BACKWARD PEAK IN HIGH-ENERGY n-p ELASTIC SCATTERING*

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A sharp backward peak in n-p scattering has been observed by Palevsky et al.[1] at 2.04-BeV and 2.85-BeV lab energy. Two different explanations for this peak have been proposed: (i) Phillips[2] has suggested that the narrow peak is due to a strong and destructive interference between the pion term and a slowly varying background; (ii) Muzinich[3] has suggested that the peak is due to ρ exchange where the ρ meson behaves as a Regge pole. In Phillips' explanation the background is put in phenomenologically, while in Muzinich's explanation the pion contribution is disregarded. In our present approach, we consider both the ρ-meson exchange and the pion exchange in the t channel as well as in the u channel. We calculate the amplitudes due to these exchanges using second-order perturbation theory. However, for both ρ and π we introduce form factors in the amplitudes. We also consider the electric coupling constant of ρ meson with nucleon as a parameter to be determined by fitting with the experimental distribution. We shall show later that our approach, though perturbative, actually includes Muzinich's treatment of ρ as a Regge pole.

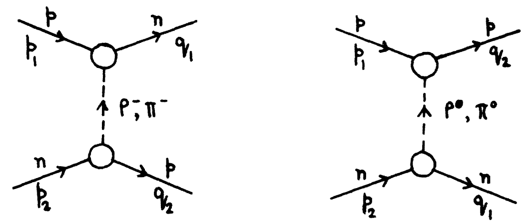

FIG. 1. Feynman diagrams for n-p scattering due to ρ and π exchanges in t and u channels.

The diagrams we are considering are shown in Fig. 1. The ρ-N interaction is taken as[4,5] $ig_e\bar{\psi}\gamma_\mu\vec{\tau}\psi\vec{\rho}^\mu + (g_m/4m)\bar{\psi}\sigma_{\mu\nu}\vec{\tau}\psi(\partial_\mu\vec{\rho}^\nu - \partial_\nu\vec{\rho}^\mu)$, and the π-N interaction as $ig_{\pi N}\bar{\psi}\gamma_5\vec{\tau}\psi\vec{\pi}$. The amplitudes can be written down using the rules for Feynman diagram.[6] To illustrate our calculation, we write down the ρ-exchange amplitude due to its electric coupling:

$$M_\rho^e = g_e^2\left[\frac{\bar{u}(q_1)\xi_{1'}^\dagger\gamma_\mu\tau_i\xi_1 u(p_1)\bar{u}(q_2)\xi_{2'}^\dagger\gamma^\mu\tau_i\xi_2 u(p_2)}{\Delta^2+m_\rho^2}F_\rho(\Delta^2) - \frac{\bar{u}(q_2)\xi_2^\dagger\gamma_\mu\tau_i\xi_1 u(p_1)\bar{u}(q_1)\xi_{1'}^\dagger\gamma^\mu\tau_i\xi_2 u(p_2)}{\bar{\Delta}^2+m_\rho^2}F_\rho(\bar{\Delta}^2)\right], \tag{1}$$

where ξ_1 and ξ_2 are the initial, and $\xi_{1'}$ and $\xi_{2'}$ are the final isotopic spin states; $F_\rho(\Delta^2)$ and $F_\rho(\bar{\Delta}^2)$ are the form factors; $t=-\Delta^2=-(q_1-p_1)^2$, $u=-\bar{\Delta}^2=-(q_2-p_1)^2$. The total amplitude M is related to the S matrix by

$$S_{fi}=\delta_{fi}-i(2\pi)^4\delta^4(p_1+p_2-q_1-q_2)\left(\frac{m^4}{p_{10}p_{20}q_{10}q_{20}}\right)^{1/2}M, \tag{2}$$

and to the c.m. differential cross section by

$$(d\sigma/d\Omega)_{\text{c.m.}}=(m^2/2\pi W)^2|M|^2. \tag{3}$$

M will be the sum of a number of terms like M_ρ^e.

