ROLE OF ρ IN HIGH-ENERGY n-p CHARGE EXCHANGE

R. J. N. Phillips
Atomic Energy Research Establishment, Harwell, England
(Received 26 August 1963)

A narrow forward peak was recently found[1] in n-p charge-exchange scattering at 2.04 and 2.85 GeV. An explanation was suggested[2] in terms of one-pion-exchange interference effects. Alternatively, Muzinich[3] has now shown that the peak may be fitted by a single dominant Regge pole associated with the ρ meson.

This note pursues the latter explanation further. Muzinich simply fitted the differential cross section at one energy, but his model also predicts the total cross-section difference $\sigma_T(pp) - \sigma_T(np)$, and the energy dependence of both; we find it does not fit all the data satisfactorily. Also this model, or any modified form in which the ρ trajectory plays a major role, requires the residue function to change sign between $t = 0$ and $t = m_\rho^2$, with physical consequences ($-t$ is the momentum transfer squared, m_ρ is the ρ-meson mass).

Assuming just the ρ Regge pole, n-p charge-exchange scattering is dominated at small t by the spin-averaged amplitude[3]:

$$a = \frac{1}{2}\left(\frac{\pi}{s}\right)^{1/2} \frac{(2\alpha+1)\Gamma(\alpha+\frac{1}{2})}{\Gamma(\alpha+1)}\beta(t) \times \left(\frac{s+2p_t^2}{p_t^2}\right)^{\alpha} \frac{1-\exp(-i\pi\alpha)}{\sin\pi\alpha}. \tag{1}$$

Here s is the total c.m. energy squared, $\alpha(t)$ is the trajectory, $\beta(t)$ is a residue function, Γ denotes the gamma function, and $p_t^2 = \frac{1}{4}t - m_N^2$. The differential c.m. cross section for small t is therefore $d\sigma/d\Omega = |a|^2$.

Muzinich factors out the threshold dependence of $\beta(t)$,

$$\beta(t) = b(t)[2p_t^2/m_\rho^2]^{\alpha(t)}, \tag{2}$$

and assumes that $b(t)$ is a constant, while $\alpha(t)$ is linear, in the region of interest, $0 \geq t \geq -0.1$ (GeV/c)^2. He can then fit $d\sigma/d\Omega$ with a wide choice of parameters, to wit: $\alpha(0) = 0.7$, 0.5, or 0.3 with slope $d\alpha/dt = 1.5$, 1.7, or 2.0 (GeV/c)$^{-2}$, respectively. But we can restrict $\alpha(0)$ for other reasons.

(i) Energy dependence of total cross sections.[4]—This has been discussed by many authors already. $\alpha(0)$ for the ρ trajectory is estimated to be near 0.3 from pion-nucleon data,[5] and near 0.4 from nucleon-nucleon data,[6,7] with considerable uncertainty.

(ii) Energy dependence of differential cross section.—Palevsky *et al.*[1] measured the ratio

$$\frac{(d\sigma/d\Omega)(t=0;\ 2.85\ \text{GeV})}{(d\sigma/d\Omega)(t=0;\ 2.04\ \text{GeV})} = 0.94 \pm 0.08, \tag{3}$$

for c.m. cross sections. For the present model this implies

$$\alpha(0) = 0.27 \pm 0.18, \tag{4}$$

which is consistent with the estimates in (i).

(iii) Magnitude of $\sigma_T(pp) - \sigma_T(np)$.—The isospin dependence of nucleon-nucleon scattering, plus the optical theorem, gives

$$\sigma_T(pp) - \sigma_T(np) = (4\pi/k)\,\mathrm{Im}\,\alpha(t=0), \tag{5}$$

$$= (4\pi/k)\cos\tfrac{1}{2}\pi\alpha[d\sigma/d\Omega(0)]^{1/2}, \tag{6}$$

where k is the relative momentum, and Eq. (6) follows from the usual assumption that $b(0)$ is real. Now at 2.85 GeV we have[1] $d\sigma/d\Omega(0) = 2.9 \pm 0.6$ mb and can estimate $\sigma_T(pp) - \sigma_T(np) = 4 \pm 2$ mb from the data in this region.[8,9] Hence,

$$\alpha(0) = 0.77 \pm 0.12, \tag{7}$$

which substantially disagrees with Eq. (4).

It must be admitted that the $\sigma_T(np)$ data are not all very accurate or consistent with one another. But if we assume a smooth energy variation (which the Regge-pole hypothesis implies), there is much less uncertainty; our estimate of $\sigma_T(pp) - \sigma_T(np)$ depends on such a smooth interpolation of many data.

Thus Muzinich's model as it stands seems too restrictive to fit all the data. Nevertheless, it is important to confirm this finding with more accurate measurements.

