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NUCLEON AXIAL-VECTOR FORM FACTOR

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High-energy neutrino experiments at CERN and Brookhaven will soon be able to give some information on the nucleon axial-vector form factor, through separate measurements of $\nu n \to pl$ and $\bar{\nu}p \to n\bar{l}$ cross sections.[1] We report here a dispersion-theoretical estimate of the rate of decrease of this axial-vector form factor F with increasing momentum transfer. F is defined by

$$(E_N E_{\bar{N}}/M^2)^{1/2}\langle 0|A_\mu|N\bar{N}\rangle = \bar{v}_{\bar{N}}\{\gamma_5\gamma_\mu F(s) - 2i\gamma_5 K_\mu G(s)\}u_N, \qquad (1)$$

where A_μ = weak axial-vector current, $K_\mu = \frac{1}{2}(P_{\bar{N}} + P_N)_\mu$, M = nucleon mass (with pion mass μ taken to be unity), and $s = 4K^2$. Here and in the following, the trivial isotopic spin dependences will be suppressed.

Among the intermediate states that can contribute to the imaginary part of F, we will keep only the one with the lightest mass, i.e., the 3π state, and will further assume that this is approximated by the $\pi\rho$ state. We will use the generalized unitarity and N/D formulation appropriate for 3-particle intermediate states only as a guide,[2] but will treat the ρ as if it were stable as much as possible. Under such an assumption one needs the matrix elements $\langle\pi\rho|N\bar{N}\rangle$ and $\langle 0|A_\mu|\pi\rho\rangle$.

(a) The $\langle\pi\rho|N\bar{N}\rangle$ amplitude. – For the discontinuities across the dynamical cuts of this amplitude we will use Born approximation with one nucleon exchange. This is a strong assumption,[3] with which, however, the problem simplifies considerably, because it contributes to only the following three invariant amplitudes:

$$2(E_N E_{\bar{N}} E_\pi E_\rho/M^2)^{1/2}\langle\pi\rho|N\bar{N}\rangle = \bar{v}_{\bar{N}}\{i\gamma_5\eta^*\cdot K\alpha(s,\cos\theta) + i\gamma_5\eta^*\cdot P\beta(s,\cos\theta) + i\gamma_5\sigma_{\mu\nu}Q_\mu\eta_\nu^*\lambda(s,\cos\theta)\}u_N, \qquad (2)$$

where η = polarization vector of ρ, $Q_\mu = \frac{1}{2}(P_\rho - P_\pi)_\mu$, $P_\mu = \frac{1}{2}(P_N - P_{\bar{N}})_\mu$, and θ = c.m. scattering angle. From invariance considerations, the matrix element $\langle 0|A_\mu|\pi\rho\rangle$ is of the form

$$2(E_\pi E_\rho)^{1/2}\langle 0|A_\mu|\pi\rho\rangle = \eta\cdot KQ_\mu A(s) + \eta_\mu B(s) + \eta\cdot KK_\mu C(s). \qquad (3)$$

The form factor C may be relatively large because it receives contribution via a one-pion state. But it is clear that C cannot contribute to any $J = 1$ state. From Eqs. (1), (2), and (3),

$$\mathrm{Im}F(s) = (q/2\sqrt{s})\lambda^J(s)B(s) \quad (s > 42), \qquad (4)$$

where

$$\lambda^J(s) \equiv \int_{-1}^{1}\lambda(s,\cos\theta)\frac{Mq}{p}\cos\theta\frac{d\cos\theta}{8\pi},$$

and q and p = magnitudes of ρ and N c.m. momenta, respectively. We now write a set of N/D equations for $\lambda^J(s)$. We will denote by the subscript 1 the $N\bar{N}$ channel with $J = 1$, $\tau = 1$, and parity +; and by subscripts 2 and 3 the two mutually orthogonal $\pi\rho$ states with these same quantum numbers, such that $\lambda^J(s)$ corresponds to the reaction $1 \to 2$. Thus

$$\lambda^J(s) = \sum_{j=1}^{3} N(s)_{1j}D^{-1}(s)_{j2}, \qquad (5)$$

$$N(s)_{ij} = \frac{1}{\pi}\sum_K\int_L \frac{[\mathrm{disc}ND^{-1}(s')]_{ik}D(s')_{kj}}{s'-s}ds', \qquad (6)$$

$$D(s)_{ij} = \delta_{ij} + \frac{1}{\pi}\int_R \frac{q'^3 N(s')_{ij}}{2\sqrt{s'}(s'-s)}ds', \qquad (7)$$

where L denotes the dynamical cuts and R the physical cuts. In Eq. (7), a subtraction may be required. We will solve these equations by the first iterations in a determinantal approximation, replacing the discontinuities across the dynamical cuts by the Born-approximation contributions.