As can be seen from Eq. (1), if $\xi_1=p$, $\xi_2=n$, $\xi_{1'}=n$, and $\xi_{2'}=p$, then the τ's in the first term in Eq. (1) give a factor 2 (using $\tau_i^{(1)}\tau_i^{(2)}=2\tau_-^{(1)}\tau_+^{(2)}+2\tau_+^{(1)}\tau_-^{(2)}+\tau_3^{(1)}\tau_3^{(2)}$), and those in the second term give a factor of -1. Thus, the relative sign of the two terms is positive. In the case of scattering of identical nucleons, there will not be any factor of two and the relative sign is negative.[7]

We have assumed $F_\rho(\Delta^2)=e^{-\Delta^2/\Omega}$ and $F_\rho(\bar{\Delta}^2)=\chi$, where Ω and χ are considered as adjustable parameters.[8] The electric coupling constant g_e has also been taken as an adjustable parameter. The ratio g_m/g_e is considered approximately known from the form-factor data. The contributions of the ρ meson to the form factors G_{eV} and

G_{MV} of Hand, Miller, and Wilson[9] are, respectively, $[fg_e/(m_\rho^2+q^2)+(q^2/4m^2)fg_m/(m_\rho^2+q^2)]$ and $[fg_e/(m_\rho^2+q^2)-fg_m/(m_\rho^2+q^2)]$, where f is the coupling constant of ρ^0 with the electromagnetic field.[10] From their result, we obtain $g_m/g_e=-5.71$. The results of our calculation are shown in Figs. 2(a) and 2(b) for lab energy 2.04 BeV. The parameters used by use are $g_e^2/4\pi=0.384$, $\Omega=2.50\mu^2$, $\chi=\frac{1}{3}$, and $g_m/g_e=-5.5$. We have taken the π-N coupling constant to be $f^2=(g_{\pi N}^2/4\pi)(4m^2/\mu^2)^{-1}=0.06$. For the pion form factor, we have used the empirical formula obtained by Ferrari and Selleri[11]

$$F_\pi(\delta^2)=0.72[1+(\delta^2+\mu^2)/4.73\mu^2]^{-1}+0.28 \qquad (\delta^2=\Delta^2,\bar{\Delta}^2).$$

We now want to show how Muzinich's treatment of ρ as a Regge pole is included in our approach. The differential cross section in the c.m. system obtained by Muzinich[3] can be written as

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{c.m.}}=[\gamma(s,t)]^2\frac{1}{s}\frac{[4m(T_L+m)]^2}{(\Delta^2+m_\rho^2)^2}, \tag{4}$$

where

$$\gamma(s,t)=\frac{b\sqrt{\pi}}{2t_0}\left[\frac{(2\alpha+1)\Gamma(\alpha+\frac{1}{2})(\Delta^2+m_l^2)}{\Gamma(\alpha+1)\cos\frac{1}{2}\pi\alpha}\right]e^{(\alpha-1)K};$$

T_L = lab incident energy, and $K=\ln[2(s-2m^2)/t_0]$. On the other hand, in our perturbative approach, the ρ exchange in t channel due to electric coupling gives the following differential cross section for small values of Δ^2:

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{c.m.}}=\left(\frac{g_e^2}{4\pi}\right)^2F_e^2(\Delta^2)\frac{1}{s}\frac{[4m(T_L+m)]^2}{(\Delta^2+m_\rho^2)^2}. \tag{5}$$