(iv) Comparison at $t = 0$ and $t = m_\rho^2$.—At $t = m_\rho^2$, the ρ-meson pole, the renormalized Born approximation is exact, and we have

$$a(t = m_\rho^2) = -[2s^{1/2}/(m_\rho^2 - t)]f_v^2/4\pi, \tag{8}$$

where $f_v^2/4\pi$ is the ρNN vector coupling constant, estimated[10] to be ≈ 2. Only the leading power of s is kept in Eq. (8); tensor coupling is therefore absent. Hence,

$$b(m_\rho^2) = -\tfrac{2}{3}(f_v^2/4\pi)m_\rho^2(d\alpha/dt)(m_\rho^2) \approx -1. \tag{9}$$

On the other hand, $b(0)$ must be positive to satisfy Eq. (5) and the cross-section data.[8] From the differential cross section,[1] we find the magnitude

$$b(0) \approx 1.5, \tag{10}$$

for the case $\alpha(0) = 0.3$. Any modified model in which the ρ trajectory plays a major part will also require $b(0) > 0$.

Hence, $b(t)$ cannot simply be a constant or vary exponentially. There are also wider implications. Consider the functions $\eta_1(t)$, $\eta_2(t)$, $\eta_\pi(t)$, etc., introduced by Gell-Mann[11] to factorize the residues of a Regge pole like ρ. The spin-averaged amplitudes of NN, πN, and $\pi\pi$ scattering have essentially the residues η_1^2, $\eta_1\eta_\pi$, and η_π^2; the function $b(t)$ of Eq. (2) corresponds in fact to $-\eta_1^2$. The residues are presumed to be analytic functions of t with cuts from the physical threshold at $t = 4m_\pi^2$ to $+\infty$; it is suggested[11] that the factor functions η_1, η_2, etc. are also analytic with at least the same cut. Now Muzinich's model requires $\eta_1^2(0) < 0$, while $\eta_1^2(m_\rho^2) > 0$, suggesting that $\eta_1(t)$ has an additional cut. If this cut begins in the range $0 < t \leq 4m_\pi^2$, η_1 evidently vanishes at the branch point; likewise all other factor functions whose products with η_1 are real in this range must have the same branch point and vanish there. If $\mathrm{Re}\,\eta_1^2$ changes sign in $4m_\pi^2 < t < m_\rho^2$, where η_1^2 and the other residues have imaginary parts, the situation is less clear-cut; there is no branch point on the real t axis, but if these imaginary parts are small the other conclusions hold approximately.

Such a situation has physical effects. For example, $\eta_\pi^2(0) < 0$, since $\eta_1\eta_\pi$ is real and $\eta_1^2 < 0$ here; also $\eta_\pi^2(t)$ is zero or small somewhere in the range $0 < t < m_\rho^2$, since the real part changes sign and the imaginary part is zero or small. Hence, for $\pi\pi$ scattering the ρ contribution in the s channel changes sign between the resonance at $s = m_\rho^2$ and $s = 0$, implying a damped or repulsive effect at low energy. The contribution in the t channel affects high-energy cross sections; if ρ gives the only significant isospin-dependent term, $\eta_\pi^2(0) < 0$ implies $\sigma_T(\pi^+\pi^+) > \sigma_T(\pi^+\pi^0) > \sigma_T(\pi^+\pi^-)$. The ρ term in the pion form factor depends on $\eta_\pi(t)$ and is zero or small somewhere in the interval $0 < t < m_\rho^2$. The same holds for other kinds of particle. There is yet no direct evidence on these questions, apart from NN scattering with which we began.

To summarize, this note makes essentially two observations. Firstly, Muzinich's model appears not to fit all the relevant data as it stands. Secondly, any such model in which the ρ Regge pole is important[12] needs the residue function to change sign; if the residues are analytic and factorable as usually supposed, this has consequences for other physical systems.

I am grateful to Dr. P. G. Burke and Dr. B. M. Udgaonkar for helpful comments.

---

[1]H. Palevsky, J. A. Moore, R. L. Stearns, H. R. Muether, R. J. Sutter, R. E. Chrien, A. P. Jain, and K. Otnes, Phys. Rev. Letters 9, 509 (1962).

[2]R. J. N. Phillips, Phys. Letters 4, 19 (1963).

[3]I. J. Muzinich, Phys. Rev. Letters 11, 88 (1963).

[4]B. M. Udgaonkar, Phys. Rev. Letters 8, 142 (1962).

[5]G. von Dardel, D. Dekkers, R. Mermod, M. Vivargent, G. Weber, and K. Winter, Phys. Rev. Letters 8, 173 (1962).

[6]V. I. Lendyel and J. Mathews, Phys. Letters 5, 286 (1963).

[7]D. H. Sharp and W. G. Wagner, Phys. Rev. 131, 2224 (1963).

[8]A. N. Diddens, E. Lillethun, G. Manning, A. E. Taylor, T. G. Walker, and A. M. Wetherell, Phys. Rev. Letters 9, 32 (1962).

[9]The estimate in reference 2 omitted the shadow correction.

[10]J. J. Sakurai, Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962 (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 176.

[11]M. Gell-Mann, Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962 (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 533.

[12]Reference 7 suggests that the ρ Regge pole may really be dwarfed by a branch-cut contribution.