Table II. A comparison of experimental values from our results and theoretical predictions of decay ratios of the η.

Decay ratio	Experimental value	Theoretical value	Authors	Basis of theoretical calculations
$\gamma\gamma/\pi^+\pi^-\pi^0$	1.3 ± 0.4	0.6 to 1.9	Barrett and Barton[a]	Unitary symmetry model[b, c] (the uncertainty is due to the uncertainty in the experimental value of the lifetime of the π^0).
$3\pi^0/\pi^+\pi^-\pi^0$	$\leq 1.7\pm0.6$[d]	<1.73	Feinberg and Pais[e]	Similarity of the final states for K and η decays into 3π.
		1.5 to 1.73	Wali[f] and Bég[g]	
$\gamma\gamma/\pi^+\pi^-\gamma$	5.0 ± 1.6	≈ 4	Gell-Mann[b, h, i]	Unitary symmetry model.
		8	Brown and Singer[j]	Unitary symmetry model.

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[d]The equality holds if the $\pi^0\gamma\gamma$ decay is small with respect to the decay into $3\pi^0$.
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[h]See reference 8.
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κ MESON [K^* (725)] AND THE STRANGENESS-CHANGING CURRENTS OF UNITARY SYMMETRY*

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We wish to examine, within the framework of unitary symmetry, the hypothesis that the vector mesons M and $\bar{M}$ (to be identified with the observed K^* meson of mass 885 MeV) are coupled to strangeness-changing currents that are conserved "as exactly as possible." It is pointed out that this hypothesis suggests the existence of $Y=\pm1$, $T=1/2$, and $J=0^+$ mesons (with no unitary partners) whose couplings to other strongly interacting particles vanish in the limit of exact unitary symmetry. The possible connection between the conjectured scalar meson and the experimentally observed κ meson (the K^* meson of 725 MeV) is discussed.

Some time ago, it was argued that there should exist one $Y=0$, $T=1$ vector meson and two $Y=0$, $T=0$ vector mesons coupled, respectively, to the exactly conserved currents of the strong interactions: isospin, hypercharge, and baryonic charge.[1,2] Subsequently, Gell-Mann[3] and Ne'eman[4,5] have shown that the vector mesons of reference 1 can be elegantly accommodated in a higher symmetry model called the "eightfold way" (the octet version of unitary symmetry) together with the $Y=\pm1$, $T=1/2$ vector mesons M and $\overline{M}$ coupled to strangeness-changing ($F^{(4,5,6,7)}$) currents which, like the isospin ($F^{(1,2,3)}$) and hypercharge ($F^{(8)}$) currents, are generated by the gauge transformations associated with SU(3). Various experiments carried out in the last two years have conclusively established the existence of all the nine ($=1+8$) vector mesons with the desired quantum numbers.[6]

Although unitary symmetry treats ρ, φ (or more precisely a linear combination[7] of φ and ω), M, and $\overline{M}$ on the same footing as members of one unitary family, in the real world there is an important distinction among them: The sources of ρ and φ are exactly conserved (with F-type couplings of the vector-meson octet to the baryon octet), whereas M and $\overline{M}$ are coupled to currents which would be strictly conserved only in the limit of exact unitary symmetry, i.e., when $m_K - m_\pi \to 0$, $m_\Lambda - m_N \to 0$, etc. We may naturally speculate on the following question: Can we construct a strangeness-changing vector current which is exactly conserved even in the presence of finite mass differences within unitary multiplets?

The solution to this problem may be found in the answer to a similar question raised in connection with the possible conservation of the axial vector current $J_{\mu+}^{(A)} = i\overline{N}\gamma_5\gamma_\mu\tau_+N$. Although $J_{\mu+}^{(A)}$ is not divergenceless because of the finite nucleon mass, it is possible to construct an axial vector vertex $\Gamma_\mu^{(A)}(p',p)$ between the real neutron and the proton state that satisfies the continuity equation[8,9]:

$$\Gamma_\mu^{(A)} = F(q^2)[i\gamma_5\gamma_\mu - 2m_N\gamma_5 q_\mu/q^2], \qquad (1)$$

where $q \equiv p'-p$. The presence of a pole at $q^2=0$ [provided $F(0)\neq 0$] implies the existence of a zero-mass pseudoscalar meson. By interpreting this meson as the $m_\pi^2 \to 0$ limit of the observed finite-mass pion, we have succeeded in deriving the famous Goldberger-Treiman[10] relation for the $\pi^\pm$ lifetime.[8,9]