Consider first $N(s)_{22}$, corresponding to $\pi\rho$ scattering. The Born approximation with one-pion exchange is of the form

$$4(E_\pi E_\rho E_{\pi'} E_{\rho'})^{1/2}\langle\pi\rho|\pi'\rho'\rangle = M(s,\cos\theta)q\cdot\eta q'\cdot\eta^*. \quad (8)$$

The relevant amplitude is given by

$$M(s)_{22} \equiv -\int_{-1}^{1} M(s,\cos\theta)\frac{\sin^2\theta\cos\theta}{16\pi}d(\cos\theta). \quad (9)$$

We make one subtraction in $D(s)_{22}$ at $s=42$, then in the first iteration,

$$N(s)_{22} = -\gamma_{\rho\pi\pi}{}^2 D(42)_{22}\left\{\frac{1}{\pi}\int_{L_1}\frac{F(s')ds'}{s'-s} + \frac{1}{\pi}\int_{L_2}\frac{F(s')ds'}{s'-s}\right\}, \quad (10)$$

where

$$F(s) = \frac{1}{32q^2}\sin^2\theta_0\cos\theta_0;$$

$$\cos\theta_0 = \frac{1}{q^2}(2m_\rho{}^2 + 1 - s + 2q^2). \quad (11)$$

The positions of the cuts are shown in Fig. 1. Substituting this solution into Eq. (7) for $D(s)_{22}$, one finds the integral term to be much smaller than the constant term, and $D(s)_{22}$ is very slowly varying over a wide range of s. Similarly one finds the off-diagonal term $D(s)_{23}$ to be typically two orders of magnitude smaller than $D(s)_{22}$. Furthermore, it turns out that the Born amplitudes in Eq. (2) do not contribute to the reaction $1\to3$. Thus the influence of channel 3 on the reaction $1\to2$ is negligible. If one consistently neglects also the $N\bar{N}$ state in summing over intermediate states in view of its higher mass, one can approximate Eqs. (5) and (6) by

$$\lambda^J(s) = N(s)_{12}/D(s)_{22}, \quad (12)$$

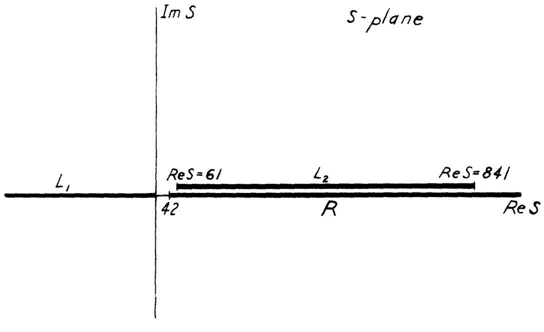

FIG. 1. The one-pion-exchange and elastic cuts for $M(s)_{22}$.

$$N(s)_{12} = \frac{1}{\pi}\int_L \frac{[\mathrm{disc}\lambda_{\mathrm{B.A.}}{}^J(s')]D(s')_{22}}{s'-s}ds'. \quad (13)$$

Since we have found $D(s)_{22}$ to be very slowly varying, an approximate solution is obtained in the form

$$\lambda^J(s) = \lambda_{\mathrm{B.A.}}{}^J(s)\frac{D(s_{\mathrm{av}})_{22}}{D(s)_{22}}. \quad (14)$$

We will take s_{av} to be around zero.

(b) For $B(s)$ we will write the dispersion relation

$$B(s) = \frac{1}{\pi}\int_{42}^{\infty}\frac{\mathrm{Im}\,B(s')}{s'-s}ds' \quad (15)$$

(with possibly a subtraction).

From Eq. (4) one sees that $B(s)$ should have the same phase as $\lambda^{J*}(s)$ over the physical cut in our approximation. From Eq. (14), since $\lambda_{\mathrm{B.A.}}{}^J$ is real, $B(s)$ should have the same phase as $D(s)_{22}$. Thus a solution to Eq. (15) is

$$B(s) = B(0)D^*(0)_{22}/D^*(s)_{22}. \quad (16)$$

We have taken a possible multiplicative polynomial to be unity, because $D(s)_{22}$ increases only logarithmically as $s\to\infty$, so that any other choice would lead to $B(s)\to\infty$ as $s\to\infty$. So finally one has

$$\mathrm{Im}\,F(s) = (q/2\sqrt{s})B(0)\lambda_{\mathrm{B.A.}}{}^J(s) \times |D(0)_{22}|^2/|D(s)_{22}|^2. \quad (17)$$

Since $\lambda_{\mathrm{B.A.}}{}^J(s)\to s^{-1}\ln s$ as $s\to\infty$, the integral of $\mathrm{Im}\,F(s)$ over the physical cut is actually convergent in our approximation. We will therefore try an unsubtracted dispersion relation for $F(s)$:

$$F(s) = \frac{1}{\pi}\int_{42}^{\infty}\frac{\mathrm{Im}\,F(s')}{s'-s}ds'. \quad (18)$$

The requirement $F(0)=g_A$ fixes the unknown parameters in Eq. (17), and the resulting behavior of $F(s)$ is shown in Fig. 2. If one approximates the curve by a formula of the type $F(s)=(1-b^2s/12)^{-2}$ for $|s|<150$, one finds $b\approx0.22\times10^{-13}$ cm.

