SU(4) ASSIGNMENTS FOR THE VECTOR RESONANCES*

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Two possible interpretations of the 730-MeV $K\pi$ resonance[1] (κ) have been proposed. Nambu and Sakurai[2] considered a scheme into which the κ could naturally be inserted should its spin and parity be 0^+. Minami considered a 1^- spin and parity assignment for the κ and discussed a scheme in which there are two octuplets of vector mesons.[3] The purpose of this note is to consider an alternative scheme, based on the 1^- assignment, in which all the vector resonances are interpreted as forming one multiplet corresponding to the adjoint (regular) representation of SU(4).

(I) In the case of the stable baryons, the stable scalar bosons, and to a lesser extent, the baryon resonances, it seems fairly well established that the dominant symmetry, if any, is that of SU(3). For the vector resonances, however, the problem of φ-ω mixing complicates the phenomenological assignment of a symmetry. From a theoretical point of view, of the known particles the vector resonances seem the nearest to forming a self-contained (bootstrap) system.[4] It therefore seems possible that this system should exhibit a different, and even greater, symmetry. Here, we consider the possibility of that symmetry being that of the group SU(4).

(II) Since SU(4) is of rank three, the interactions of the vector mesons, in our scheme, conserve three additive quantum numbers. We take them to be hypercharge Y and the third component of isotopic spin T_3, as in the SU(3) case, and a third quantity, which we call supercharge, Z. Arguments have been given that require the assignment of the vector resonances to the adjoint representation, which, for SU(4), is 15-dimensional.[4]

We construct the 15 particles by the method of Gell-Mann.[5] We consider four fictitious basic fields: p and n with $Z=0$, $Y=1$, $T_3=\pm1/2$; Λ with $Z=Y=T_3=0$; and X with $Z=1$, $Y=T_3=0$. With this assignment for X, the members of the 15-dimensional representation are

$$\begin{aligned}
&\rho^+ = p\bar{n},\\
&\rho^0 = (p\bar{p} - n\bar{n})/\sqrt{2},\\
&\rho^- = n\bar{p},\\
&A = (\Lambda\bar{\Lambda} - X\bar{X})/\sqrt{2},\\
&B = (p\bar{p} + n\bar{n} - 2\Lambda\bar{\Lambda})/\sqrt{6},\\
&K^{*+} = p\bar{\Lambda}, \quad \bar{K}^{*-} = \Lambda\bar{p},\\
&K^{*0} = n\bar{\Lambda}, \quad \bar{K}^{*0} = \Lambda\bar{n},\\
&\kappa^+ = p\bar{X}, \quad \bar{\kappa}^- = X\bar{p},\\
&\kappa^0 = n\bar{X}, \quad \bar{\kappa}^0 = X\bar{n},\\
&\lambda = \Lambda\bar{X}, \quad \bar{\lambda} = X\bar{\Lambda}.
\end{aligned} \tag{1}$$

The quantum numbers of the resonances are easily inferred from those of p, n, Λ, and X. The ω and φ are linear combinations of A and B; in contrast with the SU(3) case, both appear in the adjoint representation. The only undetected resonances in the scheme are λ and $\bar{\lambda}$, $Z=\pm1$ singlets with charge and strangeness zero. It should be noted that the three $Z=+1$ ($Z=-1$) particles form the SU(3)-3 ($\bar{3}$) multiplet, which in the eightfold way cannot be physically realized with integral strangeness.[5] In the SU(4) case, however, introduction of the quantity Z allows the existence of the 3 and $\bar{3}$, with $Z=\pm1$, as we have seen by explicit construction.

(III) We expect the new quantity supercharge to be a property only of the vector mesons; i.e., the baryons and pseudoscalar mesons have $Z=0$. We therefore expect small cross sections for producing κ and λ singly, compared with those for the other vector mesons, since these processes do not conserve supercharge. The observed cross section for κ production is about one-tenth that for K^* production.[1] Since κ and λ decays do not conserve supercharge either, we expect smaller widths for them. Again this is in agreement with the experimental result for κ, which is[1]

$$\Gamma_\kappa < \Gamma_{K^*}/3.$$

These properties make difficult the problem of observing the λ. In addition, because λ decay does not conserve supercharge, we are not able to estimate the branching ratio between $\lambda \to 2\pi$ and $\lambda \to 3\pi$. In spite of nonconservation of supercharge in both production and decay processes, it is not necessarily fruitless to introduce the quantity. Presumably, it is conserved in the interactions of the vector mesons among themselves and is thus meaningful to the extent that the vector mesons form a self-consistent (bootstrap) system.[4]

(IV) From Eq. (1) we may derive mass formulas analogous to the Gell-Mann-Okubo (GMO) results in the SU(3) case. Assuming p and n to be degenerate, we have

$$2(\varphi+\omega+\rho)=3(\kappa+K^*), \quad (2)$$

$$\lambda=\kappa+K^*-\rho, \quad (3)$$

$$\rho+3B=4K^*, \quad (4)$$

where we have used the particle symbol for the square of the particle mass. We note that Eq. (2) is independent of the amount of mixing of A and B necessary to produce ω and φ, while Eq. (4) is just the GMO formula for the SU(3)-8 contained in the SU(4)-15. The three mass relations are the result of neglecting three mass-splitting terms consistent with T_3, Y, and Z conservation occurring in the 20 and in the 84, in the decomposition of $15\times\overline{15}$, which is

$$15\times\overline{15}=84+45+\overline{45}+20+15_S+15_A+1.$$

Similarly, the GMO formula arises from neglecting a term in the 27, in the decomposition of $8\times\overline{8}$.[5,6]

Inserting the experimental values for the resonance energies in Eq. (2) gives a discrepancy of $11\pm3\%$ between the left- and right-hand sides.[7] Using Eq. (3), we obtain for the λ energy

$$M_\lambda=870\text{ MeV}, \quad (5)$$

while inserting Eq. (2) in Eq. (3) gives

$$M_\lambda=950\text{ MeV}. \quad (6)$$

If the κ should be found to be 1^-, a search in this area for the λ would seem reasonable.

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THREE-PION AND FOUR-PION DECAYS OF THE INTERMEDIATE BOSON*

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It has been observed[1] that the predominant decay of the hypothetical intermediate vector boson W of the weak interactions would be $W\to2\pi$ if the W mass is in the neighborhood of the ρ-meson mass (750 MeV). However, recent experiments in CERN[2] indicate that the mass of the W meson might be greater than 1.4 BeV, opening up the possibility of other pion modes being important. We estimate here two decay modes $W\to3\pi$ and $W\to4\pi$ assuming they are dominated by $W\to\rho+\pi$ and $\omega+\pi$, respectively. The branching ratios between these modes and $W\to e+\nu$ are tabulated.

(1) $W\to3\pi$.—If we assume the $\Delta I=1$ rule, this proceeds through the axial-vector current. If the W mass is large enough, this can go via the decay ρ and π, with subsequent decay of the ρ. The W-ρ-π vertex has been studied by Woo[3] using dispersion relations. The vertex part can be described by three form factors:

$$(0|(\Box+m_W^{\;2})W_\mu|\rho\pi)=\rho\cdot Kq_\mu A(s)+\rho_\mu B(s)+\rho\cdot KK_\mu C(s), \quad (1)$$

where

$$q_\mu=\tfrac{1}{2}(p_\rho-p_\pi),\quad K_\mu=\tfrac{1}{2}(p_\rho+p_\pi),$$