Similarly, we can demonstrate that the exact conservation of the strangeness-changing vector current in the presence of a finite K-π mass difference would require the existence of a zero-mass $Y=1$, $T=1/2$ scalar meson. To see this, we first note that a vector current that satisfies the continuity equation can be constructed between the real K^+ and the real π^0 state as follows:

$$F(q^2)\left[(p_K+p_\pi)_\mu - \frac{(p_K-p_\pi)_\mu(m_K^2-m_\pi^2)}{(p_K-p_\pi)^2}\right]. \qquad (2)$$

As before, the second term in (2) implies the existence of a $Y=1$, $T=1/2$ scalar meson of zero mass if $F(0)\neq 0$. Since we know of no zero-mass meson of strangeness unity, we again interpret this ideal situation as the $m^2 \to 0$ limit of a $Y=1$, $T=1/2$ scalar meson of finite mass, which we call κ. For a slowly varying $F(q^2)$, we have, in place of (2),

$$(p_K+p_\pi)_\mu - \frac{(p_K-p_\pi)_\mu(m_K^2-m_\pi^2)}{(p_K-p_\pi)^2+m_\kappa^2}, \qquad (3)$$

which is completely identical to the structure of the $K_{\mu 3}$ form factor suggested by Bernstein and Weinberg.[11,12]

The hypothesis that the M meson be coupled to a current that is conserved as exactly as possible prompts us to identify (3) as the vertex function for the $M^-\pi^0K^+$ interaction with the π^0 and the K^+ on the mass shell. We then obtain a new "Goldberger-Treiman" relation:

$$\gamma(m_K^2 - m_\pi^2) = fg, \qquad (4)$$

where our definitions of the coupling constants γ, f, and g correspond to the equivalent Lagrangian densities

$$i\gamma M_\mu^-[\pi^0\partial_\mu K^+ - \partial_\mu\pi^0K^+],$$
$$ifM_\mu^-\partial_\mu\kappa^+, \text{ and } g\kappa^-\pi^0K^+. \qquad (5)$$

We now propose, for illustrative purposes, a dynamical model which enables us to calculate the coupling constants f and g. Consider the decay $\kappa^+ \to K^+ + \pi^0$. We assume that this process is dominated by the M meson (which is possible since the divergence of the interacting M_μ field does not vanish[13]). We then obtain, for the $\overline{\kappa}K\pi$ coupling constant, the relation

$$g = \gamma f(p_K+p_\pi)_\mu \left[\delta_{\mu\nu} + \frac{(p_K+p_\pi)_\mu(p_K+p_\pi)_\nu}{m_M^2}\right]\frac{(p_K-p_\pi)_\nu}{(m_\kappa^2-m_M^2)}$$
$$= \gamma f\frac{(m_K^2-m_\pi^2)}{m_M^2}. \qquad (6)$$

Solving for f and g, we have

$$f^2/4\pi = m_M{}^2/4\pi,$$
$$g^2/4\pi = (\gamma^2/4\pi)(m_K{}^2 - m_\pi{}^2)^2/m_M{}^2, \qquad (7)$$

from which we may compute the decay width of the κ meson:

$$\Gamma_{\rm tot}(\kappa \to K + \pi) = 3\Gamma(\kappa^+ \to K^+ + \pi^0) = \frac{3}{2}\frac{g^2}{4\pi}\frac{p_K}{m_\kappa{}^2}. \qquad (8)$$

Similarly, assuming that the M meson dominates the $\kappa\bar{N}\Lambda$ and $\kappa\bar{N}\Sigma$ form factors, we obtain

$$\frac{G_{\kappa\Lambda N}{}^2}{4\pi} = 3\frac{\gamma^2}{4\pi}\left(\frac{m_\Lambda - m_N}{m_M}\right)^2,$$
$$\frac{G_{\kappa\Sigma N}{}^2}{4\pi} = \frac{\gamma^2}{4\pi}\left(\frac{m_\Sigma - m_N}{m_M}\right)^2, \qquad (9)$$

where we have used the octet version of unitary symmetry with F-type couplings of the vector mesons to the baryons.

Note that this model does exhibit explicitly how the couplings of the κ meson to other strongly interacting particles disappear as the mass differences within unitary multiplets go to zero. In other words, the κ meson "exists" as a strongly interacting particle only in so far as unitary symmetry is violated. Note also that our expressions for the coupling constants do not involve $m_K{}^2$. The model is therefore quite consistent with the requirement that the $m_K{}^2 \to 0$ limit be a "gentle" one.