(c) In conclusion, we make two remarks. If an unsubtracted dispersion relation for $F(s)$ should prove to be invalid and a once-subtracted dispersion relation must be used, the constant $B(0)$ must be determined by other means (such as the 3π decay of the W boson). As determined from the unsubtracted dispersion relation, $\mu B(0)$ turns out to be $\approx0.7g_AM^2$, which should perhaps be con-

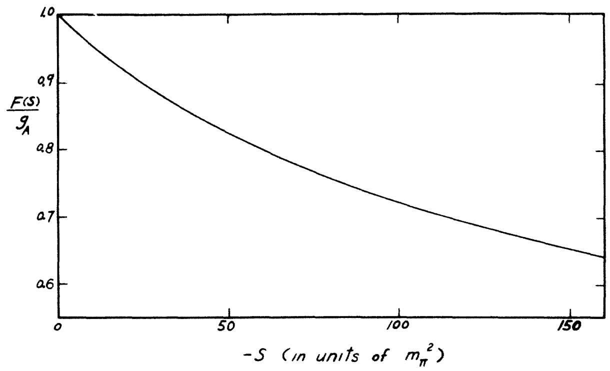

FIG. 2. The axial-vector form factor $F(s)/g_A$ as a function of the square of momentum transfer $-s$.

sidered as an overestimate in view of the neglect of contributions to g_A other than the $\pi\rho$ state.[4] Another remark is that the short cut above the real axis in Fig. 1 gives rise to a marked resonance behavior in the $\pi\rho$ scattering amplitude $M(s)_{22}$ around $s=65$. There have been considerable interest and discussion as to whether such a resonance in unstable particle scattering can manifest itself in other physical processes by coupling through unitarity.[5] So far the attention is mostly on its effect on production amplitudes; e.g., whether the resonance in πN^* scattering can enhance $\pi N \to \pi N^*$. We have here another place where a similar effect can be looked for, where the physical manifestation, if any, is through the behavior of the form factor. This is because if the resonance in $\pi\rho$ scattering should enhance the $N\bar{N}\to\pi\rho$ amplitude $\lambda^J(s)$ in the low-energy region, by Eq. (17) $\mathrm{Im}F(s)$ would also become concentrated in the low-energy region, leading to a faster drop of $F(s)$ with increasing $-s$ than we have found. In our N/D solution no such enhancement in $\lambda^J(s)$ was found because $1/D(s)_{22}$ does not show any marked resonance around $s=65$, in contrast to the full amplitude $M(s)_{22}$.

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[2]There now exists an extensive literature on the treatment of 3-particle intermediate states, particularly when two of them resonate. See R. Blankenbecler, Phys. Rev. 122, 983 (1960); S. Mandelstam et al., Ann. Phys. (N.Y.) 18, 198 (1962); J. S. Ball, W. R. Frazer, and M. Nauenberg, Phys. Rev. 128, 478 (1962); P. G. Federbush et al., Ann. Phys. (N.Y.) 18, 23 (1962); L. F. Cook and B. W. Lee, Phys. Rev. 127, 283 (1962); D. R. Harrington, Phys. Rev. 127, 2235 (1962); R. C. Hwa, Phys. Rev. 130, 2580 (1963).

[3]However, it can be shown that the Born term with N^* exchange does not contribute to the amplitude $\lambda^J(s)$ defined below.

[4]Of course, to determine $B(0)$ this way one must know the value of $\gamma_{\rho N\bar{N}}^2/4\pi$, which we have taken to be ≈ 2.

[5]R. F. Peierls, Phys. Rev. Letters 6, 641 (1961); and references quoted in reference 2.

ANALYSIS OF H^3 AND He^3 FORM FACTORS AND THE DETERMINATION OF THE CHARGE FORM FACTOR OF THE NEUTRON*

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Experiments on the elastic scattering of high-energy electrons from H^3 and He^3 have recently been carried out[1,2] at Stanford University on targets specially prepared by the Los Alamos Laboratory. These results were interpreted in a straightforward manner and provided distributions of charge densities, magnetic moment densities, and sizes of the two mirror nuclei.

It is also possible to make a much more detailed analysis of the appropriate form factors that is able to describe some of the internal dynamics of the nucleons in the ground states of H^3 and He^3. Moreover, by methods we shall describe, the charge form factor of the neutron can be found from the above-quoted experiments. The charge form factor of the neutron has heretofore been an