Comparing (4) and (5), we find that they represent

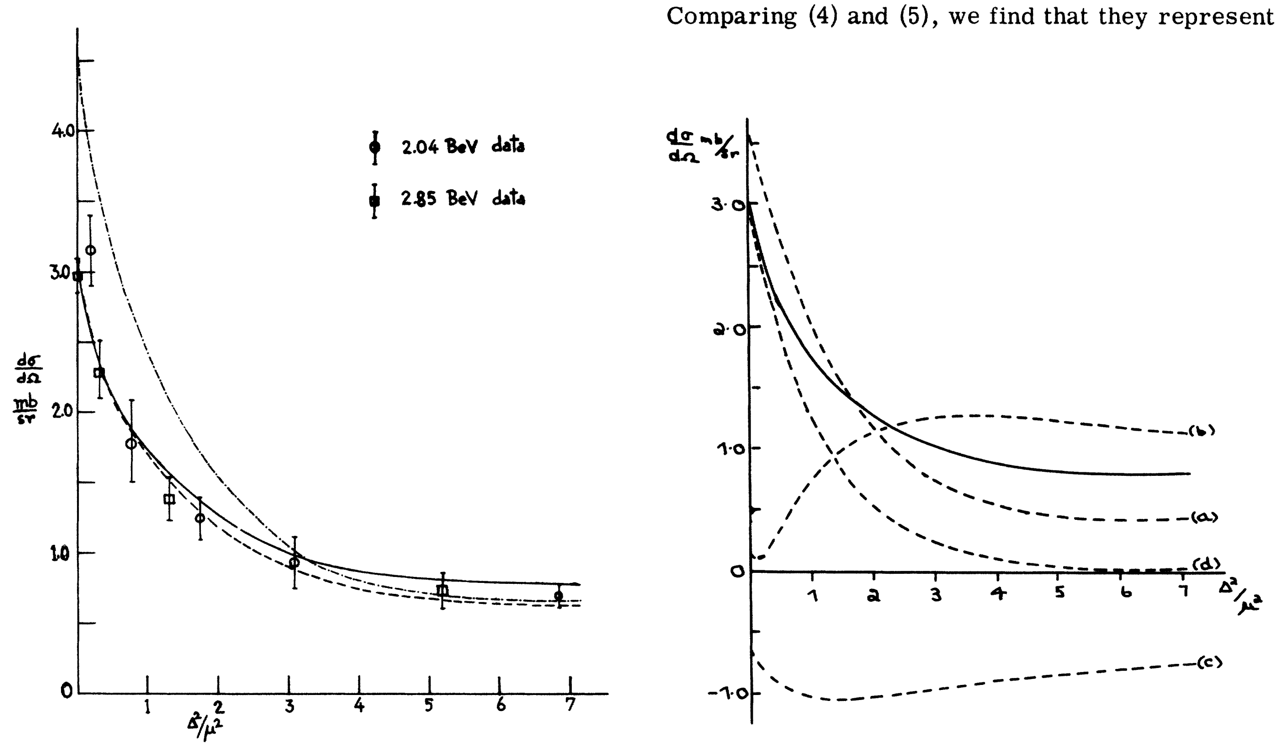

FIG. 2. (a) Continuous curve represents the calculated differential cross section for lab energy 2.04 BeV. Dashed curve represents the cross section for lab energy 2.85 BeV with ρ coupling constant changed; dot-dash curve represents the same distribution with ρ coupling constant unchanged. Experimental results are those of Palevsky et al. (reference 1). (b) Contributions of different terms to the calculated cross section (continuous curve) for lab energy 2.04 BeV. Curve (*a*) represents the contribution of the ρ-exchange amplitude alone; (*b*) represents the contribution of the π-exchange amplitude alone; and (*c*) represents the contribution of the ρ-π interference term. Continuous curve is the sum of (*a*), (*b*), and (*c*). Curve (*d*) represents the differential cross section due to ρ exchange in t channel with electric coupling.

the same angular distribution if we identify

$$g_e^2/4\pi = \gamma(s,0) \tag{6a}$$

and

$$F_\rho(\Delta^2) = \gamma(s,t)/\gamma(s,0). \tag{6b}$$

This identification is, however, significant. Equation (6a) predicts that if ρ behaves as a Regge pole, then its coupling constant is energy dependent.

We can verify this s dependence of the coupling constant using the 2.85-BeV data. With $g_e^2/4\pi = 0.317$ and the same form factors[12] as at $T_L = 2.04$ BeV, we obtain the angular distribution shown by the dashed curve in Fig. 2(a), for 2.85-BeV lab energy. On the other hand, if we use the coupling constant obtained at the lower energy 2.04 BeV, we get the angular distribution shown by the dot-dash curve in Fig. 2(a). As can be seen, the latter distribution gives too large a cross section at $\Delta^2 = 0$. An estimate of $\alpha(0)$ can be made in the following way. Equation (6a) gives

$$\frac{g_e^2(s')}{g_e^2(s)} = \frac{\gamma(s',0)}{\gamma(s,0)} = \exp\left\{\left[\alpha(0)-1\right]\ln\left(\frac{m+T_L'}{m+T_L}\right)\right\}. \tag{7}$$

Taking $T_L' = 2.85$ BeV, $T_L = 2.04$ BeV, $g_e^2(s')/4\pi = 0.317$, and $g_e^2(s)/4\pi = 0.384$, we find $\alpha(0) = 0.2$.

We can now draw the following conclusions:

(1) The ρ-nucleon coupling constant shows an s dependence,[13] thus indicating that ρ behaves as a Regge pole.[14]

(2) The narrow angular distribution of the n-p backward peak is due to a sharply decreasing form factor of the ρ meson exchanged in the t channel.