Numerically speaking, $\gamma^2/4\pi$ is about 0.7 for $\Gamma(M \to K + \pi) \approx 40$ MeV. If the observed $T = 1/2$ κ meson [K^* (725)] of the Berkeley 72-in. bubble chamber group[14-16] is to be identified with our κ meson, then the model discussed here gives

$$\Gamma_{\rm tot}(\kappa \to K + \pi) \approx 20 \text{ MeV},$$

which is somewhat larger than the reported κ width of $\Gamma \lesssim 10$ MeV. On the other hand, it is gratifying that the model does give rather small values for the scalar κ-baryon coupling constants:

$$G_{\kappa\Lambda N}{}^2/4\pi \approx 0.08, \quad G_{\kappa\Sigma N}{}^2/4\pi \approx 0.06.$$

This may account for the observed remarkably small production cross sections for κ mesons in both $\pi^- p$ and $K^- p$ collisions.

Within the framework of unitary symmetry, the possible existence of a $Y = \pm 1$, $T = 1/2$ scalar meson K' has been previously discussed by Gell-Mann.[3] The distinctive features of our κ meson are that (i) it has no "unitary partners," and (ii) it ceases to exist in the limit of exact unitary symmetry. In contrast, Gell-Mann's K' and $\bar{K}'$ mesons belong to a $J = 0^+$ unitary octet whose other members are a $T = 1$ π' meson and a $T = 0$ χ' meson. We believe that the existence of the scalar κ ($\bar{\kappa}$) meson without any unitary partner does not constitute evidence against the "not-so-badly-broken eightfold way." provided that its coupling constants go as $\sim m_K{}^2 - m_\pi{}^2$, $m_\Lambda - m_N$, etc. On the contrary, its very existence reveals that nature is making every effort to have the source of the M meson conserved as exactly as possible. As for ρ, φ, and ω, their sources are already conserved, and there is no need to postulate the existence of corresponding scalar mesons.

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MODIFIED PHASE SHIFT ANALYSIS OF PION-NUCLEON SCATTERING DATA*

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A method is presented for analyzing pion-nucleon scattering data using phase shifts for the low partial waves and a closed term to represent the effect of all higher partial waves. This closed term is calculated on the basis of the analytic properties of the scattering amplitude in the complex $\cos\theta$ plane. It represents the contribution to the scattering amplitude, in all but those lowest partial waves which are treated explicitly, of the singularities lying nearest to the physical region. The method is basically similar to that of Cziffra et al.[1] for nucleon-nucleon scattering except that those authors take only the single-pion poles into account. For pion-nucleon scattering it is necessary to include not only the contributions from the nucleon pole, the 3-3 resonance, and the ρ meson, but also from the two-pion continuum in the $T=0$ state.

In this Letter we apply the method to the π^+-p angular distribution and polarization data at 310 MeV of Foote et al.[2] The conventional phase shift analysis of these data gave ambiguous results when f-wave phase shifts were included. With the present method we are able to remove this difficulty.

Instead of attempting a conventional phase shift analysis, we write the scattering amplitude as a sum of two terms. The first term incorporates in closed form the known contributions from the singularities lying closest to the physical region. The second comes from the more distant singularities and is approximated by a finite sum of Legendre polynomials. Since the second term does not contain the nearest singularities, a smaller number of terms should be needed than in a normal partial-wave expansion.

The singularities of the invariant amplitudes for pion-nucleon scattering, in the $\cos\theta$ plane, at a fixed incident pion laboratory kinetic energy of 310 MeV, are shown in Fig. 1. The discontinuities across the cuts are determined by the absorptive parts of amplitudes for related processes. The distinction between nearby and more distant singularities is not a sharp one. We define "nearby" as on the interval $-z_1 < \cos\theta < z_2$. The choice of z_1 and z_2 is somewhat arbitrary, but the interval should presumably contain the known dominant features of the crossed channels ($z_1 = 8$, $z_2 = 5$ is a possible choice). We assume that the ef-

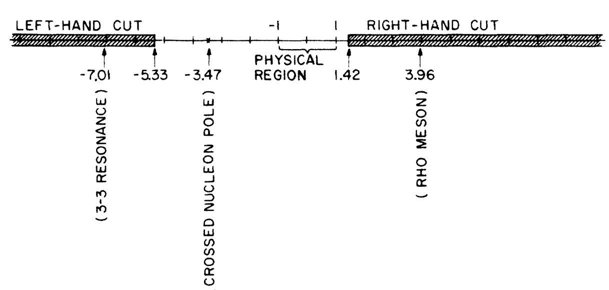

FIG. 1. Singularities of the invariant amplitudes for pion-nucleon scattering in the complex $\cos\theta$ plane, at a fixed incident pion lab kinetic energy of 310 MeV.