UNITARY SYMMETRY AND DECAY OF η MESON*

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The purpose of this note is to investigate various decay modes of the η meson in connection with the unitary symmetry model[1] where $(\pi, \eta, K, \bar{K})$ form a unitary octet. The main decay modes of the η meson are expected to be

$$\eta \to \gamma + \gamma, \tag{1}$$

$$\eta \to \pi^+ + \pi^- + \pi^0, \text{ or } \pi^0 + \pi^0 + \pi^0, \tag{2}$$

$$\eta \to \pi^+ + \pi^- + \gamma, \tag{3}$$

$$\eta \to \pi^0 + \gamma + \gamma. \tag{4}$$

Experimentally, the absolute decay rate of η is not known yet, but the branching ratios among various decay modes are found to be[2]

$$\frac{\Gamma(\eta \to \gamma + \gamma)}{\Gamma(\eta \to \pi^+ + \pi^- + \pi^0)} = 1.9 \pm 0.13, \tag{5}$$

$$\frac{\Gamma(\eta \to \pi^+ + \pi^- + \gamma)}{\Gamma(\eta \to \pi^+ + \pi^- + \pi^0)} = 0.26 \pm 0.08, \tag{6}$$

$$\frac{\Gamma(\eta \to \pi^0 + \pi^0 + \pi^0)}{\Gamma(\eta \to \pi^+ + \pi^- + \pi^0)} = 1.59 \pm 0.42, \tag{7}$$

$$\frac{\Gamma(\eta \to \text{all neutrals})}{\Gamma(\eta \to \pi^+ + \pi^- + \pi^0)} = 3.0 \pm 0.5. \tag{8}$$

The decay mode Eq. (4) has not been well established although it is expected to be small. In what follows, we compute these decay rates in lowest order with respect to the electromagnetic interactions, so that the decays Eqs. (1), (2), and (4) would be of the order α^2 while the decay Eq. (3) is of the order α (where α is the fine structure constant).

First of all, we shall consider the decay mode Eq. (1). Unitary symmetry tells us the decay matrix elements satisfy[3,4]

$$M(\eta \to \gamma + \gamma) = (1/\sqrt{3}) M(\pi^0 \to \gamma + \gamma). \tag{9}$$

When we take account of the kinematical factors, this gives

$$\frac{\Gamma(\eta \to \gamma + \gamma)}{\Gamma(\pi^0 \to \gamma + \gamma)} = \frac{1}{3}\left[\frac{m(\eta)}{m(\pi^0)}\right]^3.$$

Using the known lifetime[5] of the π^0 meson, we estimate

$$\Gamma(\eta \to \gamma + \gamma) = 142 \pm 28 \text{ eV}. \tag{10}$$

We may remark that Barrett and Barton[6] calculated the decay rate of $\eta \to 2\gamma$ to be 50 eV by using the Goldberger-Treiman method together with the unitary symmetry model; however, if one inserts the new experimental lifetime of π^0 into their expression, one finds a value close to Eq. (10).

Second, let us investigate the decay mode Eq. (2). In this case, we use the one-pion-pole diagram[7] for its evaluation. The interaction Hamiltonians which enter are

$$H_1 = \gamma \eta \pi^0 \tag{11}$$

$$H_2 = 4\pi\chi(\pi_\alpha \pi_\alpha)^2. \tag{12}$$

To estimate γ we use the unitary symmetry model and compute the transition mass between η and π^0 to get

$$\gamma = (1/\sqrt{3})\{m^2(K^+) - m^2(K^0) + m^2(\pi^0) - m^2(\pi^+)\}$$
$$\approx -[54 \text{ MeV}]^2. \tag{13}$$

This relation is an analog of the Coleman-Glashow relation.[8] Using the value $\lambda = -0.18 \pm 0.05$ of Hamilton et al.,[9] we find

$$\Gamma(\eta \to \pi^+ + \pi^- + \pi^0) = 147^{+90}_{-70} \text{ eV}. \tag{14}$$