(3) ρ exchanged in t and u channels interferes strongly and destructively with π exchanged in u and t channels, respectively. The slowly decreasing tail of the angular distribution is due to large cancellation of the ρ and π contributions by their interference term.[15]

(4) The pion form factor is important. Without it the pion contribution at higher values of Δ^2 comes out too large.[16]

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[1]H. Palevsky, J. A. Moore, R. L. Sterns, H. R. Muether, R. J. Sutter, R. E. Chrien, A. P. Jain, and K. Otnes, Phys. Rev. Letters 9, 509 (1962).

[2]R. J. N. Phillips, Phys. Letters 4, 19 (1963).

[3]I. J. Muzinich, Phys. Rev. Letters 11, 88 (1963).

[4]An equivalent interaction which differs from this by a four-divergence is $ig_a\bar{\psi}\gamma_\mu\vec{\tau}\psi\vec{\rho}^\mu + (ig_b/2m)(\bar{\psi}\vec{\tau}\partial_\mu\psi - \partial_\mu\bar{\psi}\vec{\tau}\psi)\vec{\rho}^\mu$, where $g_a = g_e + g_m$ and $g_b = g_m$. This interaction is somewhat simpler to use in the evaluation of traces.

[5]We use $a\cdot b = \vec{a}\cdot\vec{b} - a_0b_0$; the index μ takes the values 1,2,3,0. Our γ_i ($i = 1,2,3$) is Hermitian, and γ_0 is anti-Hermitian. m = nucleon mass, μ = pion mass.

[6]The propagator for the vector meson, in our notation, is $-i(g_{\mu\nu} + k_\mu k_\nu/m_\rho^2)(k^2 + m_\rho^2)^{-1}$; see T. D. Lee and C. N. Yang, Phys. Rev. 128, 885 (1962). We find a mistake in the sign of the vector propagator given in the book of N. N. Bogoliubov and D. V. Shirkov, Introduction to the Theory of Quantized Fields (Interscience Publishers, Inc., New York, 1959), Eq. (14.29). This can be checked using the commutation relation given in their Eq. (11.25). The pion propagator, in our notation, is $-i(k^2+\mu^2)^{-1}$.

[7]We find some mistakes in the literature; the sign of the interference term for single pion exchange in n-p scattering, given in reference 11, should be negative. The sign of the interference term between ρ meson and pion in identical nucleon scattering given in reference 17 should be positive, while for n-p scattering, it is negative.

[8]The reason we consider $F_\rho(\bar{\Delta}^2)$ as a parameter and not $e^{-\bar{\Delta}^2/\Omega}$ is that, in our case, Δ^2 is small ($\lesssim 7\mu^2$), while $\bar{\Delta}^2$ is very large, $\sim 200\mu^2$; thus the simple phenomenological form that we assume for the unknown function $F_\rho(\delta^2)$ for small δ^2 ($=\Delta^2$) cannot be expected to hold for very large δ^2 ($=\bar{\Delta}^2$).

[9]L. N. Hand, D. G. Miller, and R. Wilson, Rev. Mod. Phys. 35, 335 (1963). Similar form-factor analysis has been done by J. S. Levinger, Nuovo Cimento 26, 813 (1962). From Levinger's second set, we find $g_m/g_e = -3.33$.

[10]Interaction of ρ^0 with the electromagnetic field is taken as $efA_\mu\rho^\mu$.

[11]E. Ferrari and F. Selleri, Nuovo Cimento 27, 1450 (1963).

[12]Equation (6b) indicates that $F_\rho(\Delta^2)$ is s dependent. However, for the change in s we are considering, the change in $F_\rho(\Delta^2)$ is expected to be small.

[13]It is worth remarking that the value of $g_e^2/4\pi$ obtained from low-energy scattering data is much larger than the values found by us here in the BeV region. Using the π-N charge-exchange scattering length and the width of ρ meson, we obtain $g_e^2/4\pi = 0.72$.

[14]Evidence of the Regge behavior of ρ has been previously obtained by G. von Dardel et al., Phys. Rev. Letters 8, 173 (1962).

[15]The interference term between the π and the ρ, both exchanged in the same channel, is always zero.

[16]Treating pion as a Regge pole gives rise to a form factor similar to that obtained by Ferrari and Selleri. See M. M. Islam (to be published).

[17]W. K. R. Watson, Nuovo Cimento 22, 183 (1961).