind particularly those resonances which belong to multiplets in the unitary symmetry scheme SU_3. The main points we wish to note are these:

(i) In the S matrix for two or more coupled channels a resonance may appear as a pole on more than one of the unphysical Riemann sheets.

(ii) In general, only one of these poles (the dominant pole) will be near to the physical region. Under certain circumstances, however, two poles may be comparably important, in which case interference between the poles could have an observable effect on the position and shape of the resonance.

(iii) In the case of resonance multiplets of the approximate symmetry scheme SU_3, the presence of several poles on different sheets representing a single resonance allows the members of each multiplet to move into coincidence, when full symmetry is established, without any of the difficulties discussed by Oakes and Yang.[1]

The circumstances under which a resonance or bound state leads to poles on more than one Riemann sheet can be examined in terms of analyticity and unitarity of the S matrix.[2] This examination will not be given here, but we wish to note that in addition to the usual assumptions of S-matrix theory, our work requires analyticity in the coupling between different channels.[3] For the sake of brevity our discussion is given, instead, in terms of a simple resonance model based on a sum of self-energy diagrams.[4] We should, however, emphasize that our results are more general than the particular model considered. In particular, the fact that the model is S wave is not essential, and identical results hold for any angular momentum state provided similar requirements of analyticity are satisfied.

We consider a single unstable particle of mass M which has two decay modes, both into two identical particles of mass m_r ($r=1,2$) with $2m_1<2m_2<M$. The resonance model gives, as scattering amplitude for two m_1 particles,

$$A_{11}(s)=ig_1^2/[s-M^2+\sum(a_r+ib_r)], \qquad (1)$$

where g_r is a coupling constant and