As for $\eta \to \pi^0 + \pi^0 + \pi^0$, we have[10]

$$\frac{\Gamma(\eta \to \pi^0 + \pi^0 + \pi^0)}{\Gamma(\eta \to \pi^+ + \pi^- + \pi^0)} = \tfrac{3}{2}. \tag{15}$$

We see that our estimate for $\Gamma(\eta \to 2\gamma)$ and $\Gamma(\eta \to 3\pi)$ can be consistent with the experimental ratio, Eq. (5).

Third, let us study the decay mode Eq. (3). Now, we assume that the effective decay Hamiltonian responsible for $\eta \to \pi^+ + \pi^- + \gamma$ is given by a local minimal interaction. Consider the general form of the photon-three spinless boson vertex: The only local minimal interaction[11] which is compatible with both unitary symmetry and charge conjugation invariance is given by

$$H_3=\frac{-i}{\sqrt{2}}e\Lambda\epsilon_{\mu\nu\rho\sigma}A_\mu\{[\partial_\nu f_a{}^1\partial_\rho f_b{}^a\partial_\sigma f_1{}^b-\partial_\nu f_1{}^a\partial_\rho f_a{}^b\partial_\sigma f_b{}^1]-\tfrac{1}{3}[\partial_\nu f_b{}^a\partial_\rho f_c{}^b\partial_\sigma f_a{}^c-\partial_\nu f_a{}^b\partial_\rho f_b{}^c\partial_\sigma f_c{}^a]\} \tag{16}$$

in the notation given previously.[12] Latin and Greek indices refer to the unitary symmetry and to Lorentz spaces, respectively, and repeated indices are summed over 1, 2, and 3 and 1, 2, 3, and 4, respectively. If we express $f_a{}^b$ in terms of $\pi, \eta, K, \bar{K}$ as in reference 12, we get

$$H_3=e\Lambda\epsilon_{\mu\nu\rho\sigma}A_\mu\{i\partial_\nu\pi_1\partial_\rho\pi_2\partial_\sigma\pi_3+(i/\sqrt{3})\partial_\nu\eta\partial_\rho\pi_1\partial_\sigma\pi_2-(1/\sqrt{3})\partial_\nu\eta[\partial_\rho\bar{K}_+\partial_\sigma K_+-3\partial_\rho\bar{K}_0\partial_\sigma K_0]-\partial_\nu\pi_3[\partial_\rho\bar{K}_+\partial_\sigma K_++\partial_\rho\bar{K}_0\partial_\sigma K_0]\}. \tag{17}$$

This gives

$$\Gamma(\eta\rightarrow\pi^++\pi^-+\gamma)=168[\Lambda m^3(\pi)]^2\text{ eV}, \tag{18}$$

where we have used $e^2=1/137$. The experimental and theoretical value for Λ are not well established. A simple perturbation calculation (nucleon loop)[13] gives

$$\Lambda_{\text{pert}}\approx-0.7\,m^{-3}(\pi). \tag{19}$$

The experiment on single-pion photoproduction[14] indicates Λ to be about $1.5\,\Lambda_{\text{pert}}$ or more, while the experiment on double-pion photoproduction[15] as well as the electromagnetic moment of the deuteron[16] seems compatible with $\Lambda\approx\Lambda_{\text{pert}}$. If we adopt the smallest of these, the perturbation value Eq. (19), we estimate

$$\Gamma(\eta\rightarrow\pi^++\pi^-+\gamma)\approx82\text{ eV}, \tag{20a}$$

which is large compared to experiment. On the other hand, Kawarabayashi and Sato[17] estimate Λ from the decays of ω, ρ, and π^0 mesons to be $\Lambda\sim0.3\,\Lambda_{\text{pert}}$. If we adopt this value, then we get

$$\Gamma(\eta\rightarrow\pi^++\pi^-+\gamma)\sim7\text{ eV}, \tag{20b}$$

which is rather small. At any rate, to be consistent with the experimental ratio Eq. (6), our estimate requires Λ to be about one half of the perturbation value. It may be worth while to remark that Gell-Mann, Sharp, and Wagner[18] and Brown and Singer[19] made estimates of $\Gamma(\eta\rightarrow\pi^++\pi^-+\gamma)$ based upon some specific models. They obtained the following values, respectively:

$$\frac{\Gamma(\eta\rightarrow\pi^++\pi^-+\gamma)}{\Gamma(\eta\rightarrow\gamma+\gamma)}\sim\tfrac{1}{4},\ \tfrac{1}{8}. \tag{21}$$

These values are close to the experimental ratio, but we must keep in mind that there is no strong theoretical justification to prefer their model. It may be also worth while to note that if we do not assume a local minimal electromagnetic interaction, we cannot relate the interaction Hamiltonian of $\eta\rightarrow\pi^+\pi^-\gamma$ to the one of $\gamma\rightarrow3\pi$ as in Eq. (17) (see reference 11). The same is true in the case of Gell-Mann et al. in which one should include the intermediate state of a ω^0 and a φ^0 for the decay $\eta\rightarrow2\gamma$ (assuming φ^0 to be singlet), so that one cannot obtain the simple relation like Eq. (21).

Finally, let us estimate the decay rate into the mode Eq. (4) by assuming the following effective decay Hamiltonian:

$$H_4=\xi F_{\mu\nu}F_{\mu\nu}\eta\pi^0. \tag{22}$$

The value of the coupling constant ξ may be evaluated as follows: When we take the contraction of the photon lines in Eq. (22), we get an interaction between η and π^0. If this is taken to be the source of the interaction H_1 of Eq. (11), we obtain

$$\gamma=\xi\langle F_{\mu\nu}F_{\mu\nu}\rangle_0. \tag{23}$$

If we use the Feynman cut-off factor for the vacuum expectation value of the photon propagator by taking the cut-off at the nucleon mass, we compute

$$\Gamma(\eta\rightarrow\pi^0+\gamma+\gamma)\sim8\text{ eV}. \tag{24}$$

The decay rate of this mode is not known yet, but the above estimate is at least consistent with known total rate to the neutral modes. However, we have to bear in mind that our estimate depends critically upon the assumed cut-off value, and as a result, we may easily be wrong by a factor as much as 10 in our estimate. Also, we may remark that some forms of R-conjugation invariance[20] may enhance the decay $\eta\rightarrow\pi^0+\gamma+\gamma$ relatively compared to the other modes.

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DETERMINATION OF SPIN AND DECAY PARAMETERS OF FERMION STATES*

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In this Letter we discuss the decay process

$$Y^* \to Y + \pi, \tag{1}$$

where Y^* has spin J and decays into a particle with spin 1/2 (Y) and one with spin zero (π). We show how J and the amplitudes for the parity states $l = J \pm 1/2$ may be measured if the transverse and longitudinal polarization of Y are appreciable.[1] We denote these amplitudes by

$$a = \text{amplitude for } l = J - 1/2,$$
$$b = \text{amplitude for } l = J + 1/2, \tag{2}$$

defined so that the lifetime of Y^* is given by

$$\tau^{-1} = 2\pi\rho_E(|a|^2 + |b|^2);$$

and in accordance with the notation[2] for $J = 1/2$, we define the parameters

$$\gamma = (|a|^2 - |b|^2)/(|a|^2 + |b|^2),$$
$$\alpha = 2\,\mathrm{Re}\,ab^*/(|a|^2 + |b|^2),$$
$$\beta = 2\,\mathrm{Im}\,ab^*/(|a|^2 + |b|^2). \tag{3}$$

If parity is conserved in (1), $\alpha = \beta = 0$ and $\gamma = \pm 1$.